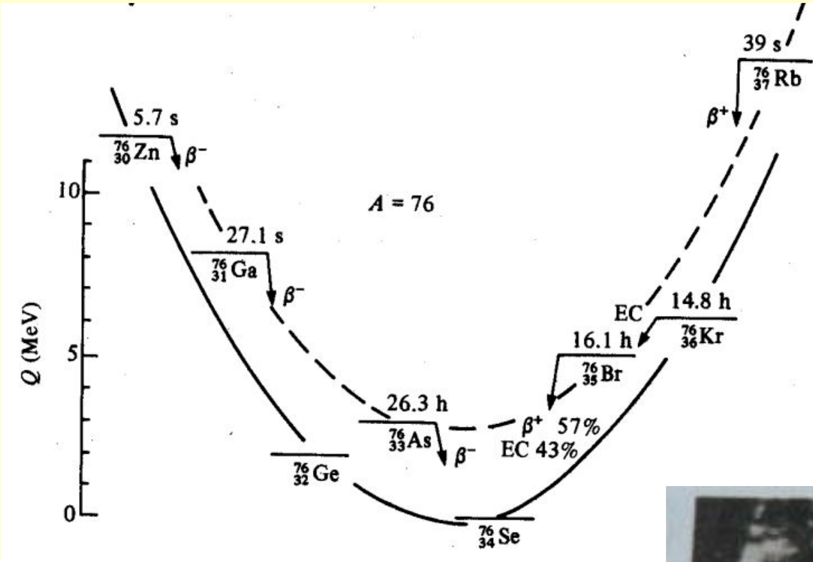
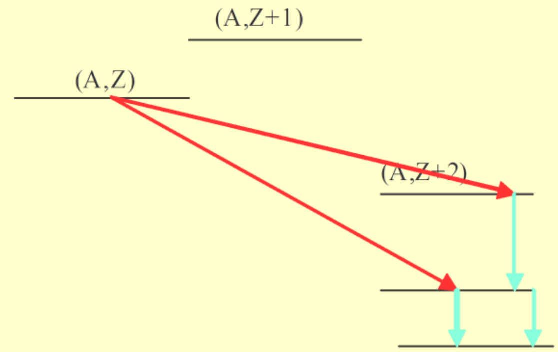
Double beta decay, neutrino and Majorana theory

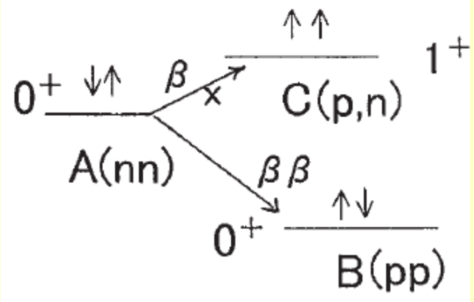
1. ***The birth of double beta decay***
2. ***From low energy nuclear physic to neutrino mass and properties***
3. ***Experimental approaches and difficulties***
4. ***Present experimental status***
5. ***The nightmare of nuclear matrix elements***
6. ***The future***
7. ***News on Nuclear Matrix Elements***
8. ***Majorana theory in solid states***

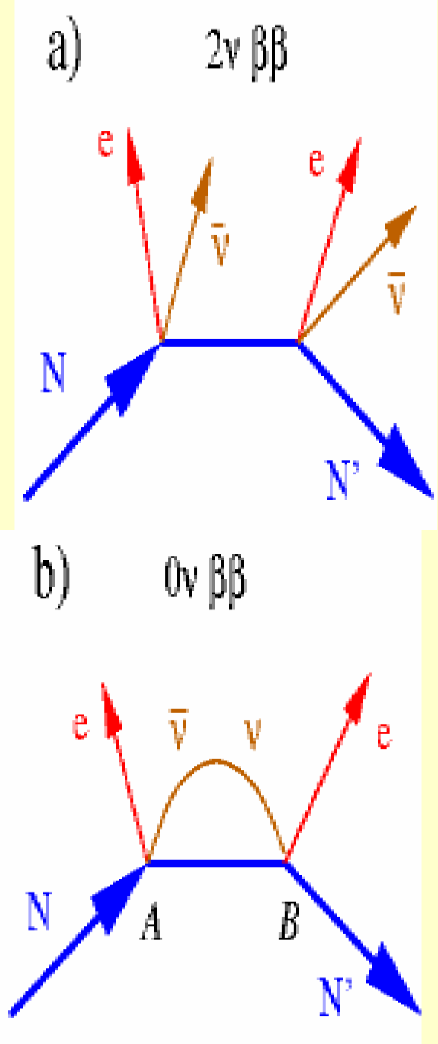
10. ***Conclusions***


5.7 s
$^{76}_{30}$Zn
β⁻
27.1 s
$^{76}_{31}$Ga
β⁻
A = 76
26.3 h
$^{76}_{33}$As
β⁻
$^{76}_{32}$Ge
$^{76}_{34}$Se
β⁺ 57%
EC 43%
16.1 h
$^{76}_{35}$Br
EC
14.8 h
$^{76}_{36}$Kr
β⁺
39 s
$^{76}_{37}$Rb
Q (MeV)
10
5
0


