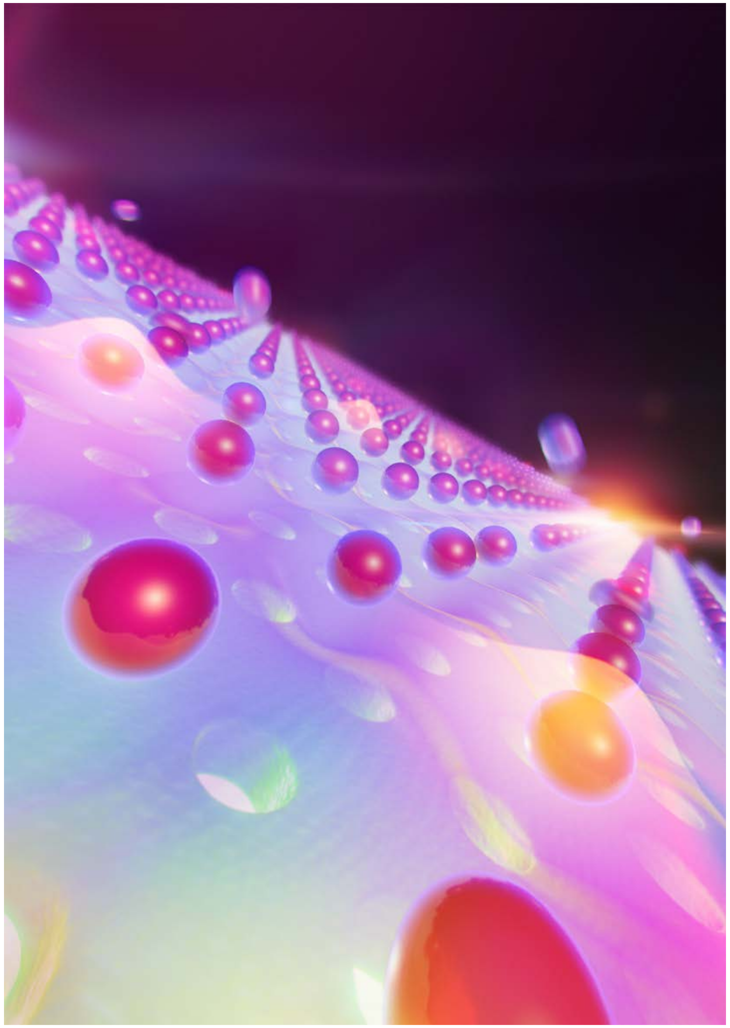


Quantum optics with atoms in waveguides

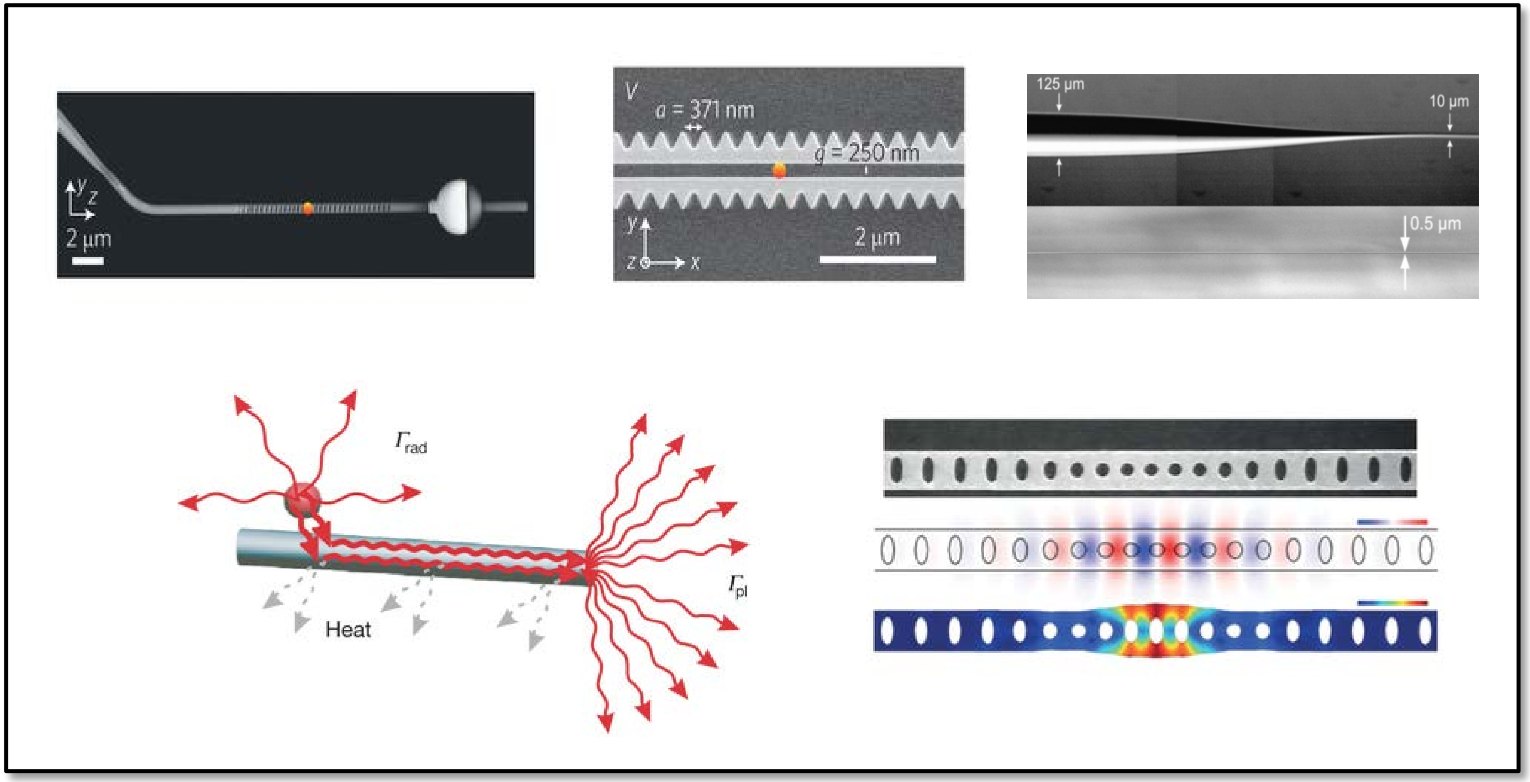
Alejandro González-Tudela
Vanessa Paulisch
Tao Shi
Yinghai Wu

Max-Planck-Institut für Quantenoptik

Jeff Kimble (Caltech)
Darrick Chang (ICFO)

