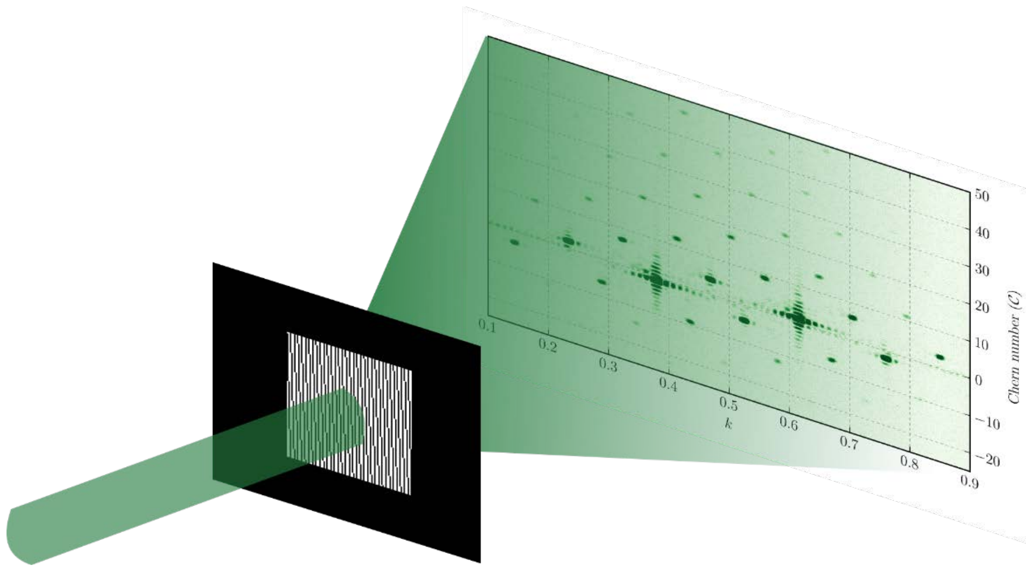
Laboratoire Kastler Brossel
Physique quantique et applications

Direct measurement of Chern numbers in the diffraction pattern of a Fibonacci chain



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COLLÈGE DE FRANCE 1530 ENS cnrs UPMC SORBONNE UNIVERSITÉS IFRAF erc nano-K DES ATOMES FROIDS AUX NANOSCIENCES

Introduction

Quasicrystals are materials which are non-periodic but present long-range order

Shechtman *et al.*, PRL (1984)
Levine & Steinhardt, PRB (1986)

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- quenched Al alloys
- metal/semiconductor epitaxy
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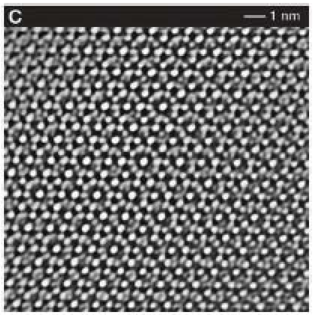
Natural (khatyrkite) ⟶



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Experiments with photonic, phononic quasicrystals, optical elements,….

Wide range of applications (mechanical properties, optical gratings, …)

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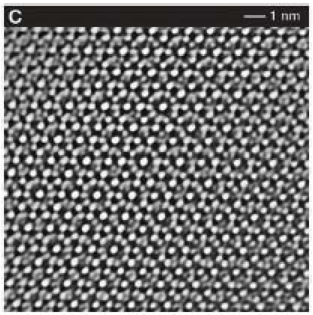

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Experiments with photonic, phononic quasicrystals, optical elements,....

Wide range of applications (mechanical properties, optical gratings, ...)

Rich physical/mathematical properties : fractal structure, topological features, ...

Outline

Fibonacci chain

Phason degree of freedom

Experimental results

Fibonacci chain

Why the Fibonacci sequence ?

basic example of 1D quasicrystal

Fibonacci numbers are defined by :

$$F_n = F_{n-1} + F_{n-2} \text{ (with } F_0 = 1,\ F_1 = 1)$$

Fibonacci chain

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Finite length Fibonacci sequence is defined by :

$$S_n = S_{n-1} S_{n-2} \text{ (with } S_0 = B,\ S_1 = A)$$

Example :

$S_7 =$ ABAABABAABAABABAABABA 21 letters

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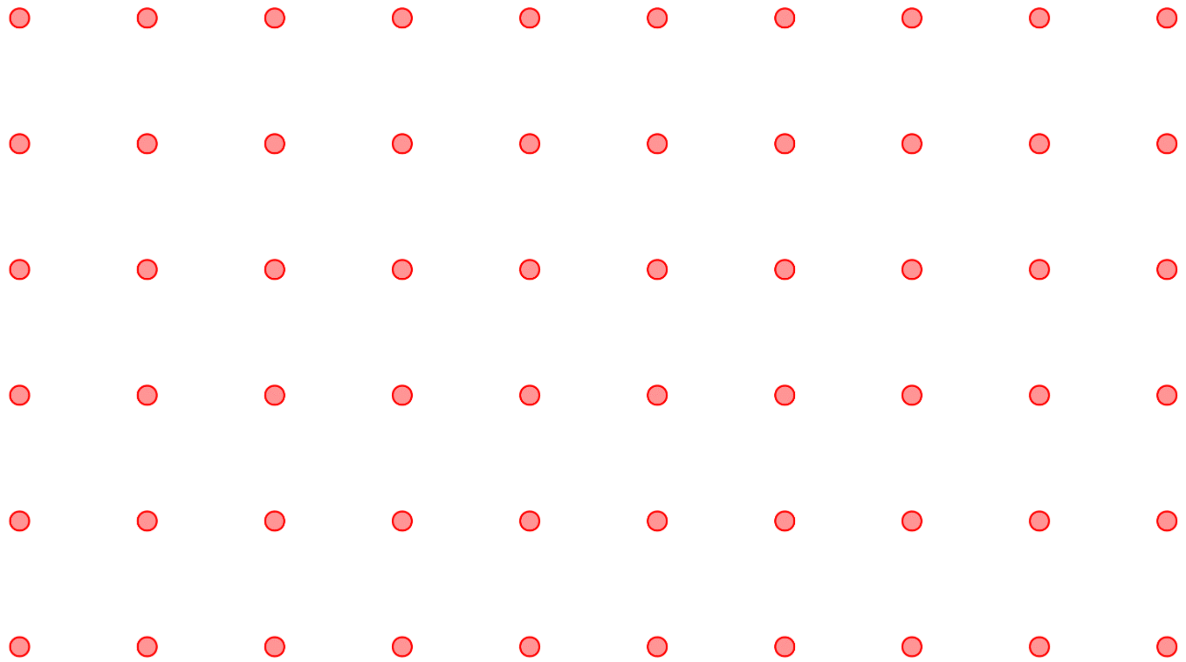
Golden mean :

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \frac{1+\sqrt{5}}{2} \equiv \tau \approx 1.618$$

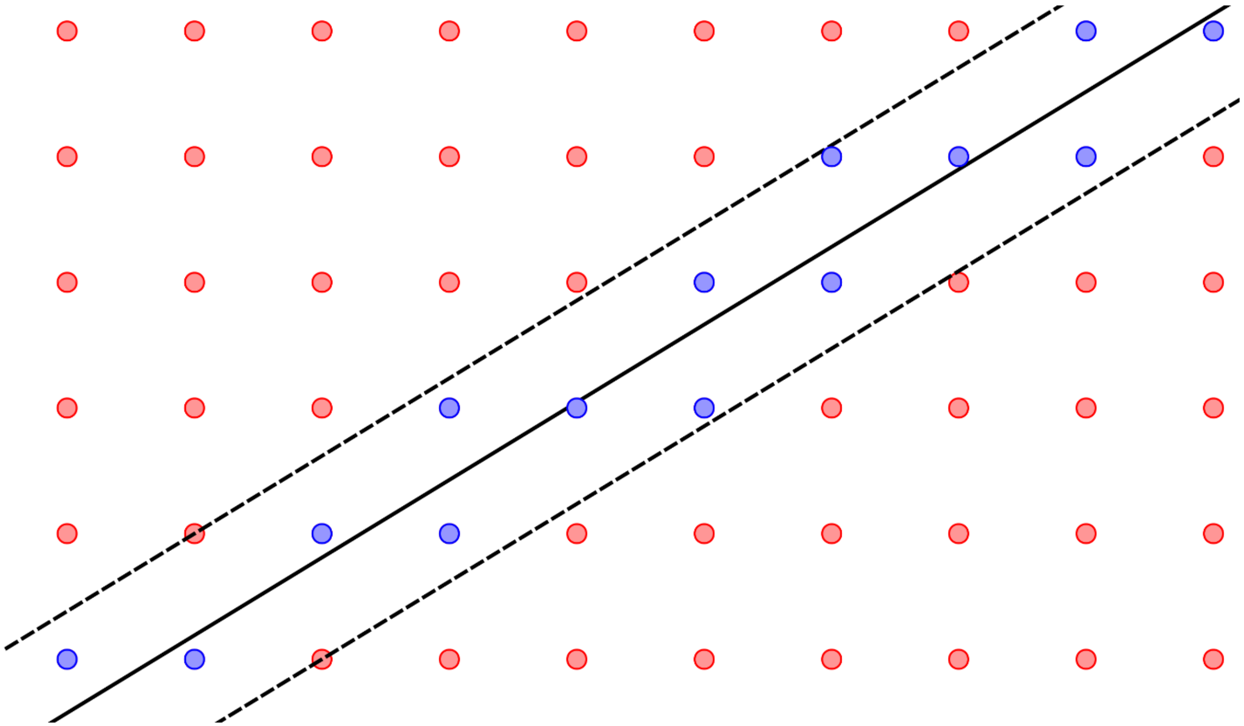
Cut and project

Starting point : square 2D lattice



Cut and project

Cut



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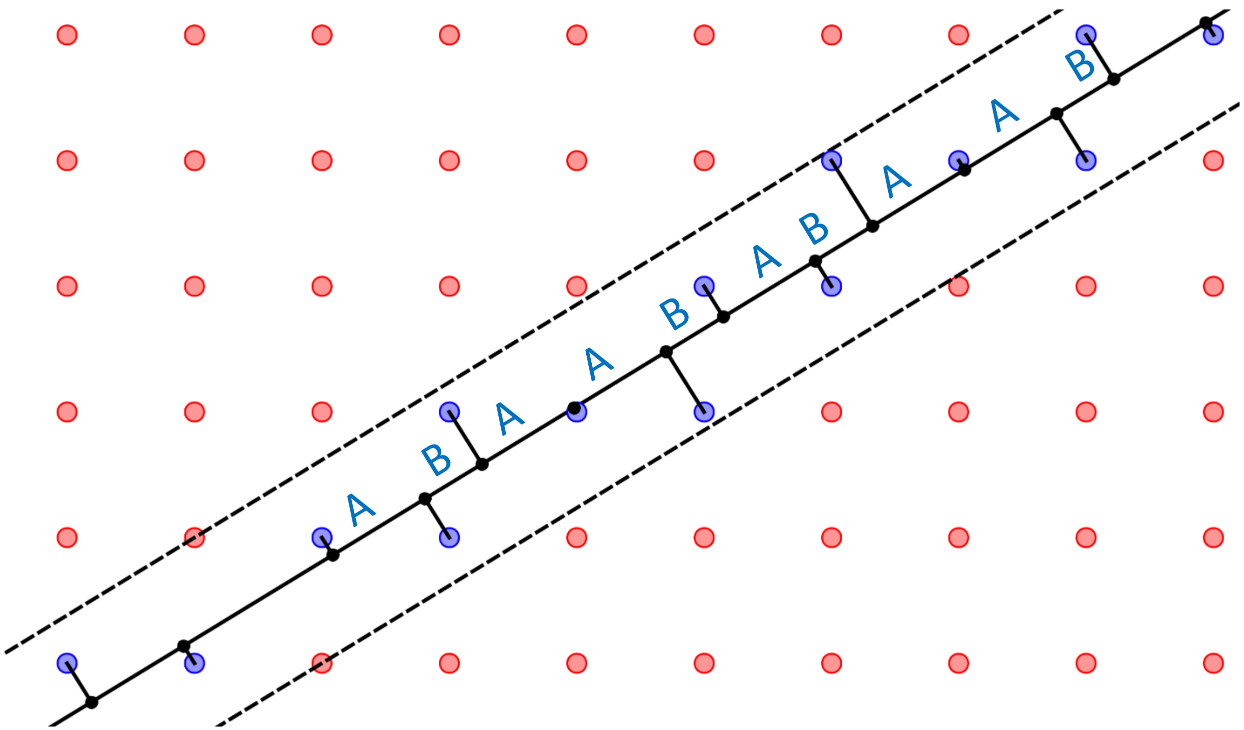