(A,Z+1)
(A,Z)
(A,Z+2)

Double beta decays





1935 M.Goeppert-Mayer, P.R. 48 (1935) 512 T>10^{20}

Majorana, Nuovo Cimento 14 (1937) 171
Symmetric Theory of Electron and Positron

G. Racah, Nuovo Cimento 14 (1937) 322

W.H. Furry Lepton number nonconservation

1967: ^{130}Te, Geochemical Ogata and Takaoka,
Kirsten et

1987: ^{82}Se, Direct counting Moe et al

.

1987 : Source=detector ^{48}Ca, ^{76}Ge

1. $(A,Z) \Rightarrow (A,Z+2) + 2e^- + 2\bar{\nu}_e$

Two neutrino double beta decay Allowed by the standard model
Found in eleven nuclei to ground state and in two to excited state

2. $(A,Z) \Rightarrow (A,Z+2) + 2e^- + \chi$ (…2,3 χ)

Emission of a massless Goldston boson named Majoron (Gelmini and Roncadelli)

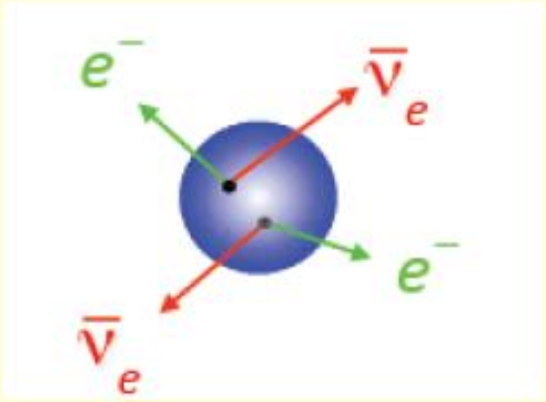
3. $(A,Z) \Rightarrow (A,Z+2) + 2e^-$

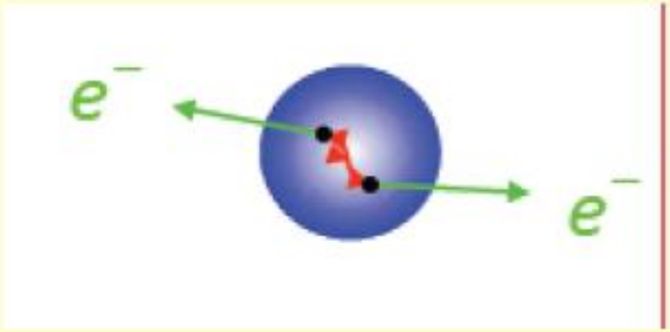
Neutrinoless double beta decay. The two electrons share the total transition energy $E_1 + E_2 \Rightarrow \Delta E \Rightarrow$ a peak should appear in the sum spectrum of the two electrons

