• Emiters (atoms, quantum dots, …)
• Structured materials
• Large couplings atom-light
OUTLINE

- Theoretical framework
  - Markovian
  - Exact


- Bound states

Shi, Wu, González-Tudela, JIC, arxiv: 1512.07238

- Multi-photon states

González-Tudela, Paulisch, Kimble, JIC, arxiv: 1602.?????
1. THEORETICAL FRAMEWORK
EMITTERS IN A WAVEGUIDE

ATOMS NEAR 1D WAVEGUIDES

Chang, Demler, Gorskov, Lukin, Zoller, …

a variety of phenomena

DESCRIPTION:

- **Scattering matrix**: entanglement, transmission, losses …
- **Atomic dynamics**: polaritons, bound states, many-body behavior …
- **Photon dynamics**: Multiphoton states, single/multi-mode, …
- **Propagation effects**: retardation, dispersion,…
\[ H = H_{\text{atoms}} + H_{\text{waveguide}} + H_{\text{interaction}} + H_{\text{dissipation}} \]
Input-output:
- Markovian limit.
- Atomic dynamics.

Path integral:
- Exact.
- Atomic dynamics.
ATOMS NEAR 1D WAVEGUIDES
INPUT-OUTPUT

(cavity QED: Gardiner, 1980's)

CONDITIONS:

- Linear dispersion relation: $H_{\text{waveguide}}$
- Flat coupling constant: $H_{\text{interaction}}$
- No atomic retardation effects

METHOD:

- Solve a master equation for the atoms
- Initial state of the waveguide => several driving fields
- Compute Fourier Transforms
- Analytical formulas for scattering

$$S_{p_1,...,p_n \leftarrow k_1,...,k_n} = FT\left( \langle \varphi_{\text{atoms}} | T \left[ o(t_1)\ldots o^+(t_1') \ldots \right] | \varphi_{\text{atoms}} \rangle \right)$$

$$o(t) = e^{iH_{\text{eff}}t} o e^{-iH_{\text{eff}}t}$$
EXAMPLE 1: single polariton propagation in EIT configuration
(check)

Polariton is absorbed if the pulse length is smaller than that of the transparency window
EXAMPLE 2: multi-photon propagation
atom-atom interactions: $C = 0, 0.2$

Initial state: $|g_{...g}\rangle |\epsilon_k\rangle$
$N = 200$
$\Gamma_{1d} = 1$  $\Omega = 2$
$\Gamma_{out} = 3$  $\epsilon = 10^{-6}$

Interactions change the propagation and produce bunching/antibunching
OTHER EXAMPLES:

- Entanglement generation
- Rydberg, dipole-dipole interactions
- Excitation probabilities
- Emission and absorption
EXACT FORMALISM:
- Arbitrary dispersion relation: $H_{\text{waveguide}}$
- Arbitrary coupling constant: $H_{\text{interaction}}$
- Retardation effects

METHOD:
- Express amplitude as a path integral
- Integrate out the waveguide modes
- New action with time-delayed kernels
- Fourier transform the action

\[
\langle \Psi_{\text{out}} | e^{-iHt} | \Psi_{\text{in}} \rangle = \int D[\beta_i] e^{-iS_{\text{int}}[\beta_i]} \int D[\alpha_j] e^{-iS_f[\alpha_j] - iS_{\text{int}}[\alpha_j, \beta_i]} \\
= \int D[\beta_i] e^{-iS_{\text{eff}}[\beta_i]}
\]

Fourier Transform
EXAMPLE 1: propagation of a single photon with two atoms

Initial state: $|g...g\rangle |1_k\rangle$
$N = 2$
$\Gamma_{1a} = 1$
$\Gamma_{\text{out}} = 0$

$kd = 2\pi n$
EXAMPLE 2: excitation probability second emitter

Initial state: $|g...g\rangle |1_k\rangle$

- $N = 2$
- $\Gamma_{1d} = 1$
- $kd = 2\pi n$
- $\Gamma_{out} = 0$

**Excitation probability**

- $d = 0$
- $d = 0.01$
- $d = 1$

**Diagram**

- $d$
- $\Gamma_{1d} = 1$
2. MULTIPHOTON BOUND STATES

Shi, Wu, González-Tudela, JIC, arxiv: 1512.07238
**IMPURITY IN A 1D WAVEGUIDE**

**SINGLE-PHOTON BOUND STATE**

**BOUND state**

John and Wang (1990)

\[ |\Psi_1\rangle = c_e |e\rangle |0\rangle + c_g |g\rangle |1\rangle \]

- **Interpretation:** band-gap, energy conservation

- **Consequences:** cavity QED, dipole-dipole interactions, …


- **Alternative experimental realization:** atoms in optical lattices

Interpretation: Atom creates a potential, where photons condense

Description:
- Analytical approach (up to three excitations)
- Phenomenological Ansatz (any dimension)
- DMRG
- Non-perturbative regimes

Alternative experimental realization: atoms in optical lattices
IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

Hamiltonian

\[ H = \Delta |e\rangle\langle e| + \sum_k \epsilon_k a_k^\dagger a_k + \sum_k g_k (a_k^\dagger |g\rangle\langle e| + h.c.) \]

We look for proper eigenstates in the thermodynamic limit

\[ |B_N\rangle = c_e |e\rangle |\Psi_{N-1}^e\rangle + c_g |g\rangle |\Psi_N^g\rangle \]
IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

PARAMETER REGIMES:

- **Jaynes-Cummings regime**: $\Omega \rightarrow \infty$

  $|B_N\rangle \sim c_e |e\rangle |N-1\rangle + c_g |g\rangle |N\rangle$

- **Perturbative regime**: $|\Delta| \rightarrow \infty$

  Adiabatic elimination: the atoms create a potential where photons condense

- **All regimes in 1D**: solution up to three excitations
IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