Workshop in Honour of Peter Zoller
Quantum simulations with cold matter and photons
UL Brussels, February 7th, 2016

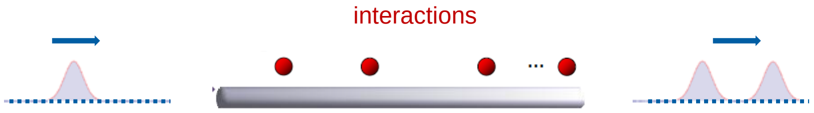
EMITTERS & NANO-STRUCTURES



- Emiters (atoms, quantum dots,…)
- Structured materials
- Large couplings atom-light

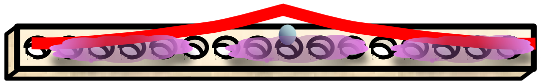
OUTLINE

- Theoretical framework
  - Markovian
  - Exact



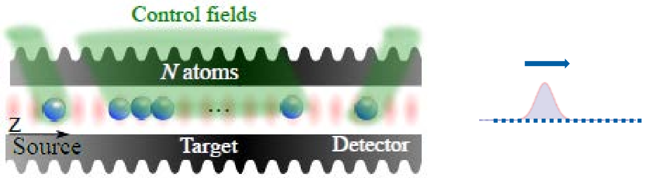
Caneva, Manzoni, Shi, Douglas, JIC, Chang, New J. Phys. **17**, 113001 (2015)
Shi, Chang, JIC, Phys. Rev. A **92**, 053834 (2015)

- Bound states



Shi, Wu, González-Tudela, JIC, arxiv: 1512.07238

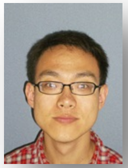
- Multi-photon states



González-Tudela, Paulisch, Chang, Kimble, JIC, Phys. Rev. Lett. **115**, 163603 (2015)
González-Tudela, Paulisch, Kimble, JIC, arxiv: 1602.?????

1. THEORETICAL FRAMEWORK
EMITTERS IN A WAVEGUIDE

Caneva, Manzoni, Shi, Douglas, JIC, Chang, New J. Phys. **17**, 113001 (2015)
Shi, Chang, JIC, Phys. Rev. A **92**, 053834 (2015)

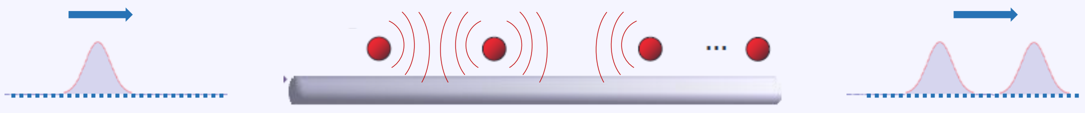


Tao
Shi

Darrick
Chang

Caneva
+ Manzoni
Douglas

ATOMS NEAR 1D WAVEGUIDES



Chang, Demler, Gorskov, Lukin, Zoller, …

a variety of phenomena

DESCRIPTION:

- Scattering matrix: entanglement, transmission, losses …
- Atomic dynamics: polaritons, bound states, many-body behavior …
- Photon dynamics: Multiphoton states, single/multi-mode, …
- Propagation effects: retardation, dispersion,…

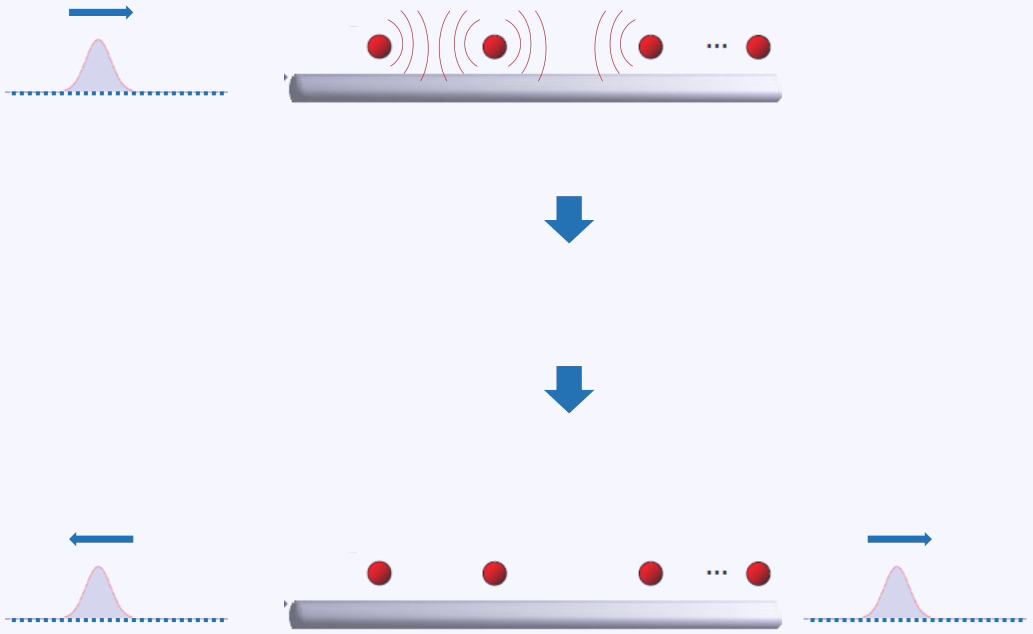
ATOMS NEAR 1D WAVEGUIDES
THEORETICAL FRAMEWORK

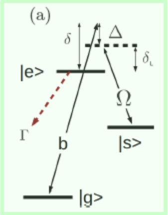
$$H = H_{\text{atoms}} + H_{\text{waveguide}} + H_{\text{interaction}} + H_{\text{dissipation}}$$

$$|\Psi(0)\rangle = |\phi_{\text{light}}\rangle\,|\varphi_{\text{atoms}}\rangle$$

$$|\Psi(t)\rangle$$

$$|\Psi(t \to \infty)\rangle$$

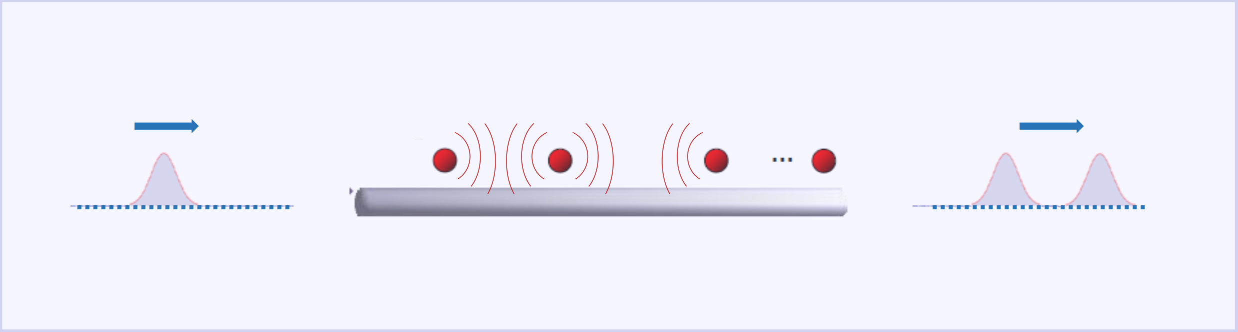




atomic structure

ATOMS NEAR 1D WAVEGUIDES

THEORETICAL FRAMEWORK



- Input-output:
  - Markovian limit.
  - Atomic dynamics.