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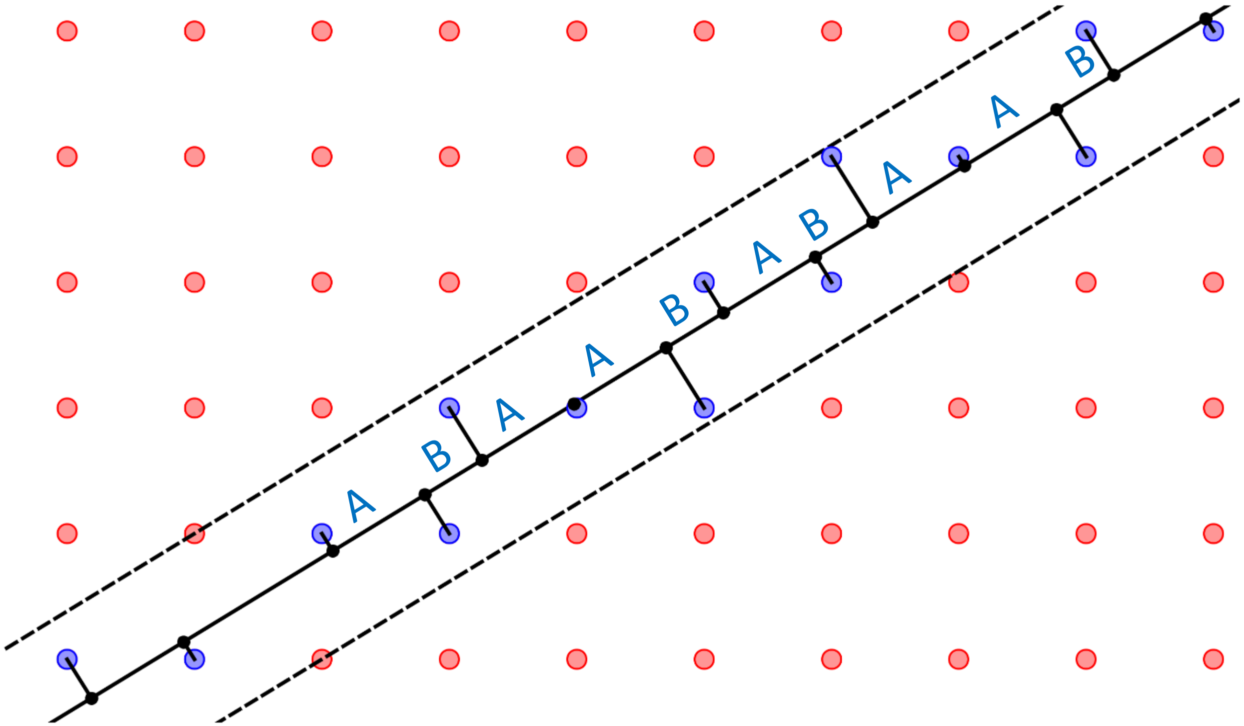
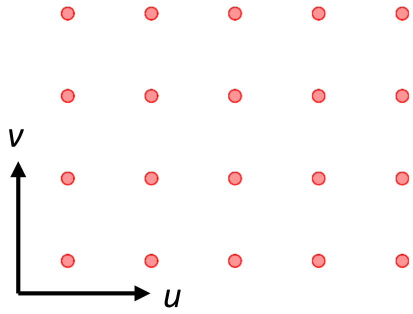
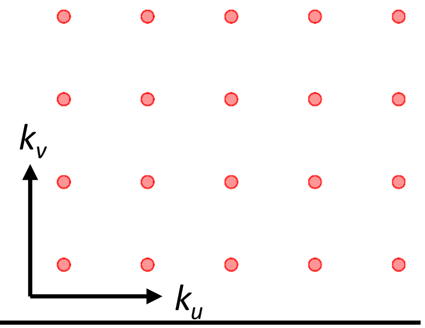


For any rational slope a 1D periodic structure is created

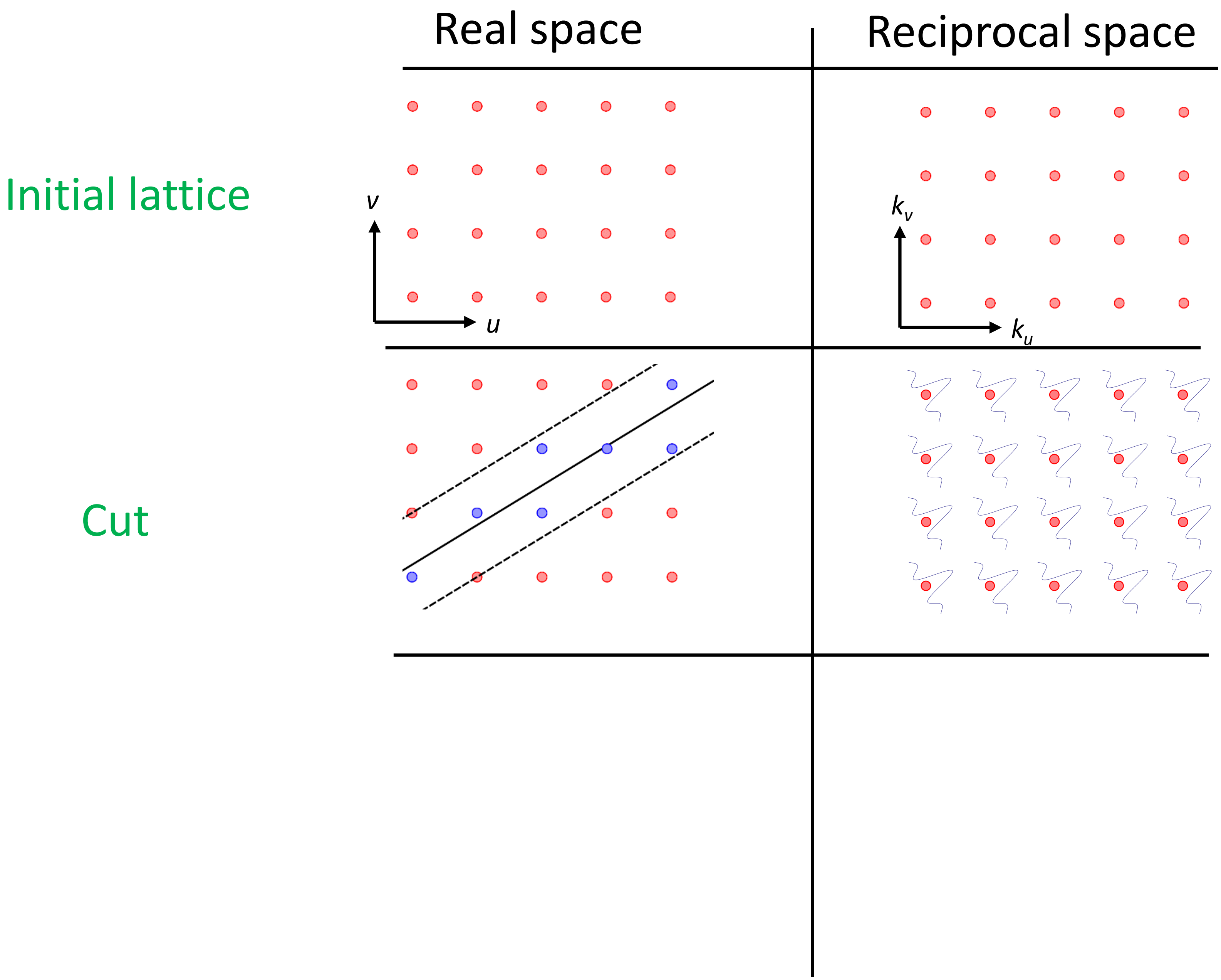
For an irrational slope, one gets a quasi-periodic structure