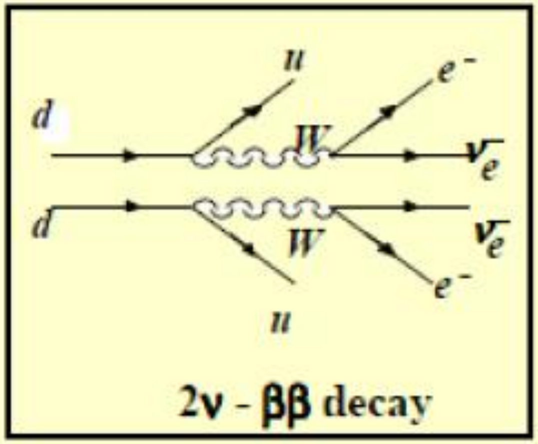
Other possible "$\Delta L=2$" decays

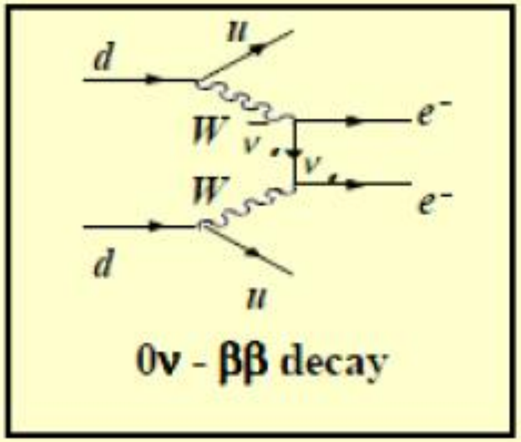
- **Double positron decay => $\beta^+ \beta^+$**
- **Positron decay + Electron Capture => EC- β^+**
- **Double electron capture => EC-EC**

Two neutrino and neutrinoless double beta decay









Measurement of the CP phase in the 0ν ββ−decay

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} \; |M^{0\nu}|^2 \; |\langle m_\nu \rangle|^2 / m_e^2$$

Experimental challenges

Many types of detectors

- Cloud chambers
- Emulsions?
- Bubble chambers (yes for Dark Matter)
- Scintillators
- Double state liquids
- Time projectrion chambers
- Imagining devices
- Single atom counters ^{136}Xe => ^{136}Ba
- Bolometers
- Hybrid devices

How to search for ββ decay

Geochemical experiments

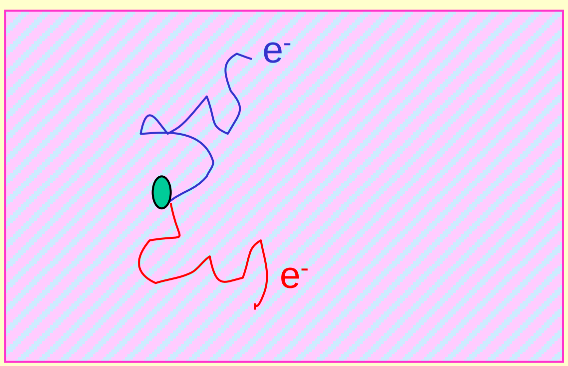
^{82}Se = > ^{82}Kr, ^{96}Zr = > ^{96}Mo, ^{128}Te = > ^{128}Xe , ^{130}Te = > ^{130}Te

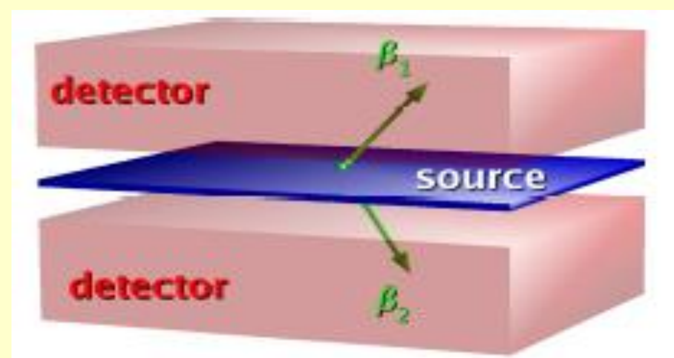
Radiochemical experiments ^{238}U = > ^{238}Pu (non confirmed)