- Path integral:
  - Exact.
  - Atomic dynamics.

ATOMS NEAR 1D WAVEGUIDES
INPUT-OUTPUT

(cavity QED: Gardiner, 1980‘s)

CONDITIONS:

- Linear dispersion relation: $H_{\text{waveguide}}$
- Flat coupling constant: $H_{\text{interaction}}$
- No atomic retardation effects

METHOD:

- Solve a master equation for the atoms
- Initial state of the waveguide => several driving fields
- Compute Fourier Transforms

- Analytical formulas for scattering

$$S_{p_1,\dots,p_n \leftarrow k_1,\dots,k_n} = FT\left(\langle \varphi_{\text{atoms}} | T\left[o(t_1)\dots o^{\dagger}(t_1')\dots\right] | \varphi_{\text{atoms}} \rangle\right)$$

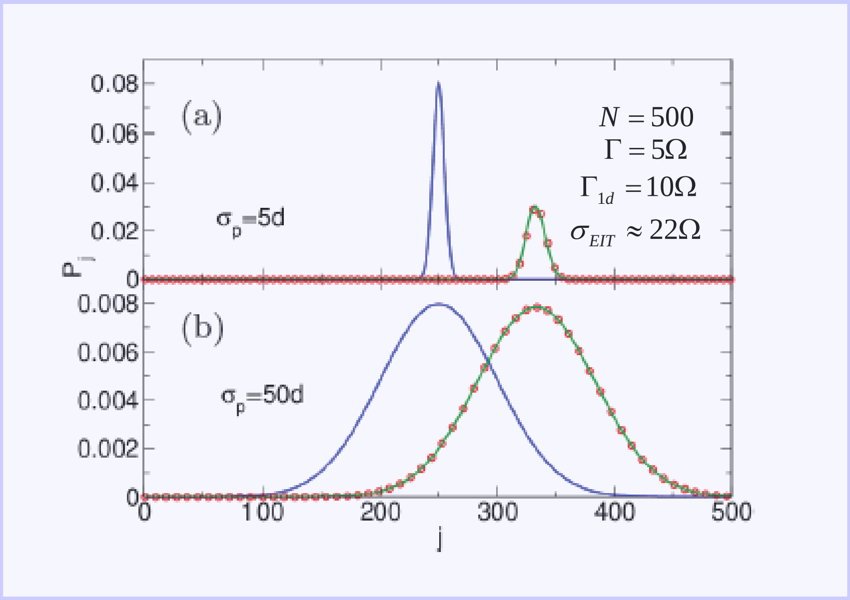
$$\nwarrow$$

$$o(t) = e^{iH_{eff}t} o e^{-iH_{eff}^{\dagger}t}$$

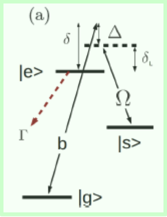
ATOMS NEAR 1D WAVEGUIDES
INPUT-OUTPUT

EXAMPLE 1: single polariton propagation in EIT configuration

(check)



Polariton is absorbed if the pulse length is smaller than that of the transparency window



atomic structure

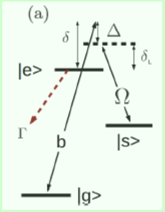
Initial state:

$$\sum_{-N/2}^{N/2} e^{ikn} e^{-n^2/4\sigma^2} \,|\, g..s_n.g\rangle \,|\, 0\rangle$$

ATOMS NEAR 1D WAVEGUIDES
INPUT-OUTPUT

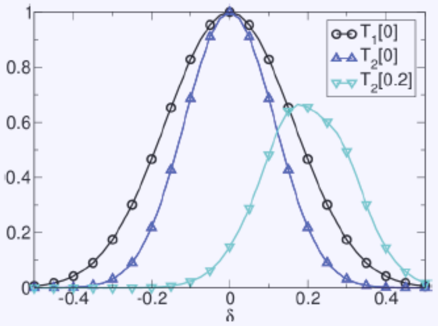
EXAMPLE 2: multi-photon propagation
atom-atom interactions: $C = 0, 0.2$

Initial state: $|g...g\rangle|\varepsilon_k\rangle$
$N = 200$
$\Gamma_{1d} = 1$
$\Gamma_{out} = 3$
$\Omega = 2$
$\varepsilon = 10^{-6}$

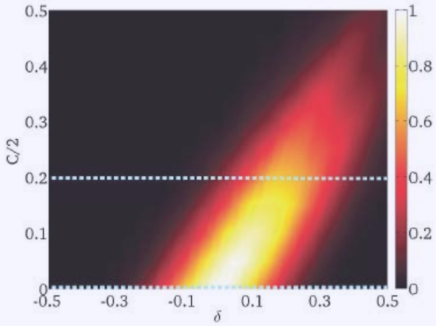


atomic structure

Transmission spectrum

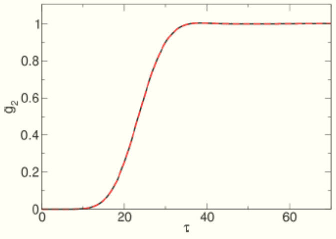


g₂(0) function



Interactions change the propagation and produce bunching/antibunching

Analytical



ATOMS NEAR 1D WAVEGUIDES

INPUT-OUTPUT

OTHER EXAMPLES:

- Entanglement generation
- Rydberg, dipole-dipole interactions
- Excitation probabilities
- Emission and absorption

ATOMS NEAR 1D WAVEGUIDES
PATH INTEGRAL

EXACT FORMALISM:

- Arbitrary dispersion relation: $H_{\text{waveguide}}$
- Arbitrary coupling constant: $H_{\text{interaction}}$
- Retardation effects

METHOD:

- Express amplitude as a path integral
- Integrate out the waveguide modes
- New action with time-delayed kernels
- Fourier transform the action

$$\langle \Psi_{out} | e^{-iHt} | \Psi_{in} \rangle = \int D[\beta_i] e^{-iS_{at}[\beta_i]} \int D[\alpha_j] e^{-iS_f[\alpha_j] - iS_{\text{int}}[\alpha_j, \beta_i]}$$

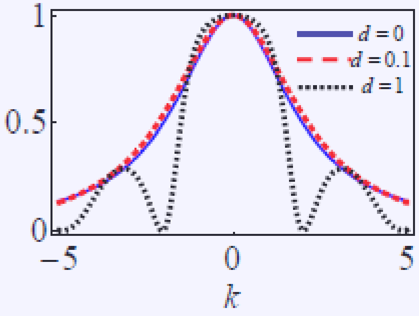
$$= \int D[\beta_i] e^{-iS_{eff}[\beta_i]}$$

Fourier Transform

ATOMS NEAR 1D WAVEGUIDES
PATH INTEGRAL

EXAMPLE 1: propagation of a single photon with two atoms

reflection coefficient

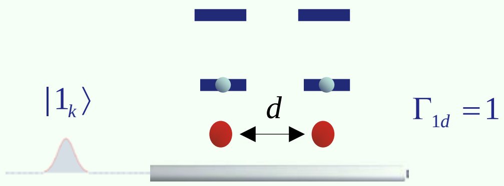


Initial state: $|g...g\rangle|1_k\rangle$

$N=2$

$\Gamma_{1d}=1$ $\quad kd=2\pi n$

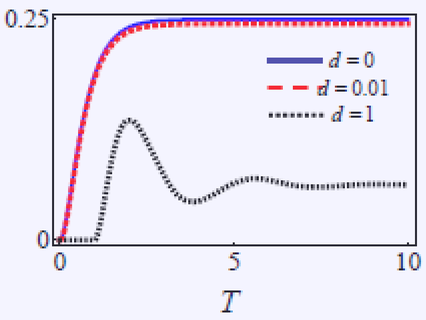
$\Gamma_{out}=0$



ATOMS NEAR 1D WAVEGUIDES
PATH INTEGRAL

EXAMPLE 2: excitation probability second emitter

excitation probability

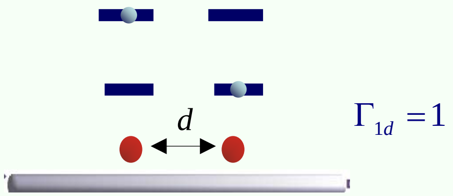


Initial state: $|g...g\rangle|1_k\rangle$

$N=2$

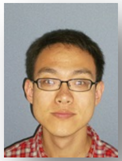
$\Gamma_{1d}=1$ $\quad kd=2\pi n$

$\Gamma_{out}=0$



2. MULTIPHOTON BOUND STATES

Shi, Wu, González-Tudela, JIC, arxiv: 1512.07238

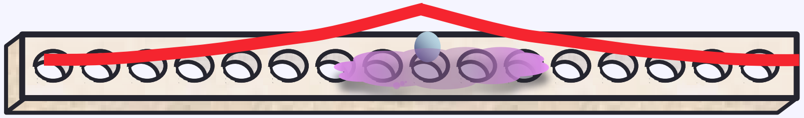

Tao
Shi


Yinghai
Wu


Alex
Gonz.-T.

IMPURITY IN A 1D WAVEGUIDE
SINGLE-PHOTON BOUND STATE

BOUND SATE



John and Wang (1990)

$$|\Psi_1\rangle = c_e\,|e\rangle\,|0\rangle + c_g\,|g\rangle\,|1\rangle$$

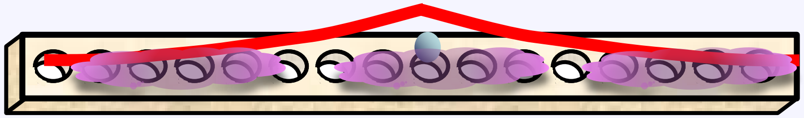
- Interpretation: band-gap, energy conservation
- Consequences: cavity QED, dipole-dipole interactions, …

  Doublas, Habibian, Hung, Gorshkov, Kimble, Chang, Nature Photonics 9, 326 (2015)
- Alternative experimental realization: atoms in optical lattices

  Vega, Porras, JIC, Phys. Rev. Lett. **101**, 260404 (2008),
  Navarrete, Vega, Porras, JIC, New J. Phys.**13**, 023024 (2011)

IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

INFINITELY MANY BOUND SATES

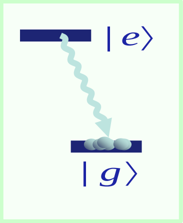


$$|B_N\rangle = c_e\,|e\rangle\,|\Psi^{e}_{N-1}\rangle + c_g\,|g\rangle\,|\Psi^{g}_{N}\rangle$$

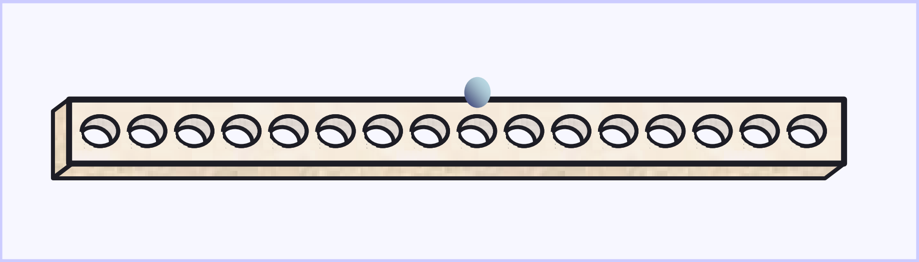
- Interpretation: Atom creates a pontential, where photons condense
- Description:
  - Analytical approach (up to three excitations)
  - Phenomenolagical Ansatz (any dimension)
  - DMRG
  - Non perturbative regimes
- Alternative experimental realization: atoms in optical lattices

IMPURITY IN A 1D WAVEGUIDE

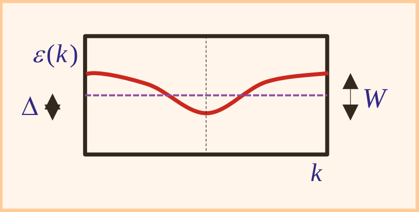
MULTI-PHOTON BOUND STATE



atomic structure



band structure



Hamiltonian

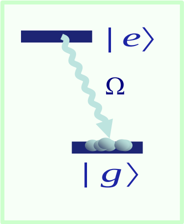
$$H = \Delta |e\rangle\langle e| + \sum_k \varepsilon_k a_k^\dagger a_k + \sum_k g_k (a_k^\dagger |g\rangle\langle e| + h.c.)$$