Fibonacci sequence is obtained by choosing a slope of $1/\tau$

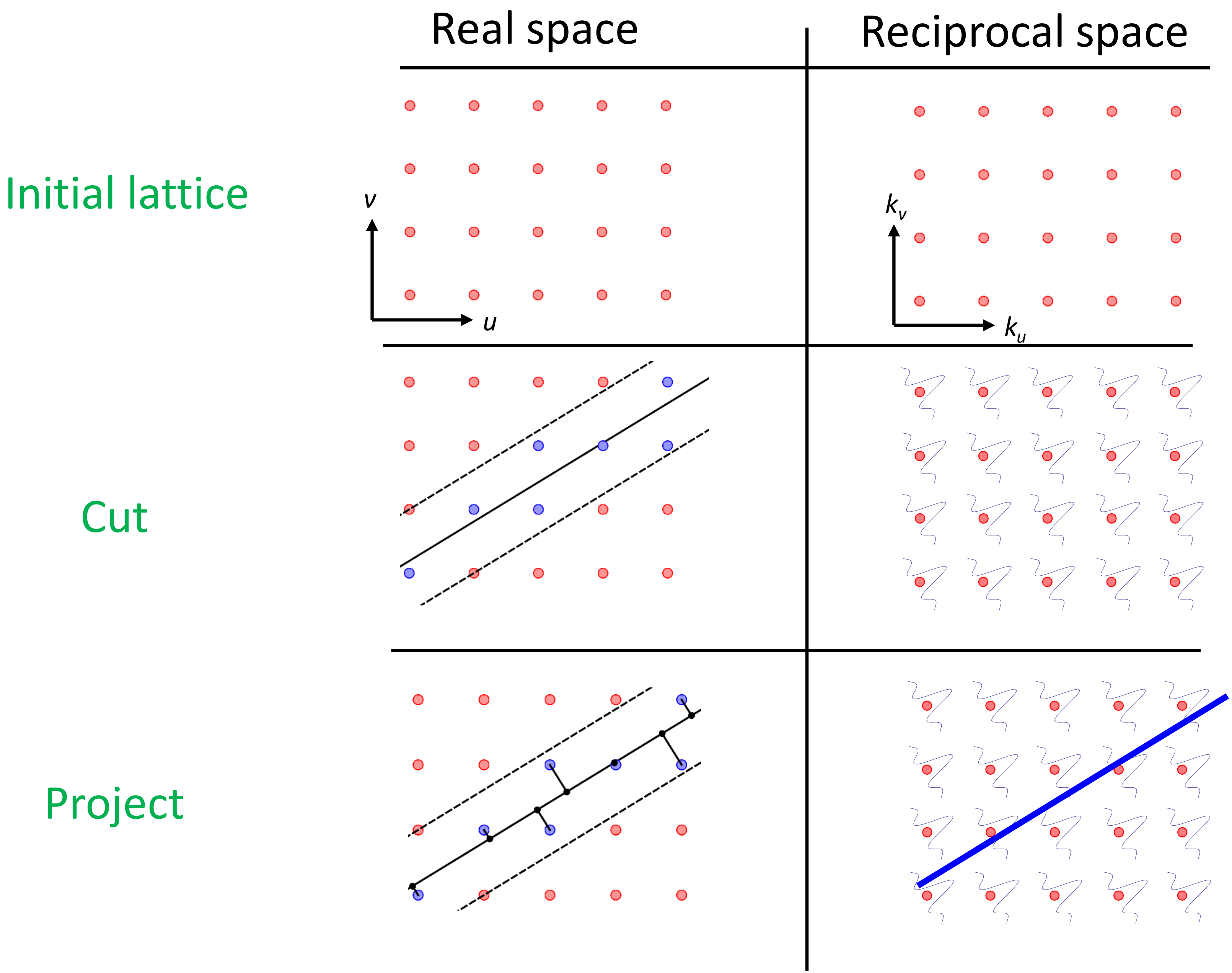
Diffraction pattern

	Real space	Reciprocal space
Initial lattice		

Diffraction pattern



Diffraction pattern



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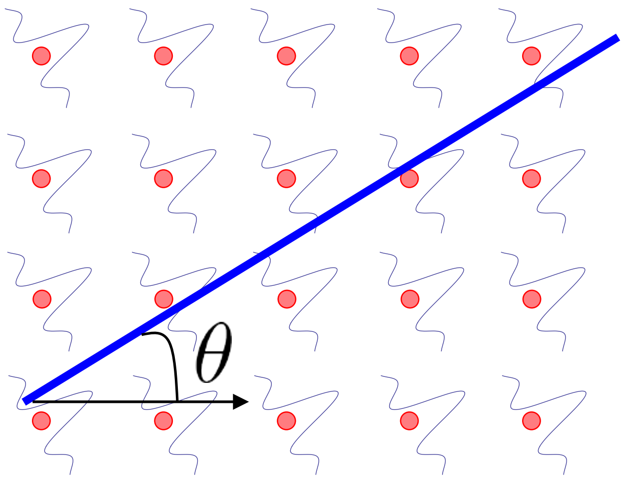


$$\tan(\theta) = \frac{1}{\tau}$$

<u>Deduce the diffraction pattern</u> :

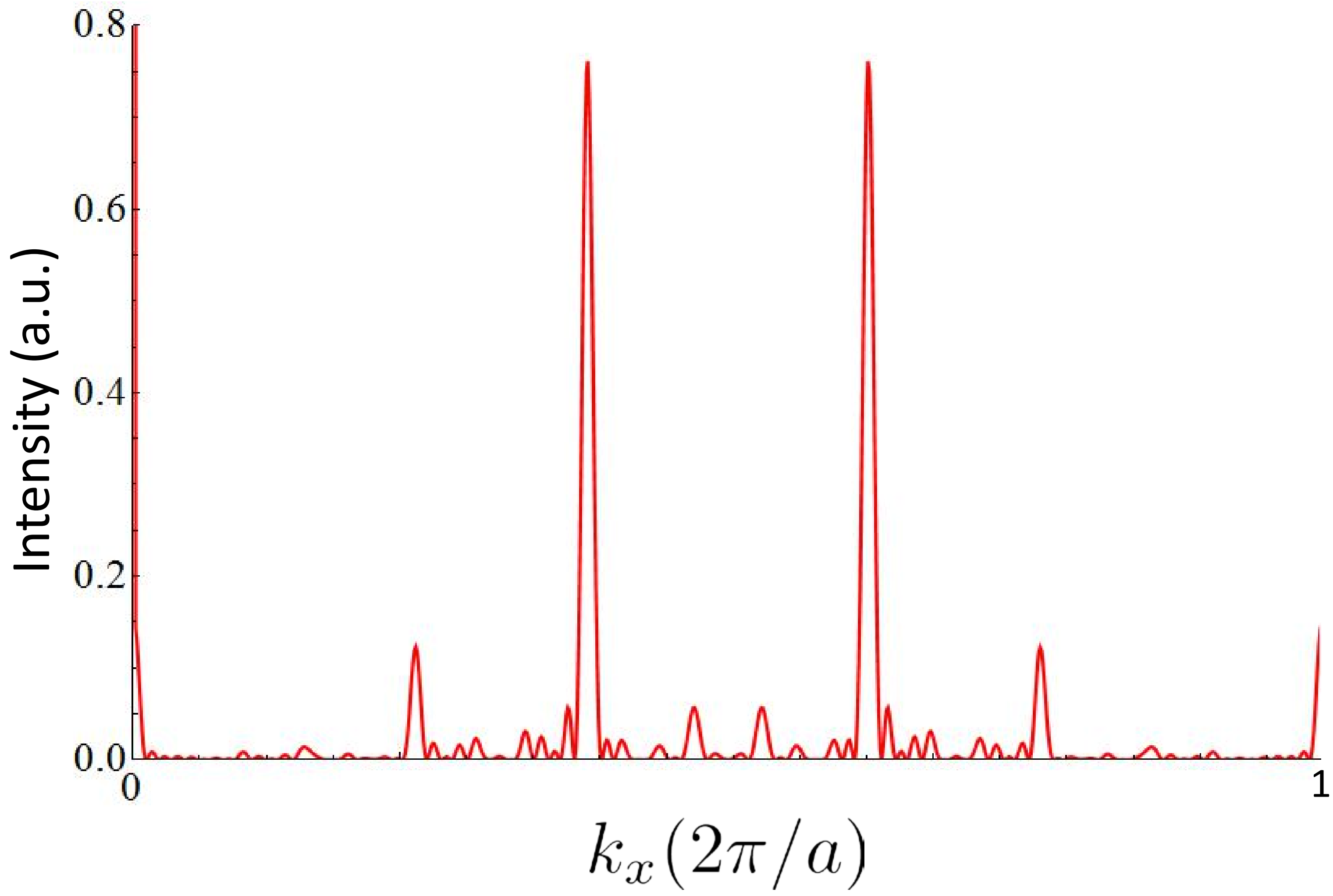
Peak positions given by two integers :

$$k_x(p,q) \propto p + \frac{q}{\tau}$$

Peaks amplitude given by their distance from the cutting line

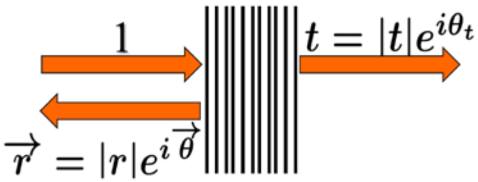
Diffraction pattern

$$k_x(p,q) \propto p + \frac{q}{\tau}$$

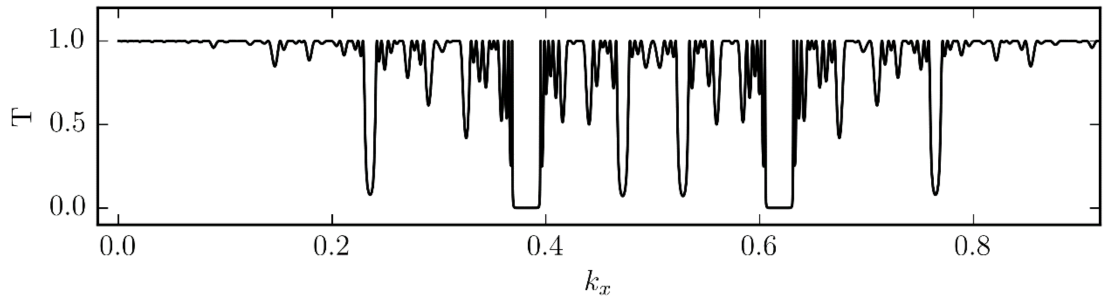


Link with scattering physics

(Levy et al. arxiv 1509.04028)



Density of states shows a series of gaps



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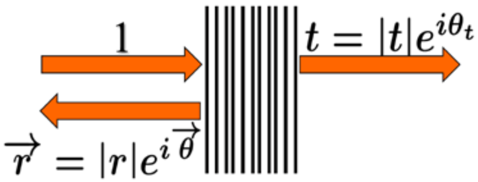

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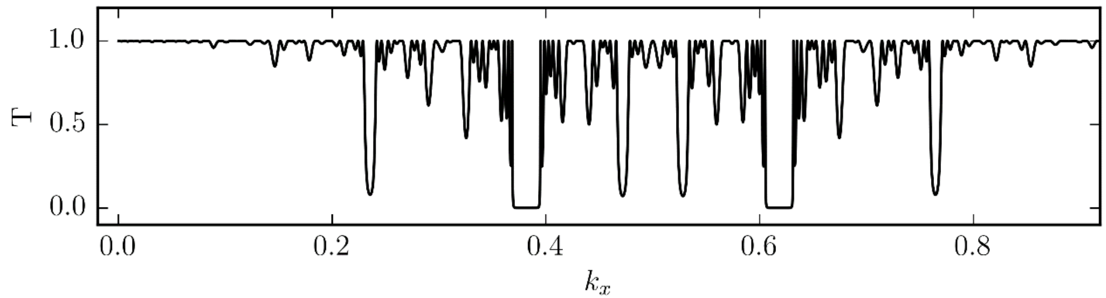

Gap Labelling Theorem Belissard (1982)

$$IDOS(k_x(\text{gap})) = p + \frac{q}{\tau}$$

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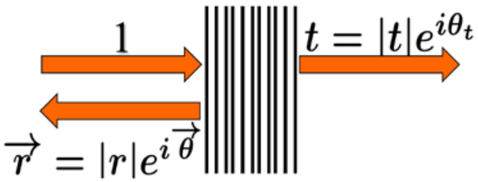

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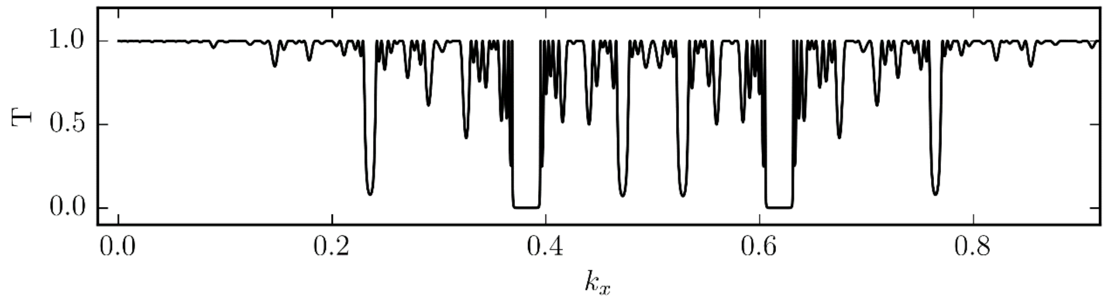

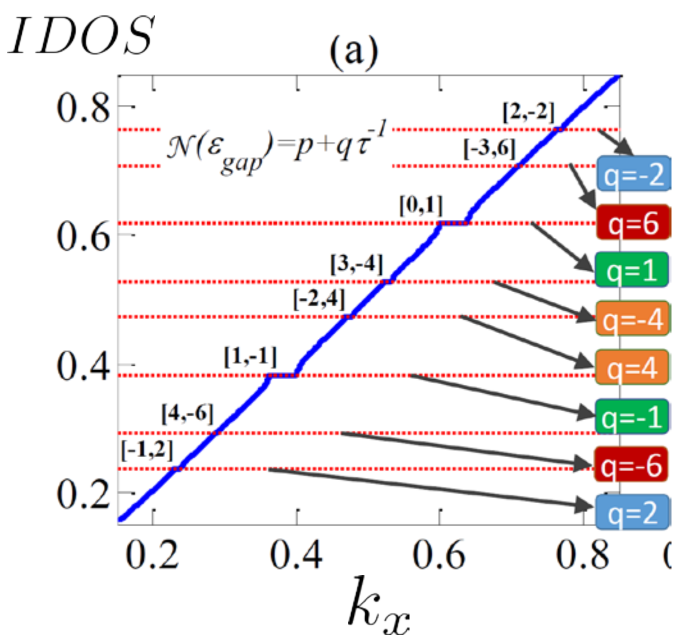

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Gaps open at the position of the diffraction peaks