SIMPLE DESCRIPTION:

Variational wavefunction:

\[ |\Psi^e_{N-1}\rangle < A^{\dagger(N-1)} |0\rangle \]
\[ |\Psi^g_{N-1}\rangle < A^{\dagger(N-1)}(A^{\dagger} + \alpha B^{\dagger}) |0\rangle \]

Generalized GP equation:

\[ \mathcal{H}_0 \left( \begin{array}{c} \varphi_A(k) \\ \varphi_B(k) \end{array} \right) + \frac{\Omega \eta k}{\sqrt{\nu}} \left( \begin{array}{c} \sqrt{N} \beta \\ \gamma \end{array} \right) = \mu \left( \begin{array}{c} \varphi_A(k) \\ \varphi_B(k) \end{array} \right) \]

Exactly solved (in terms of three parameters) in any dimension and dispersion relation
IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

NUMERICAL CERTIFICATION:

Energies

\[ \frac{\tilde{E}_3}{J} \]

\[ \frac{\tilde{E}_4}{J} \]

\[ \Delta/J = 0 \]
\[ \Delta/J = 0.2 \]
\[ \Delta/J = -0.2 \]

\( m=3 \)
\( m=4 \)
IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

NUMERICAL CERTIFICATION:

Localization length

[Graph and diagram showing the localization length as a function of Ω/|J| with ξg and ξe on the y-axis.]

atomic structure

band structure
ATMOS IN OPTICAL LATTICES:

Hamiltonian: \( H = \Delta |1\rangle_b \langle 1| + \sum_k \epsilon_k a_k^\dagger a_k + \sum_k g_k (a_k^\dagger |0\rangle_b \langle 1| + h.c.) \)

- Creation by adiabatic evolution
- Study consequences in scattering, etc
- Multi-impurities: Effective Hamiltonians
3. MULTI-PHOTON SOURCES

A. Gonzalez-Tudela, V. Paulisch, H. J. Kimble, JIC, arxiv:1602.????
MULTI-PHOTON STATES
MULTI-PHOTON STATES
• Large Purcell effects: \( P_{1d} = \frac{\Gamma_{1d}}{\Gamma_{\text{out}}} \gg 1 \)

• Infidelity: \( I = \frac{m}{P_{1d}} \)

• Complex multi-mode structure

IDEA: use collective effects + heralding
Large collective effects: \( \Gamma_{\text{eff}} = N\Gamma_{1d} \)

They can be used to:

- map atomic to photonic states
- single mode states

MULTI-PHOTON STATES
PROCEDURE

1. LOADING

\[ |s\rangle \quad |g\rangle \quad |e\rangle \]

2. TRIGGERING

\[ |s\rangle \quad |g\rangle \quad |e\rangle \]

3. EMISSION

\[ |s\rangle \quad |g\rangle \quad |e\rangle \]

How to generate the atomic (entangled states)?
SCHEME 1:
Extension of Duan, Lukin, JIC, Zoller, Nature 414, 413 (2001)

- Two-photon excitations produce errors.
- Zero-photon excitation gives low probability.

\[ I_1 \approx \varepsilon^2 \]

\[ p_1 \approx \frac{1}{\varepsilon^2} \quad \Rightarrow \quad p_m \approx \left( \frac{1}{\varepsilon^2} \right)^m \]
MULTI-PHOTON STATES
SCHEME with SOURCE AND DETECTOR

SCHEME 2:

- High detection efficiency: Detect atoms, not photons
- If the detector clicks, the process had no error

\[ I_1 = 0 \]

\[ p_1 \approx 1 - \frac{1}{P_{id}} \quad \Rightarrow \quad p_m \approx \left( 1 - \frac{1}{P_{id}} \right)^m \]

- Purcell factor limits the number of photons

Can we get a polynomial scaling in m?
MULTI-PHOTON STATES
SCHEME with ADDITIONAL LEVELS

SCHEME 3:

- Internal levels to store the excitations
- Atomic measurements (only) to merge excitations

\[ I_m \approx \frac{\text{poly}(m)}{NP_{vd}} \]

\[ p_m \approx \frac{1}{\text{poly}(m)} \]
MULTI-PHOTON STATES

SUMMARY

MULTIPHOTON GENERATION BY COLLECTIVE EFFECTS

\[ |\Psi_m\rangle = \sum_{n=0}^{m} c_n |n\rangle \]

<table>
<thead>
<tr>
<th></th>
<th>Collective Zeno</th>
<th>Heralded source + detector</th>
<th>Heralded merging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infidelity:</td>
<td>( I_m \approx m^2 / \sqrt{P_{1d}} )</td>
<td>( I_m = 1 )</td>
<td>( I_m \approx m^2 / NP_{1d} )</td>
</tr>
<tr>
<td>Probability:</td>
<td>( p_m = 1 )</td>
<td>( p_m \ll 1 / \exp(m) )</td>
<td>( p_m \ll 1 / \text{poly}(m) )</td>
</tr>
</tbody>
</table>

\( P_{1d} \): Purcell factor
\( N \): Number of atoms
\( m \): Number of photons
4. OTHER PROBLEMS
OTHER PROBLEMS
SURFACE ACOUSTIC WAVES

QUANTUM OPTICS IN WAVEGUIDES
SUMMARY & OUTLOOK

- Challenging experiments

**New regimes:**

- Large Purcell effects
- Collective phenomena
- Bound states

- Connection to cold atoms

- Quantum simulations

  - Multi-impurities

Jeff's talk