We look for proper eigenstates in the thermodynamic limit

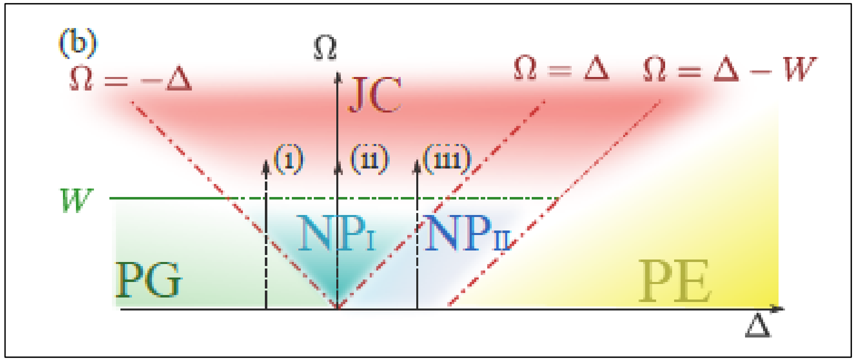
$$|B_N\rangle = c_e |e\rangle |\Psi^e_{N-1}\rangle + c_g |g\rangle |\Psi^g_N\rangle$$

IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

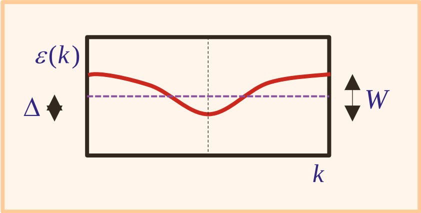
PARAMETER REGIMES:



atomic structure



band structure



- Jaynes-Cummings regime: $\Omega \to \infty$

$$|B_N\rangle \prec c_e\,|e\rangle\,|N-1\rangle + c_g\,|g\rangle\,|N\rangle$$

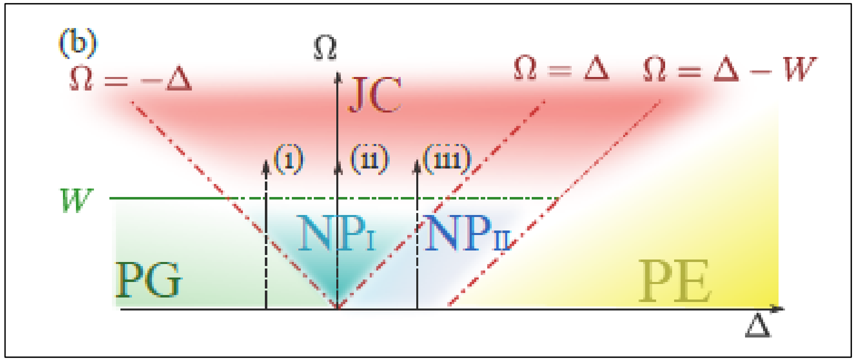
- Perturbative regime: $|\Delta| \to \infty$

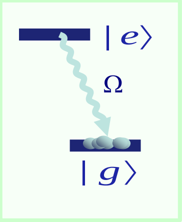
Adiabatic elimination: the atoms create a potential where photons condense

- All regimes in 1D: solution up to three excitations

IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

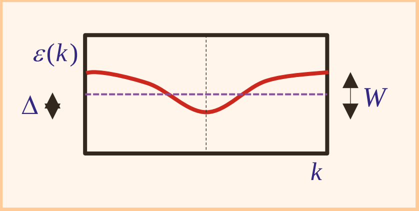
SIMPLE DESCRIPTION:





atomic structure

band structure



- Variational wavefunction:

$$|\Psi^{e}_{N-1}\rangle \prec A^{\dagger(N-1)}\,|0\rangle$$

$$|\Psi^{g}_{N-1}\rangle \prec A^{\dagger(N-1)}(A^{\dagger}+\alpha B^{\dagger})\,|0\rangle$$

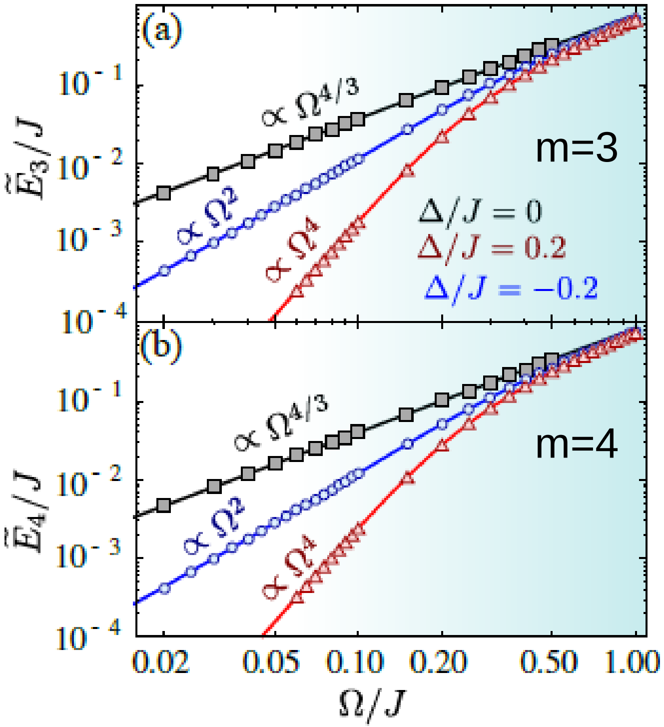
- Genralized GP equation: $\mathcal{H}_0 \begin{pmatrix} \varphi_A(\mathbf{k}) \\ \varphi_B(\mathbf{k}) \end{pmatrix} + \frac{\Omega\eta_{\mathbf{k}}}{\sqrt{V}}\alpha \begin{pmatrix} \sqrt{N}\beta \\ \gamma \end{pmatrix} = \mu \begin{pmatrix} \varphi_A(\mathbf{k}) \\ \varphi_B(\mathbf{k}) \end{pmatrix}$

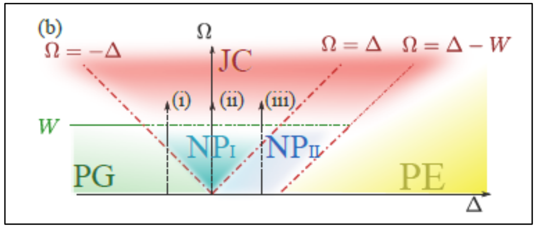
Exactly solved (in terms of three parameters) in any dimension and dispersion relation

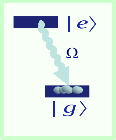
IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