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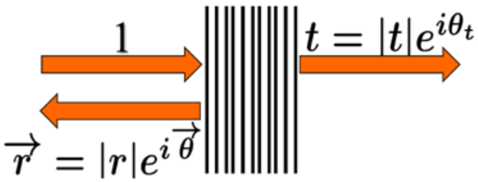

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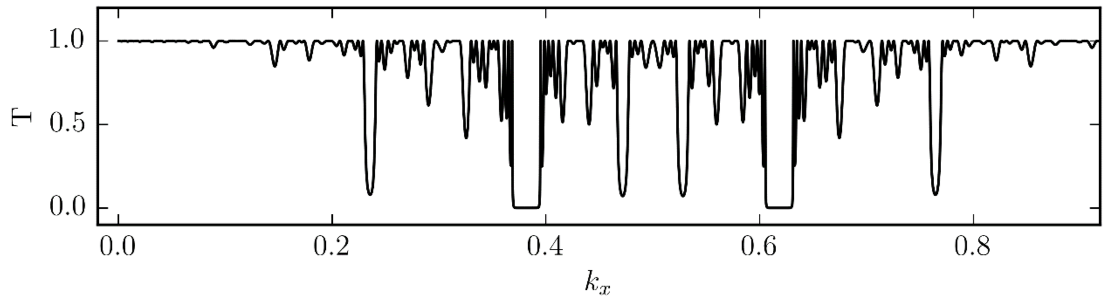

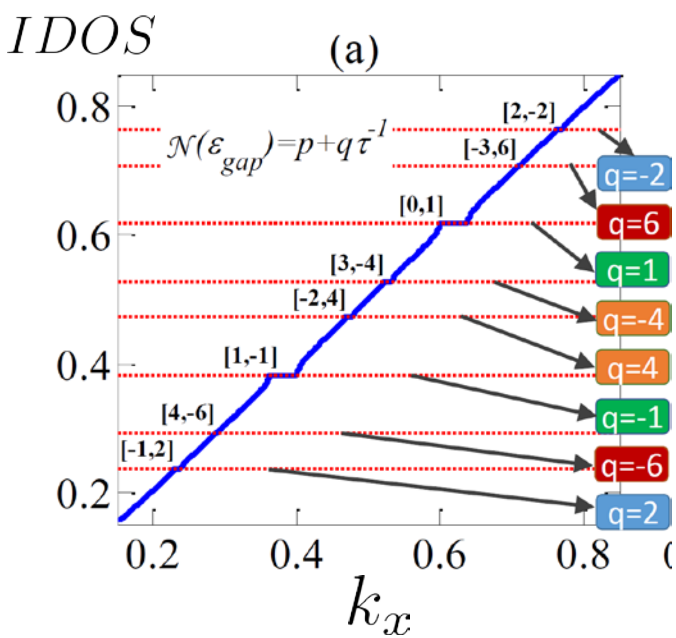

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q is a Chern number

Gaps open at the position of the diffraction peaks

Outline

Fibonacci chain

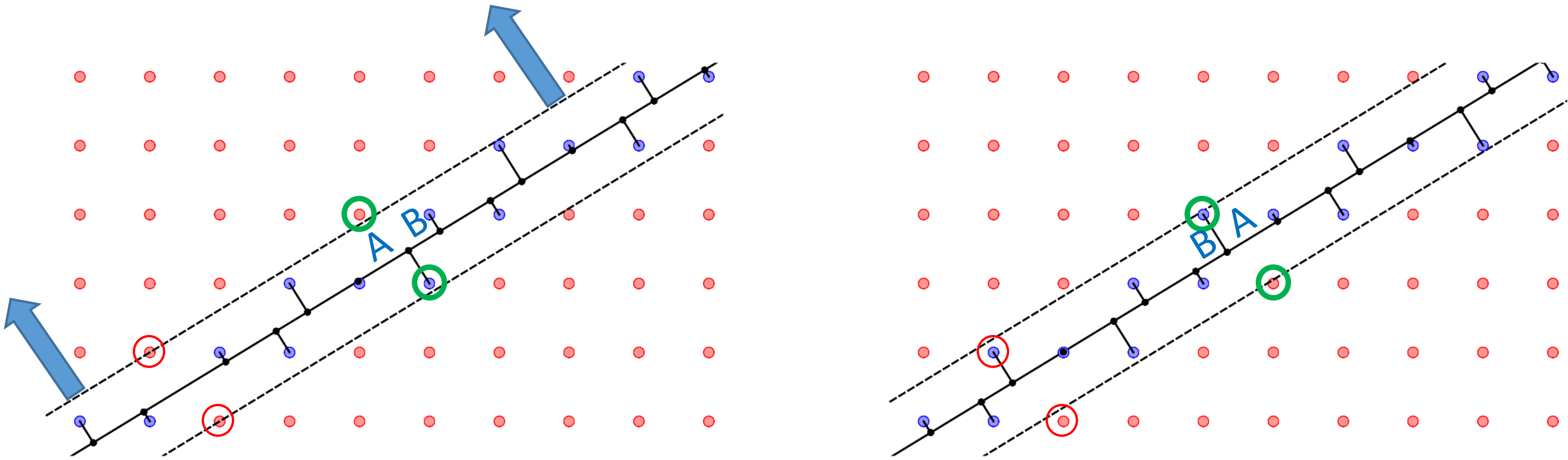
Phason degree of freedom

Experimental results

Phason degree of freedom

We consider finite-size chains of length F_n

Offset of the cut can be tuned



We associate a phase Φ with this translation with $\Phi \in [0, 2\pi]$

Scanning Φ generates F_n new chains

Phason degree of freedom

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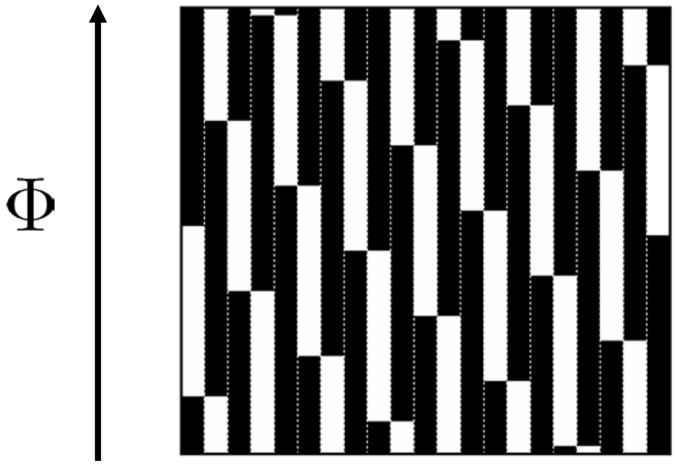

One change at a time : BAABAB $\longleftrightarrow$ BABAAB

The generated chains are different segments of the infinite chain

Example : ABAABABAABAABABAABABA.....

$\Phi = 0 \times \frac{2\pi}{F_n}$ ABAABABA Initial Fibonacci chain $F_n = 8$ $F_{n-1} = 5$

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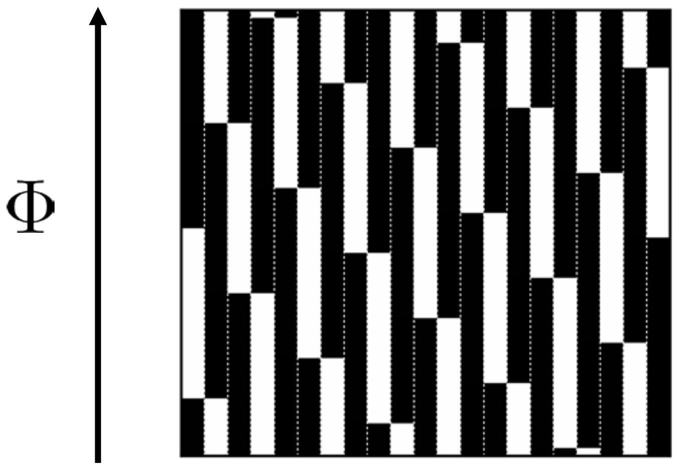

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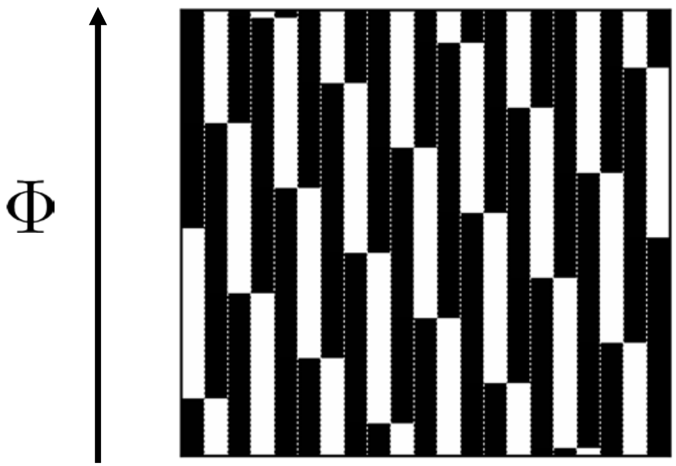

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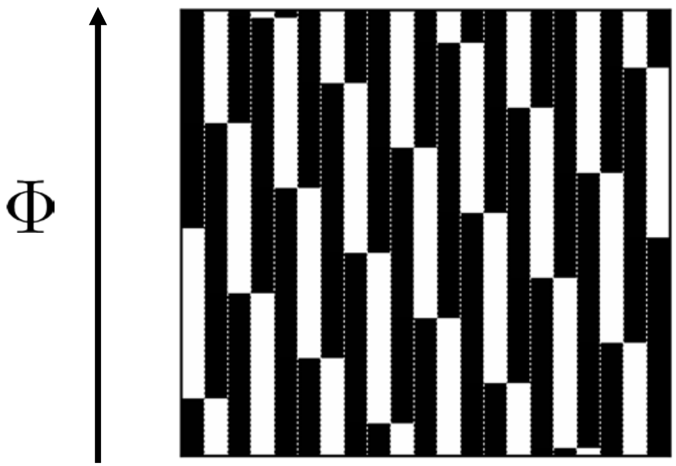

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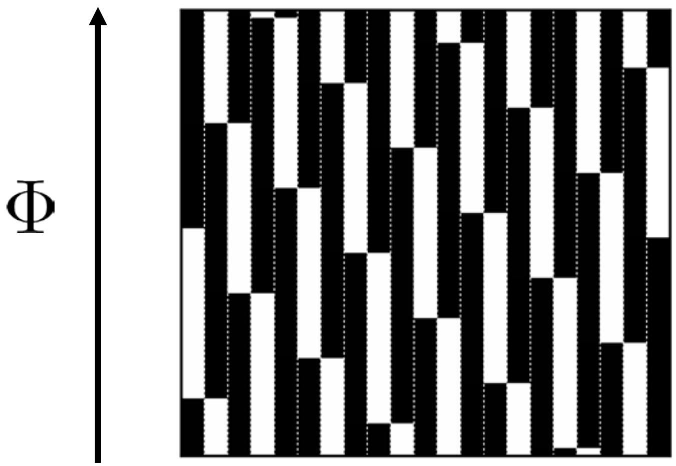

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Spatial shift is :

$$\Delta X = \{(-1)^n F_{n-1} + M F_n\} \times \frac{\Phi F_n}{2\pi}$$

$$M \in \mathbb{Z}$$

Phason degree of freedom

Scanning Φ generates F_n new chains which can be found on the Infinite chain by a shift :

$$\Delta X = \{(-1)^n F_{n-1} + M F_n\} \times \frac{\Phi F_n}{2\pi}$$

A spatial shift in real space corresponds to a phase shift in reciprocal space. Effect of the phason will appear in the phase of the diffracted field.