Direct experiments

Source = detector

Source ≠ detectors



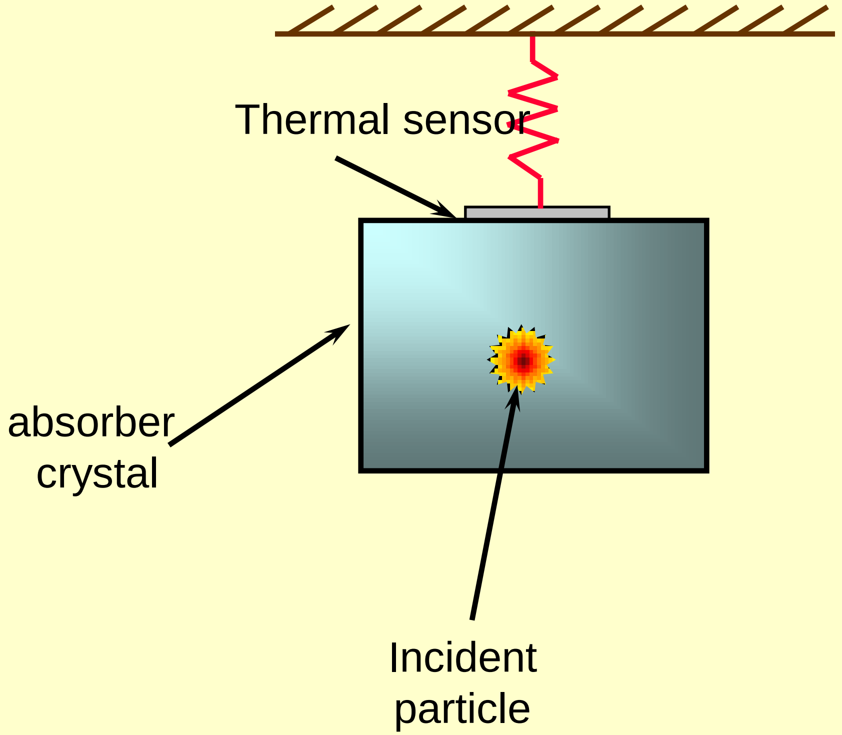


Thermal detectors

$$\Delta T = \frac{Q}{C_V}$$

$$C_V = 1944 \frac{V}{V_m} \left(\frac{T}{\Theta}\right)^3 \text{ J/K}$$

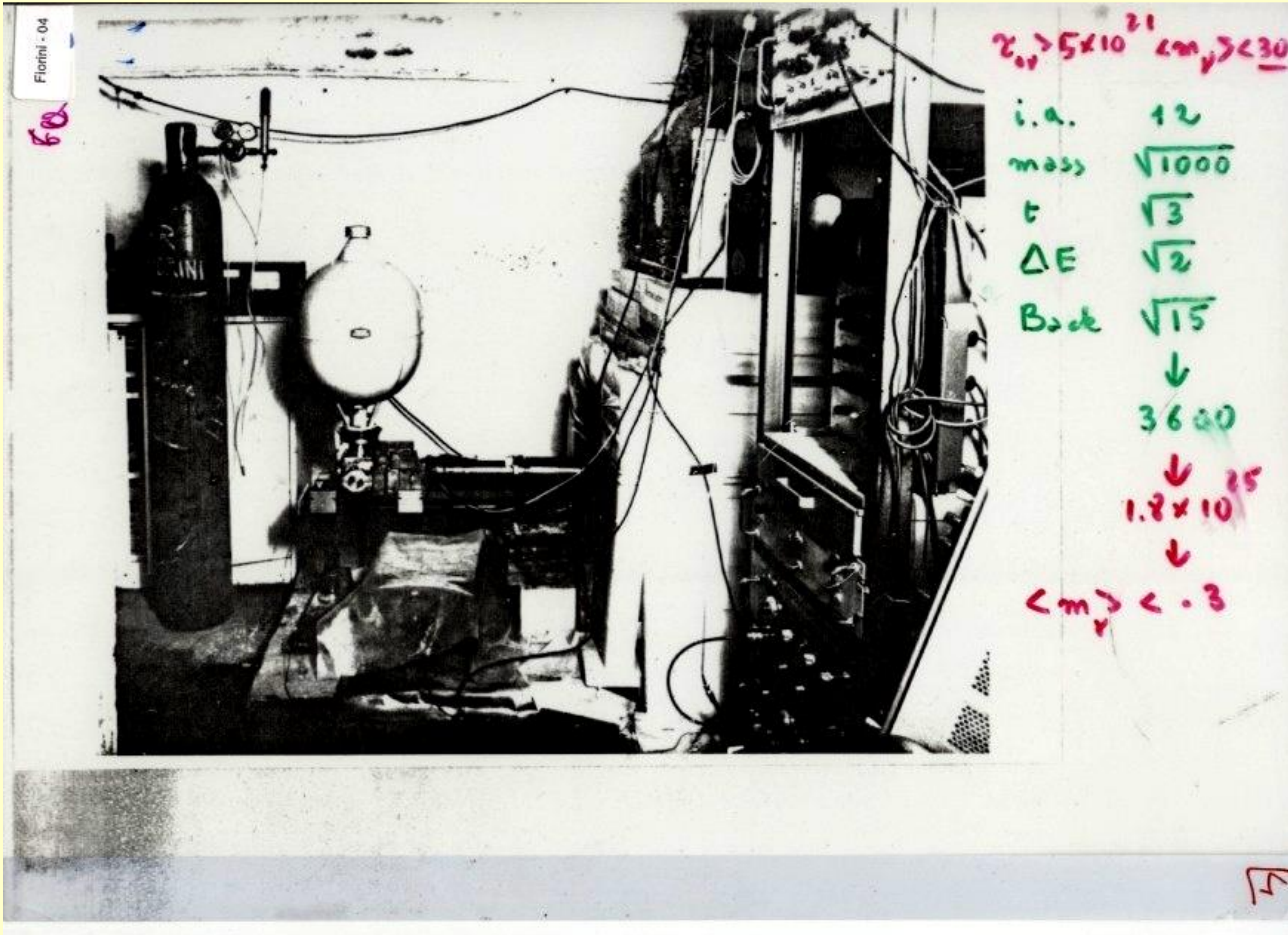
$$\Delta E_{RMS} \propto \sqrt{\frac{k_B T_0^2 C_0}{\alpha}}$$