NUMERICAL CERTIFICATION:

Energies

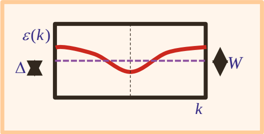






atomic structure

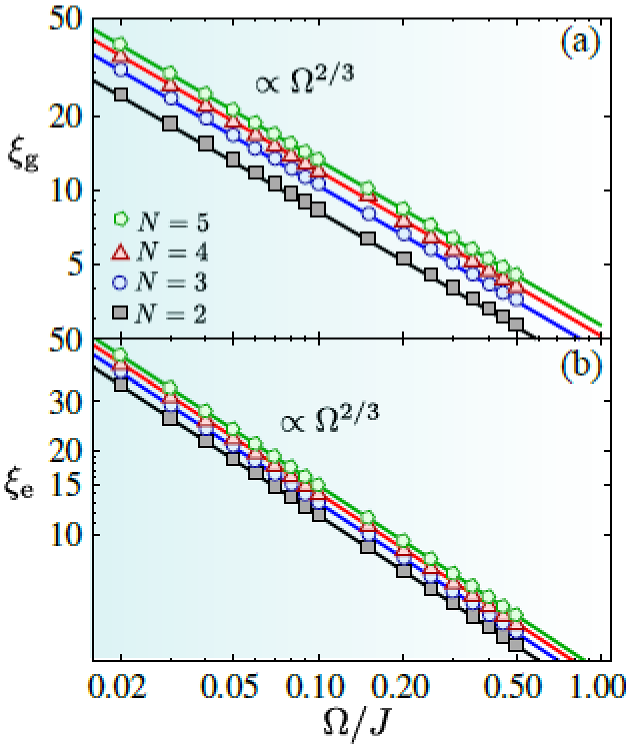
band structure

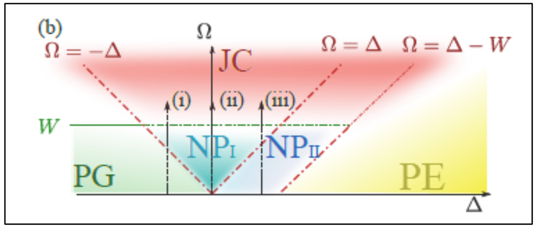


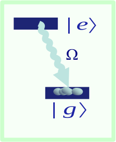
IMPURITY IN A 1D WAVEGUIDE
MULTI-PHOTON BOUND STATE

NUMERICAL CERTIFICATION:

Localization length

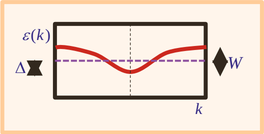






atomic structure

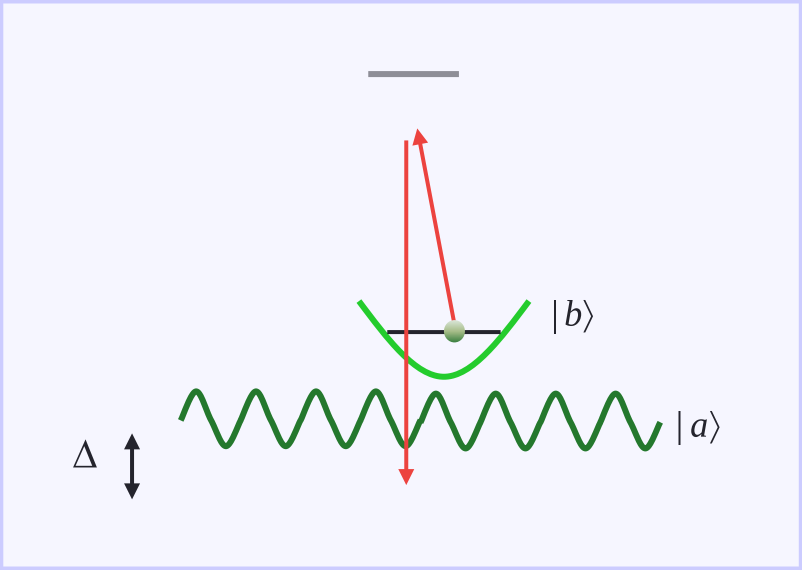
band structure

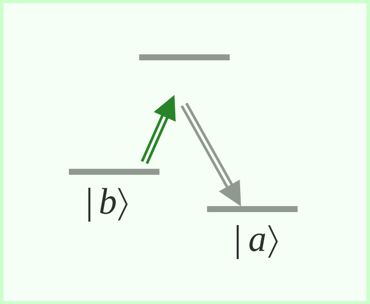


IMPURITY IN A 1D WAVEGUIDE

IMPLEMENTATION

ATOMS IN OPTICAL LATTICES:





Hamiltonian: $H = \Delta |1\rangle_b \langle 1| + \sum_k \varepsilon_k a_k^\dagger a_k + \sum_k g_k (a_k^\dagger |0\rangle_b \langle 1| + h.c.)$

- Creation by adiabatic evolution
- Study consquences in scattering, etc
- Multi-impurities: Effective Hamiltonians

3. MULTI-PHOTON SOURCES

A. Gonzalez-Tudela, V. Paulisch, D. Chang, H. J. Kimble, JIC, PRL 115, 163603 (2015)
A. Gonzalez-Tudela, V. Paulisch, H. J. Kimble, JIC, arxiv:1602.????


Alex
Gonz.-T.


Vanesa
Paulisch

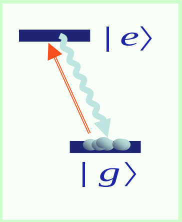

Darrick
Chang


Jeff
Kimble

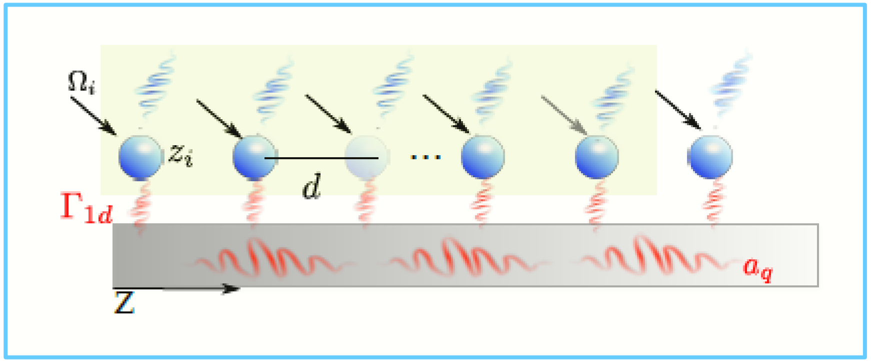
MULTI-PHOTON STATES

MULTI-PHOTON STATES

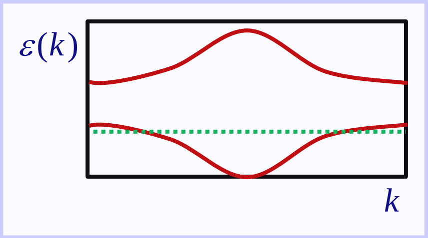
MULTI-PHOTON STATES
ATOMS IN 1D WAVEGUIDES