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For a diffraction peak at $k_x(p,q) \propto p + q\dfrac{F_{n-1}}{F_n}$,

We can show that $k_x(p,q)\Delta X = -q\Phi\ [2\pi]$

Chern numbers are encoded in the phase of diffracted field.

Link with 2D Harper model

We can find a characteristic function to define the Fibonacci sequence :

$$S_n = [\chi_1 \chi_2 ... \chi_{F_n}] \quad \text{with} \quad \chi_m = \text{sign}\left(\cos(2\pi m \tau^{-1} + \phi) - \cos(\pi \tau^{-1})\right)$$

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$$\frac{\tanh[\beta(\cos(2\pi b n + c) - \cos \pi b))]}{\tanh \beta}$$

Kraus et al. (2012)

Fibonacci : $b = \tau^{-1} \quad \text{et} \quad \beta \to \infty$

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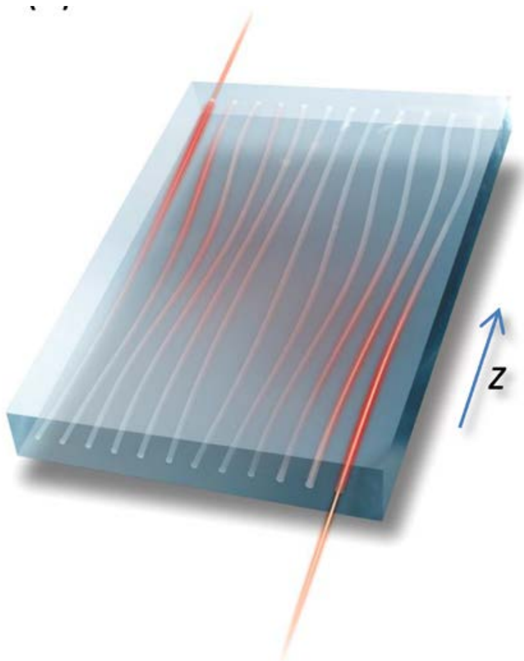
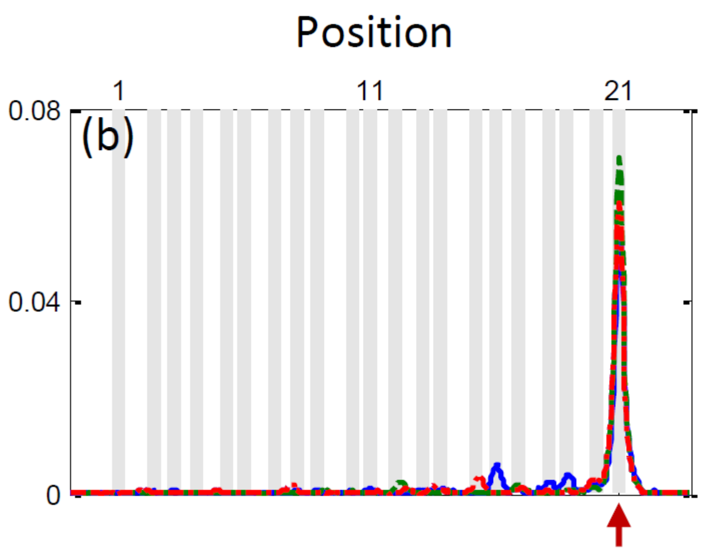
Both models can be obtained from a 2D ancestor Harper Hamiltonian :

With a flux $\alpha \equiv \tau^{-1}$ and $k_y \equiv \Phi$

Photonics experiment





Phason is scanned by changing longitudinally the coupling between guides.

Edge states propagation

Kraus et al. PRL **109** 106402 (2012)
Kraus et al. PRL **109** 116404 (2012)
Verbin et al. PRB **91** 064201 (2015)

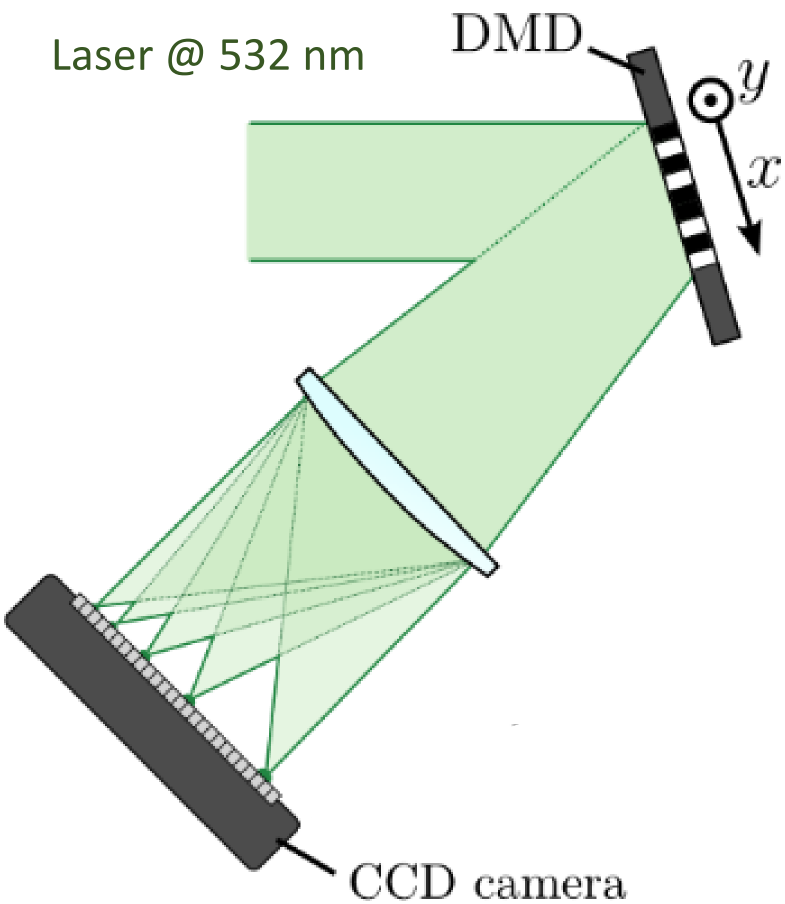
Outline

Fibonacci chain

Phason degree of freedom

Experimental results

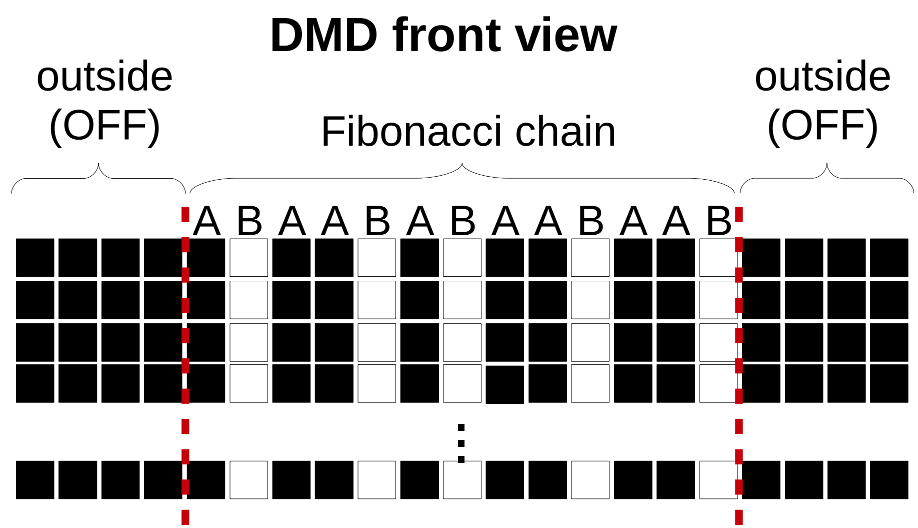
Optical setup



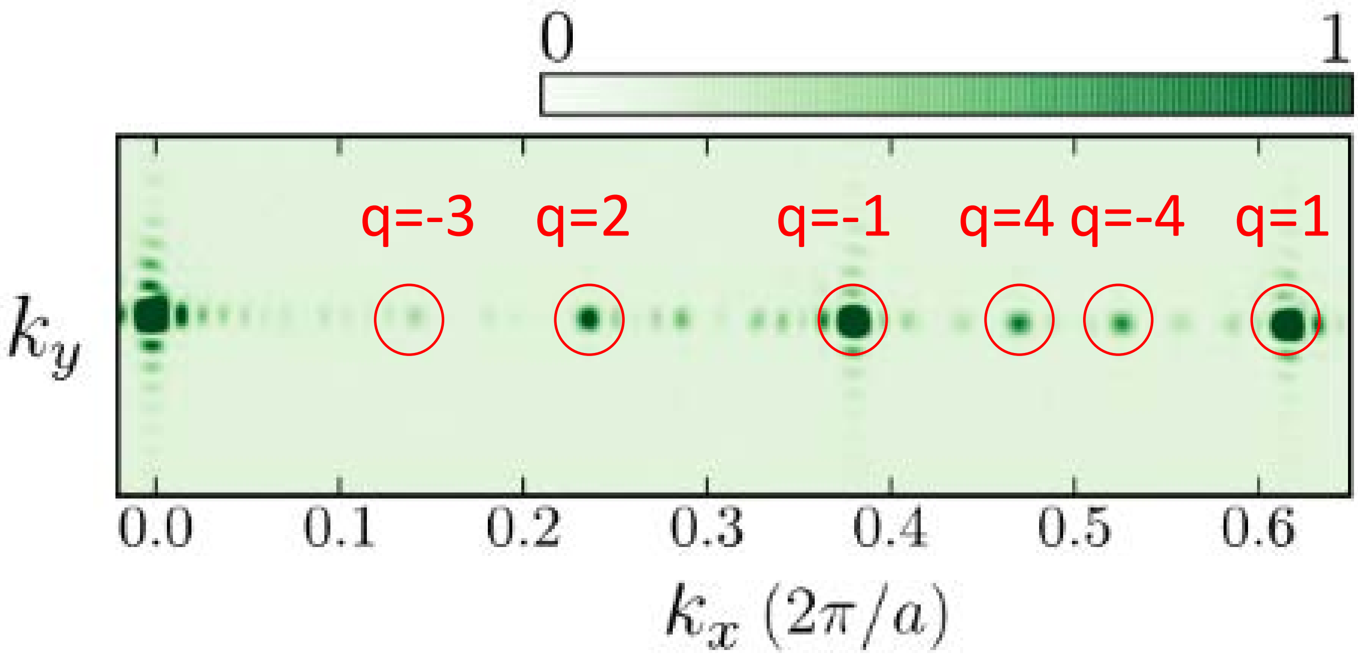
Digital Micromirror Device (DMD)

- mirror (“pixel”) size a ~ 14 µm
- 1024 × 768 pixels

Fibonacci encoding :   A   B



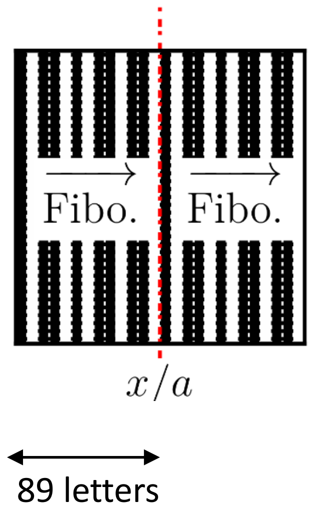
Diffraction by a single Fibonacci chain



Diffraction peaks at $k_x(p,q) = p + \frac{q}{\tau}$

Scanning the phason (1)