atomic structure



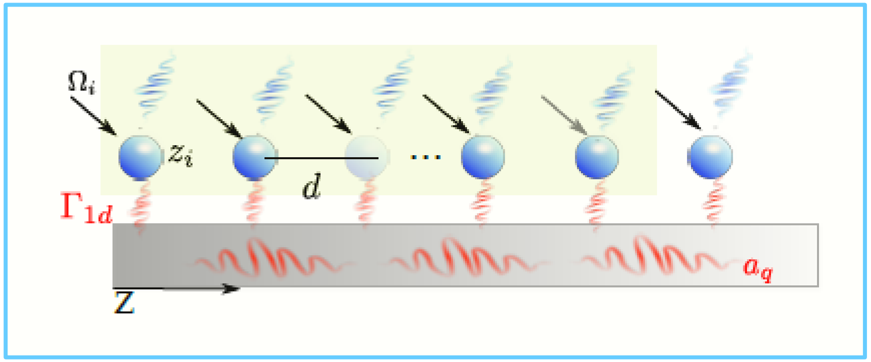
band structure

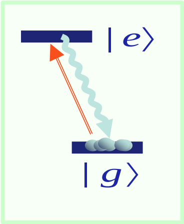


- Large Purcell effects: $P_{1d} = \dfrac{\Gamma_{1d}}{\Gamma_{out}} >> 1$
- Infidelity: $I = \dfrac{m}{P_{1d}}$
- Complex multi-mode structure

IDEA: use collective effects + heralding

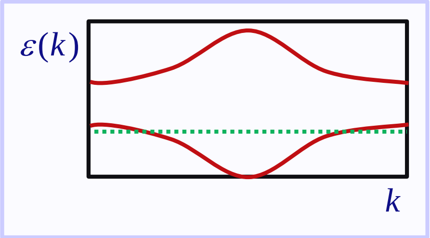
MULTI-PHOTON STATES
ATOMS IN 1D WAVEGUIDES





atomic
structure

band structure

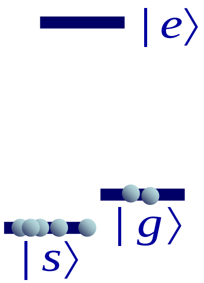


- Large collective effects: $\Gamma_{eff} = N\Gamma_{1d}$
- They can be used to:
  - map atomic to photonic states
  - *single mode* states

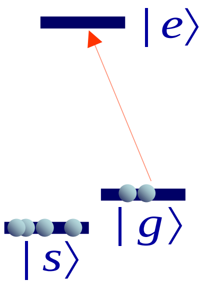
D. Porras, JIC, Phys. Rev. A **78**, 053816 (2008)

MULTI-PHOTON STATES
PROCEDURE

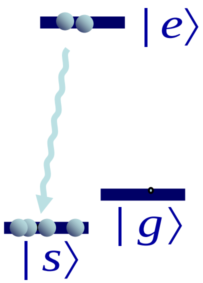
1. LOADING



2. TRIGGERING



3. EMISSION

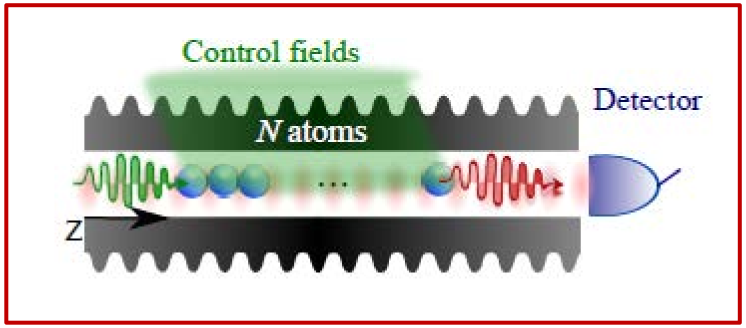


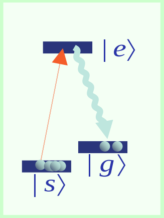
How to generate the atomic (entangled states)?

MULTI-PHOTON STATES
SIMPLE SCHEME

SCHEME 1:

Extension of Duan, Lukin, JIC, Zoller, Nature 414, 413 (2001)





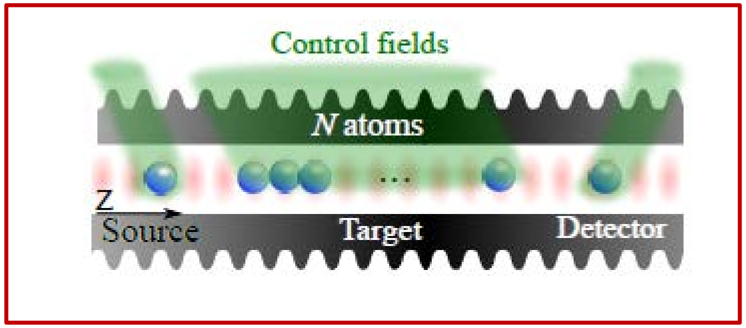
atomic structure

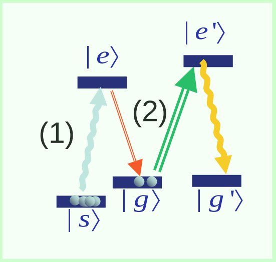
- Infidelity: $I_1 \approx \varepsilon^2$
- Success probability: $p_1 \approx \frac{1}{\varepsilon^2}$ ➡ $p_m \approx \left(\frac{1}{\varepsilon^2}\right)^m$

- Two-photon excitations produce errors.
- Zero-photon excitation gives low probability.

MULTI-PHOTON STATES
SCHEME with SOURCE AND DETECTOR

SCHEME 2:





atomic structure

- High detection efficiency: Detect atoms, not photons
- If the detector clicks, the process had no error

$$I_1 = 0$$

$$p_1 \approx 1 - \frac{1}{P_{1d}} \Rightarrow p_m \approx \left(1 - \frac{1}{P_{1d}}\right)^m$$

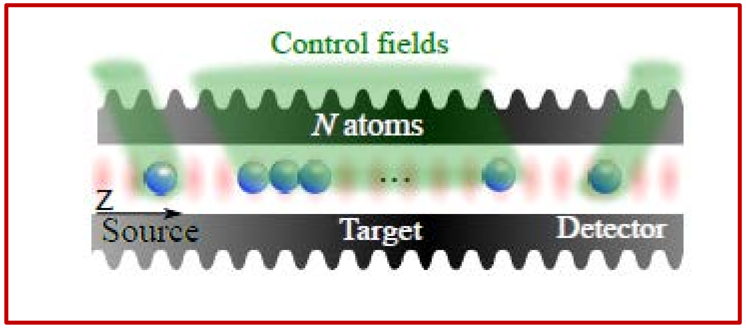
- Purcel factor limits the number of photons

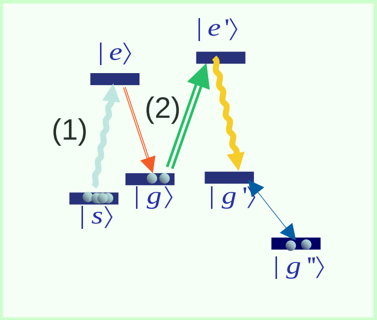
Can we get a polynomical scaling in m?

MULTI-PHOTON STATES

SCHEME with ADDITIONAL LEVELS

SCHEME 3:





atomic structure

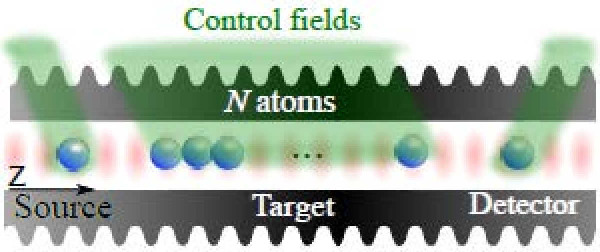
- Internal levels to store the excitations
- Atomic measurements (only) to merge excitations

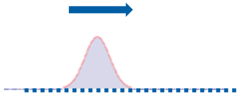
$$I_m \approx \frac{\mathrm{poly}(m)}{NP_{1d}}$$

$$p_m \approx \frac{1}{\mathrm{poly}(m)}$$

MULTI-PHOTON STATES
SUMMARY

MULTIPHOTON GENERATION BY COLLECTIVE EFFECTS





$$|\Psi_m\rangle = \sum_{n=0}^{m} c_n |n\rangle$$

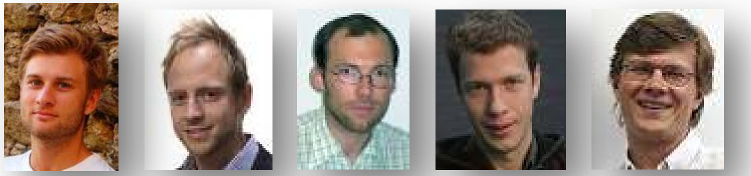
	Collective Zeno	Heralded source +dectector	Heralded merging
Infidelity:	$I_m \approx m^2/\sqrt{P_{1d}}$	$I_m = 1$	$I_m \approx m^2/NP_{1d}$
Probability:	$p_m = 1$	$p_m \prec 1/\exp(m)$	$p_m \prec 1/\text{poly}(m)$

P_{1d} :Purcel factor
N :Number of atoms
m : Number of photons

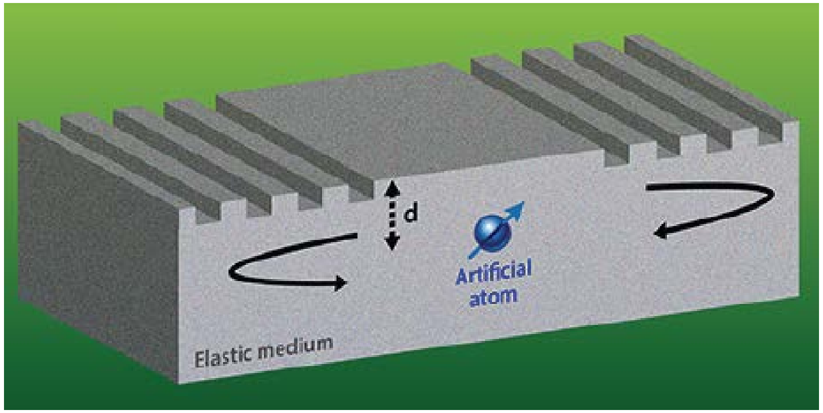
4. OTHER PROBLEMS

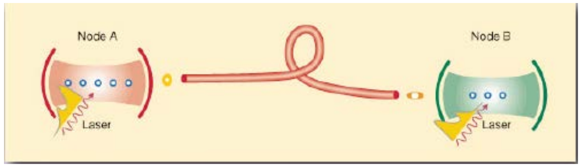
OTHER PROBLEMS
SURFACE ACOUSTIC WAVES

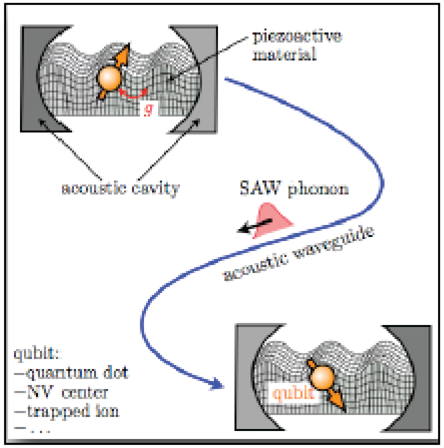
M. Schütz, E. Kessler, G. Giedke, L. Vandersypen, M. Lukin, JIC, PRX **5**, 031031 (2015)



Martin Schütz | Erik Kessler | Geza Giedke | Lieven Vanders. | Mikhail Lukin

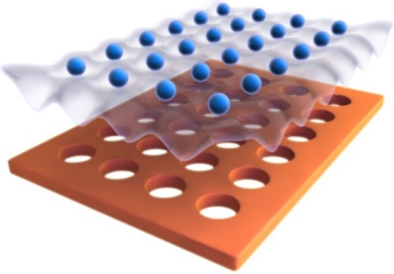




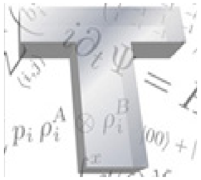


QUANTUM OPTICS IN WAVEGUIDES
SUMMARY & OUTLOOK

- Challenging experiments
- New regimes:
    - Large Purcel effects
    - Collective phenomena
    - Bound states
- Connection to cold atoms
- Quantum simulations
    - Multi-impurities



→ Jeff‘s talk




MPQ