DMD Pattern



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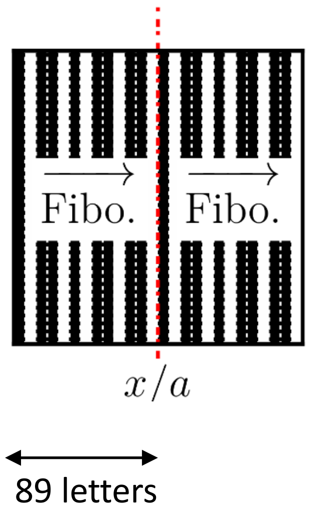
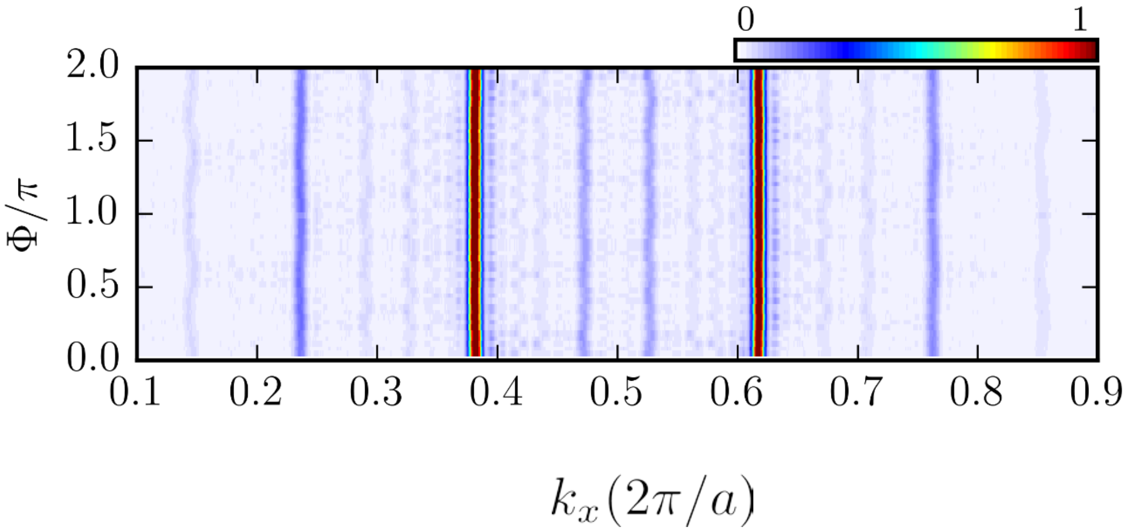


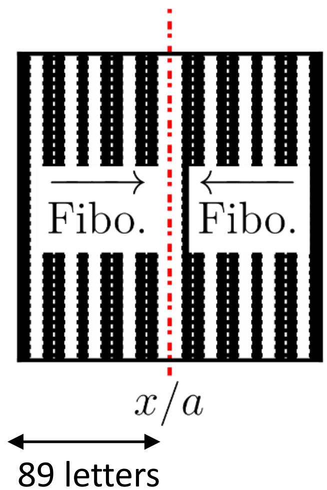
Diffraction pattern



No effect from the scan of the phason !

Scanning the phason (2)

DMD Pattern



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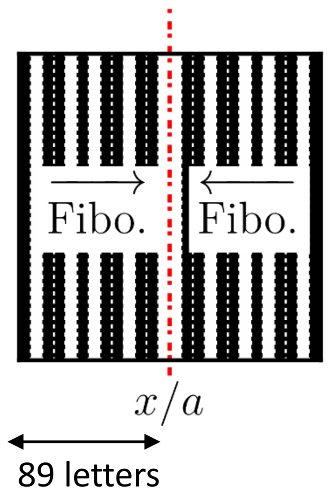
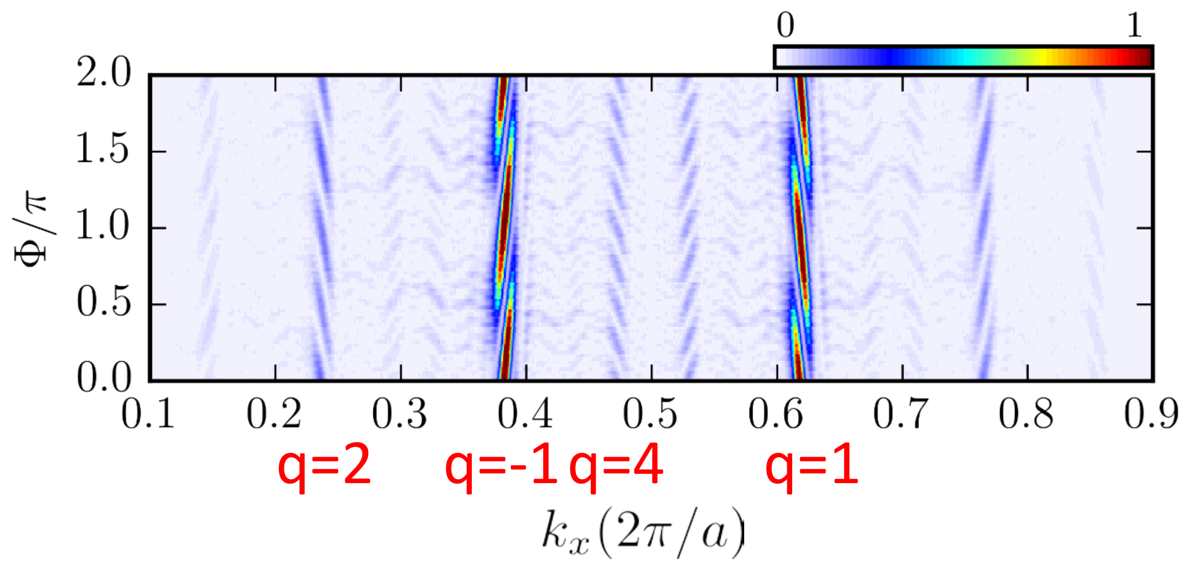


Diffraction pattern

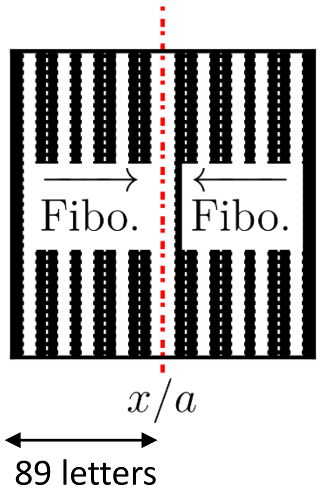


Peaks are crossed by holes

Slope of the crossing gives the Chern number q

Scanning the phason (2)

DMD Pattern



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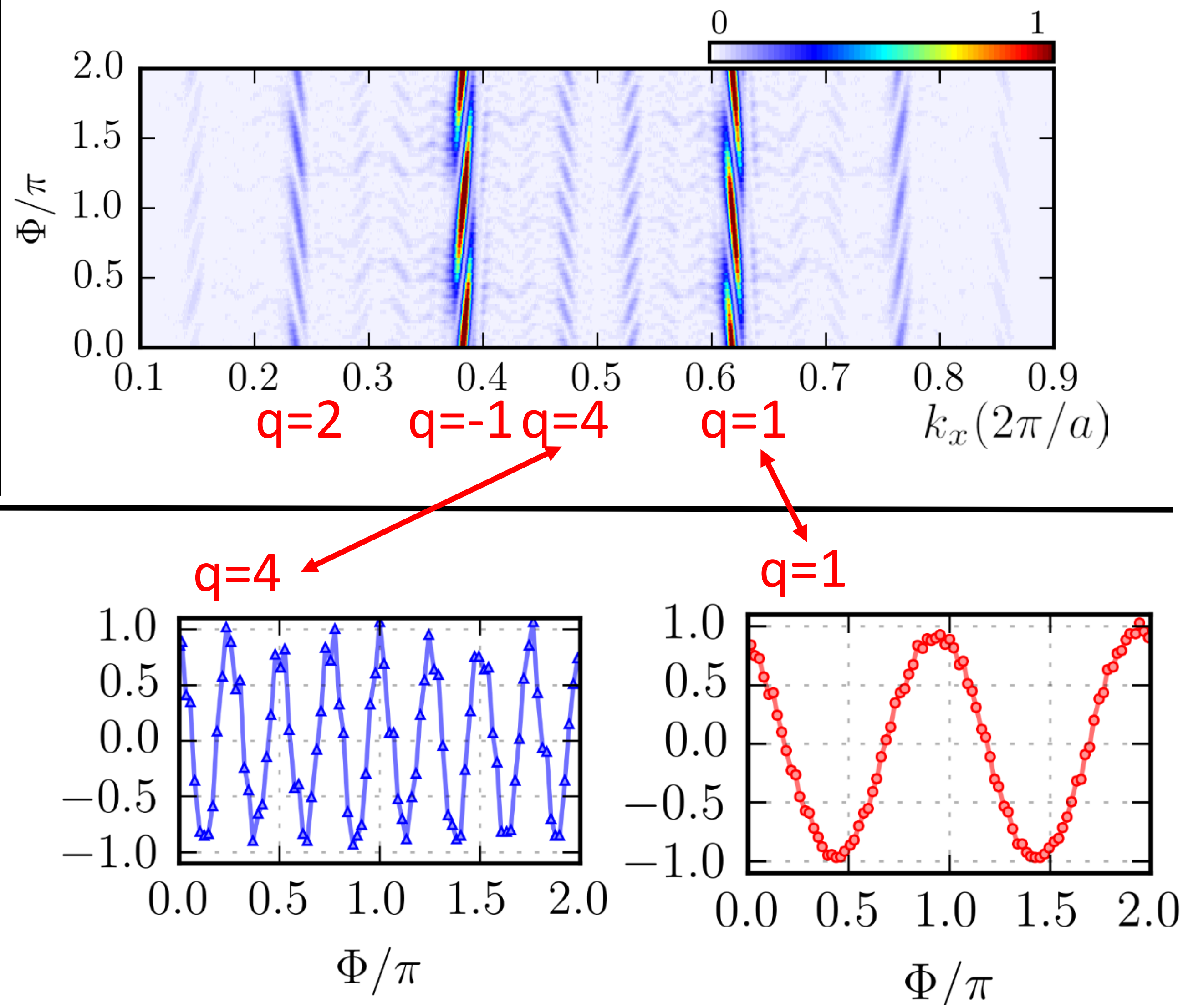
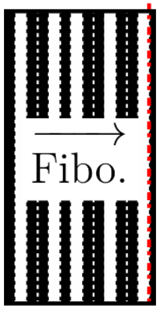


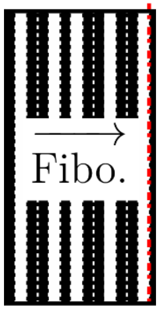
Cuts at a given k_x

Discussion



$$\mathcal{A}(k_x, \Phi) = \mathcal{A}_0(k_x)e^{-iq\Phi}$$

Discussion



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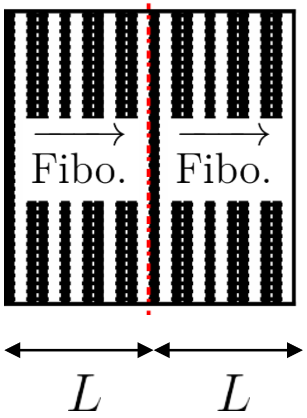


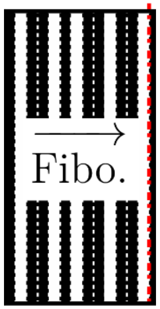
Multiple of 2π

$$I = |\mathcal{A}_0(k_x)|^2 \, |e^{-iq\Phi} + e^{-iq\Phi} e^{-ik_x L}|^2$$

$$= 4 \, |\mathcal{A}_0(k_x)|^2$$

No dependance on Φ

Discussion



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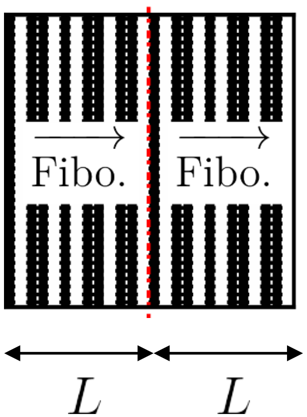

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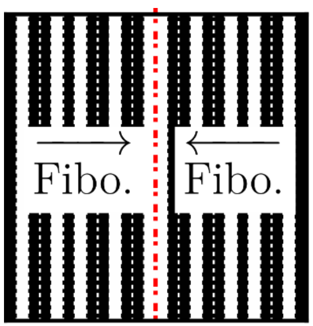


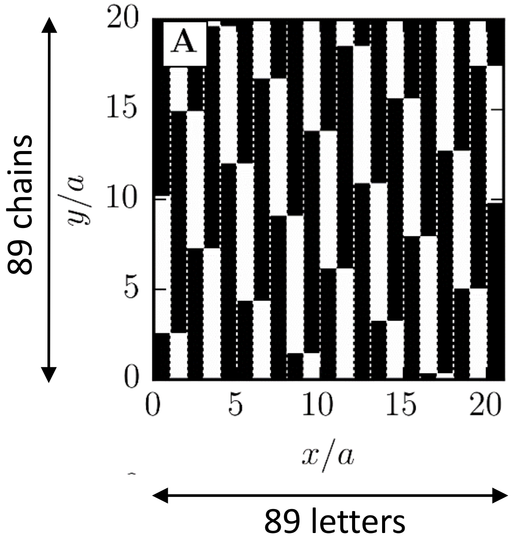
$$I = |\mathcal{A}_0(k_x)e^{-iq\Phi}e^{-ik_x L} + \mathcal{A}_0(-k_x)e^{+iq\Phi}e^{ik_x L}|^2$$

$$I = 4|\mathcal{A}_0(k_x)|^2 \cos^2(q\Phi + \phi_0)$$

Sinusoidal variation with Φ at a period π/q

2D Diffraction experiment

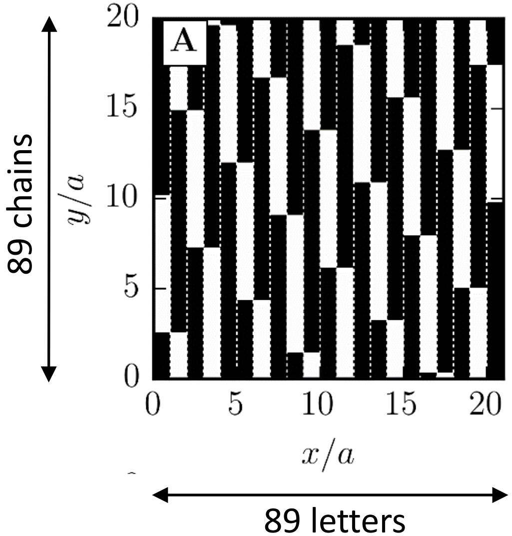
DMD Pattern



<u>y axis is associated with Φ</u>

2D Diffraction experiment

DMD Pattern



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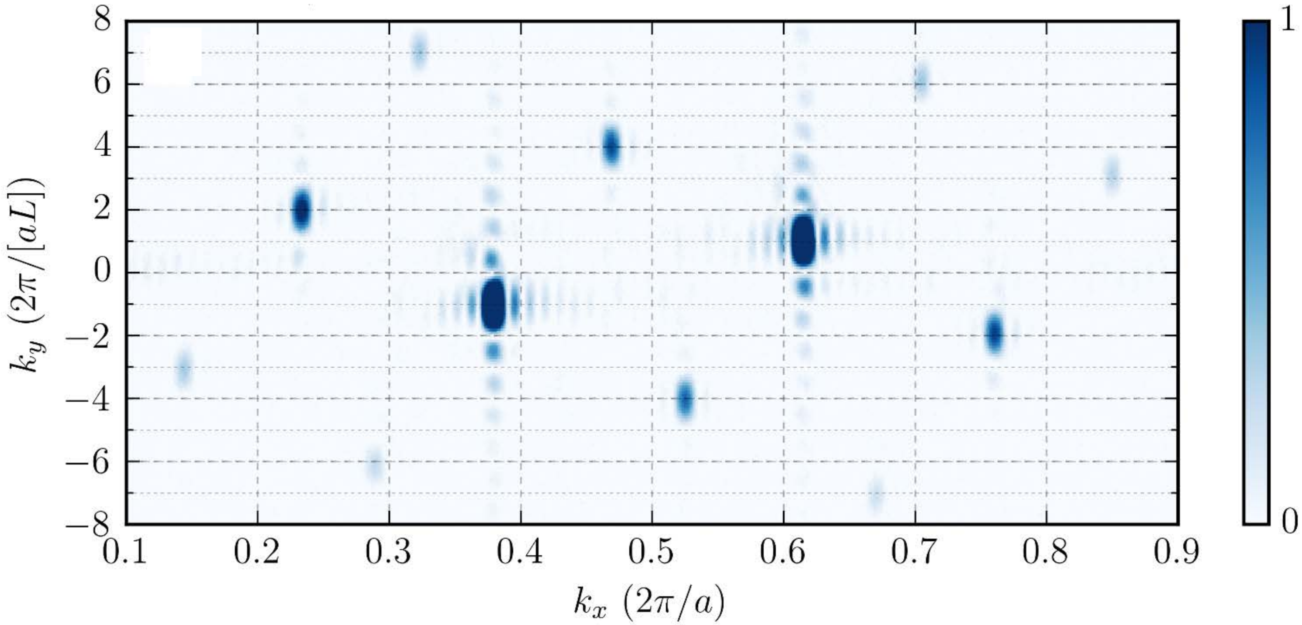
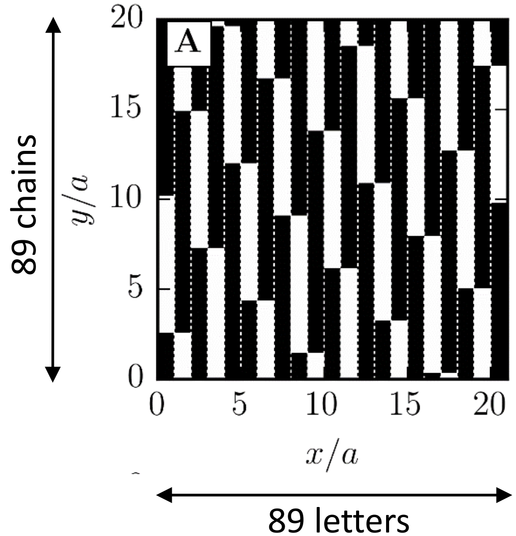
Diffraction pattern



2D Diffraction experiment

DMD Pattern



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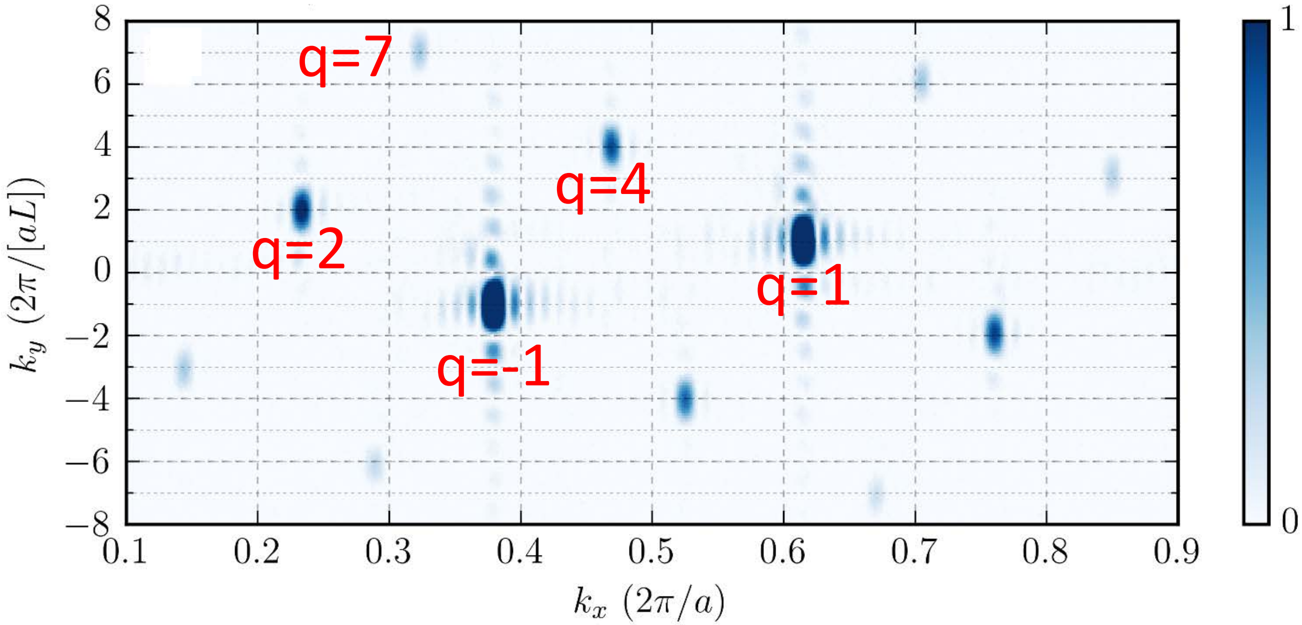


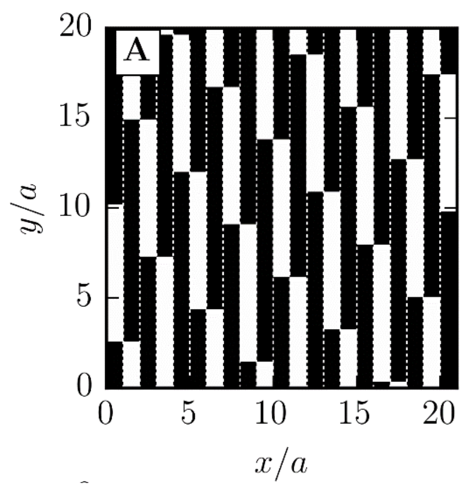
Diffraction peaks appear for the same k_x as the 1D grating

But for different k_y values

The k_y value is directly proportionnal to the Chern number

2D Diffraction experiment

DMD Pattern



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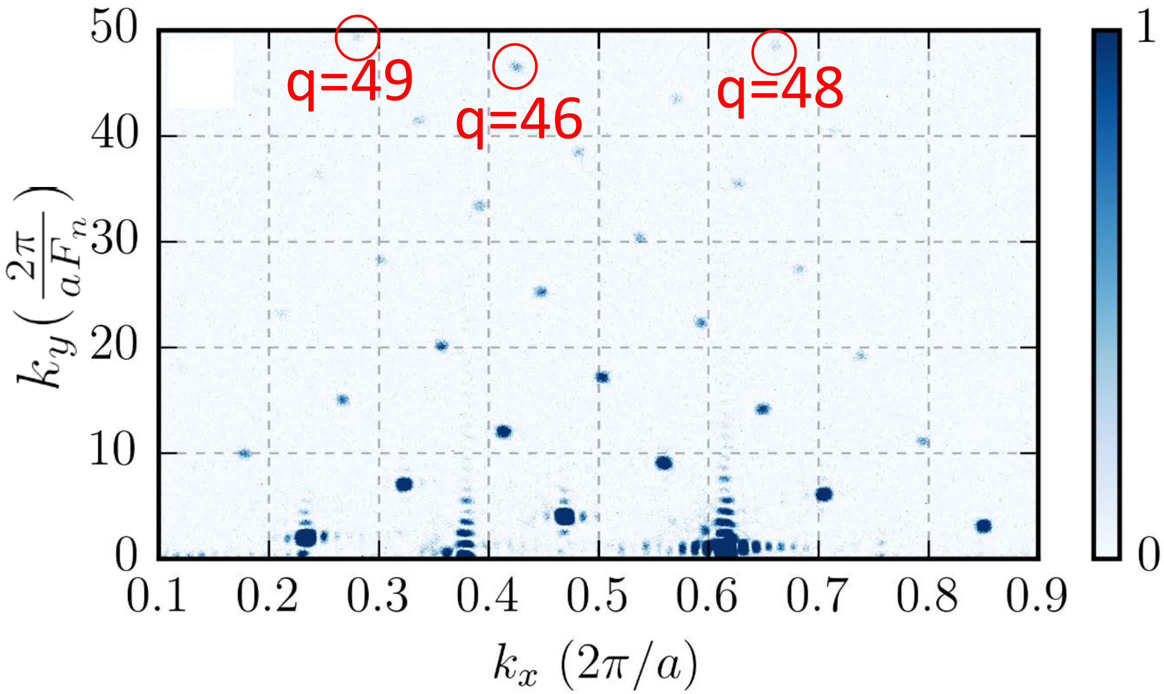
Diffraction pattern



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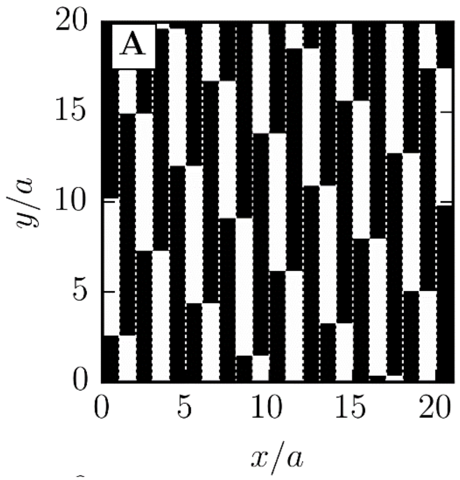
Discussion



$$I = \left| \sum_{\Phi} \mathcal{A}_0(k_x) e^{ik_y a \frac{F_n \Phi}{2\pi}} \, e^{-iq\Phi} \right|^2$$

$$= |\mathcal{A}_0(k_x)|^2 \left| \sum_{\Phi} e^{ik_y a \frac{F_n \Phi}{2\pi}} \, e^{-iq\Phi} \right|^2$$

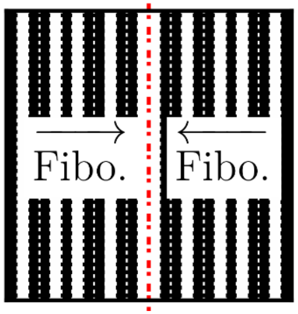
Peaks for same values of k_x as before

and for $k_y = q \times \frac{2\pi}{aF_n}$

Robustness against noise

<u>Adding noise</u> :

- choose randomly a fraction η of the lines
- choose randomly their state (A or B)
- average over many realization of the noise



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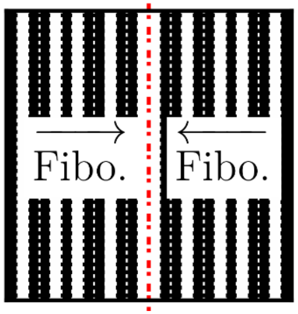

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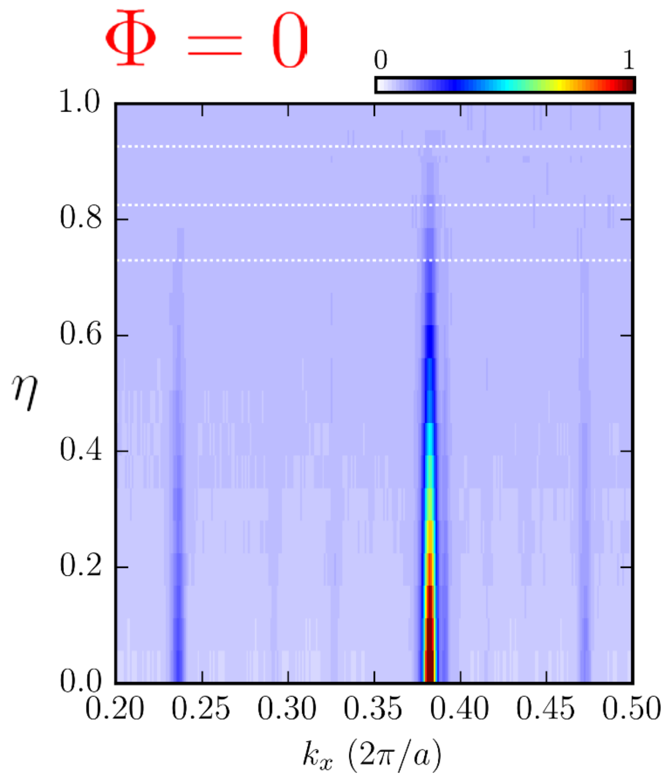

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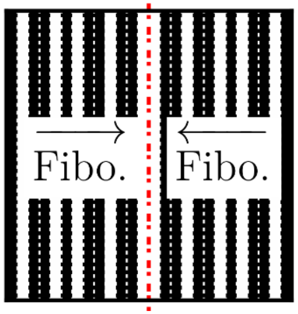
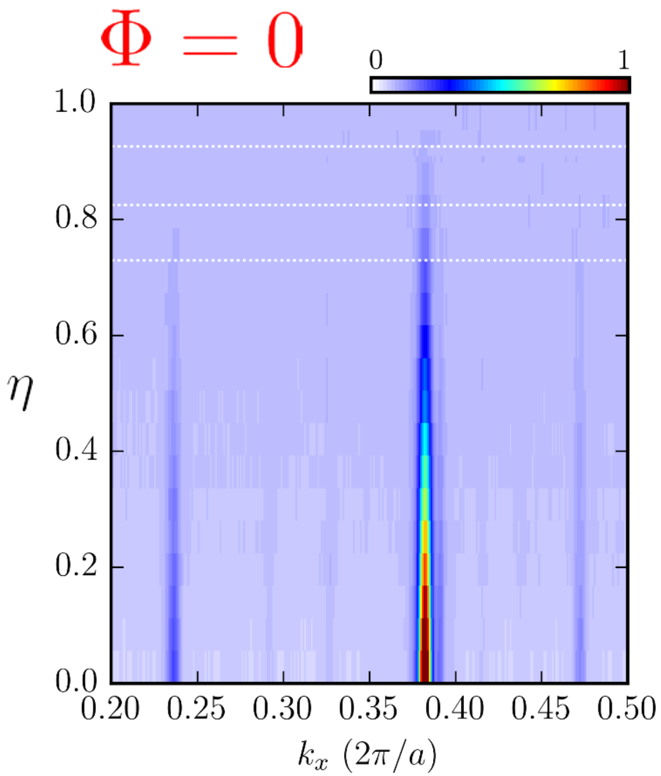
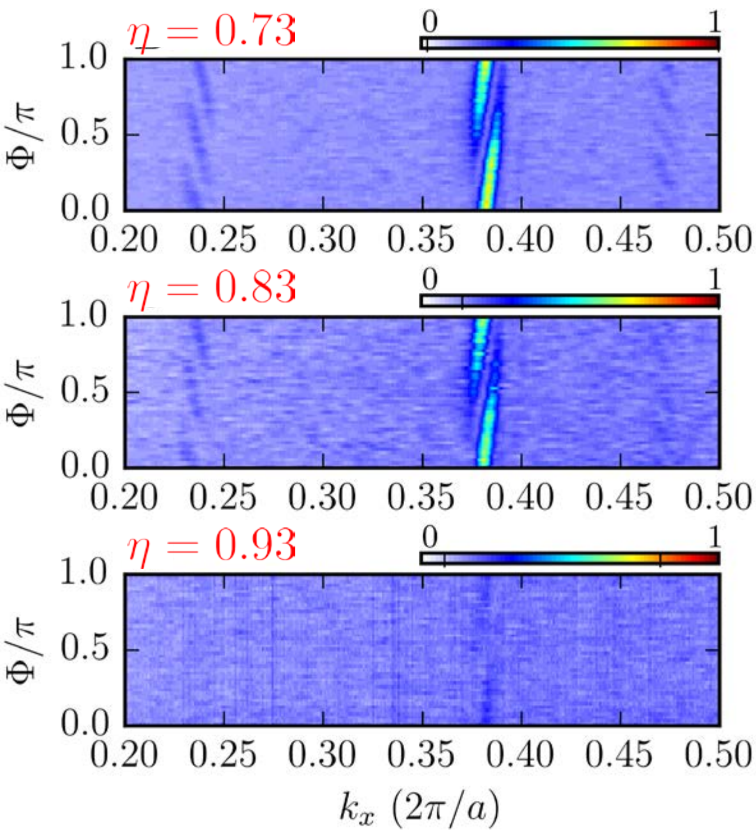




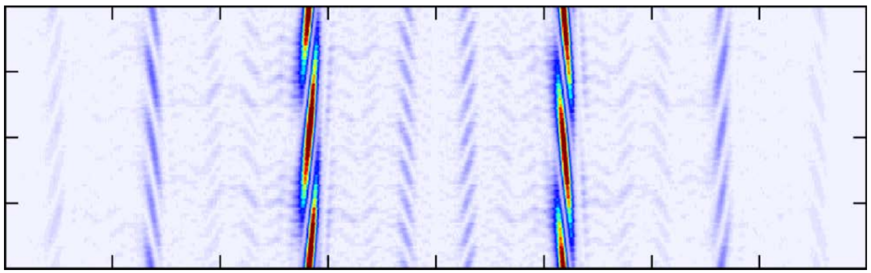


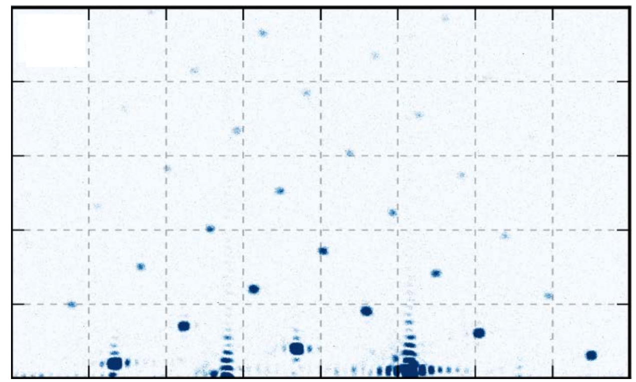
Slope of the crossing is constant and robust as the peak is still visible

Conclusion and perspectives

We measure structural properties of the Fibonacci sequence in its diffraction pattern

We can tune the phason degree of freedom
and measure Chern numbers in the diffraction pattern





Can be extended to other 1D quasiperiodic systems

Possibility to realize 2D tilings and investigate their topological properties