Energy resolution

<1 eV	**~ 1eV**	@ 6 keV
~10 eV	**~keV**	@ 2 MeV

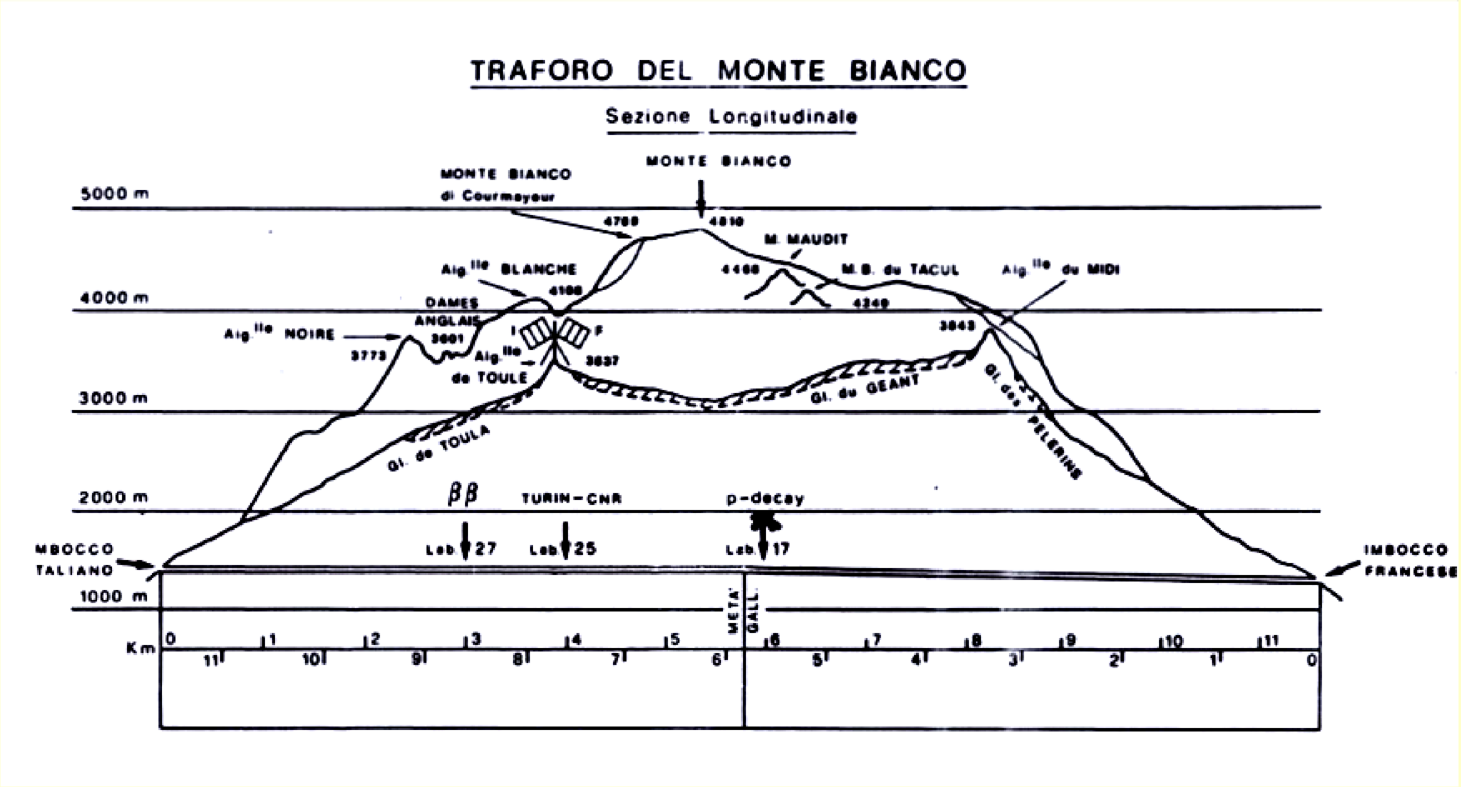
A first ββ experiment

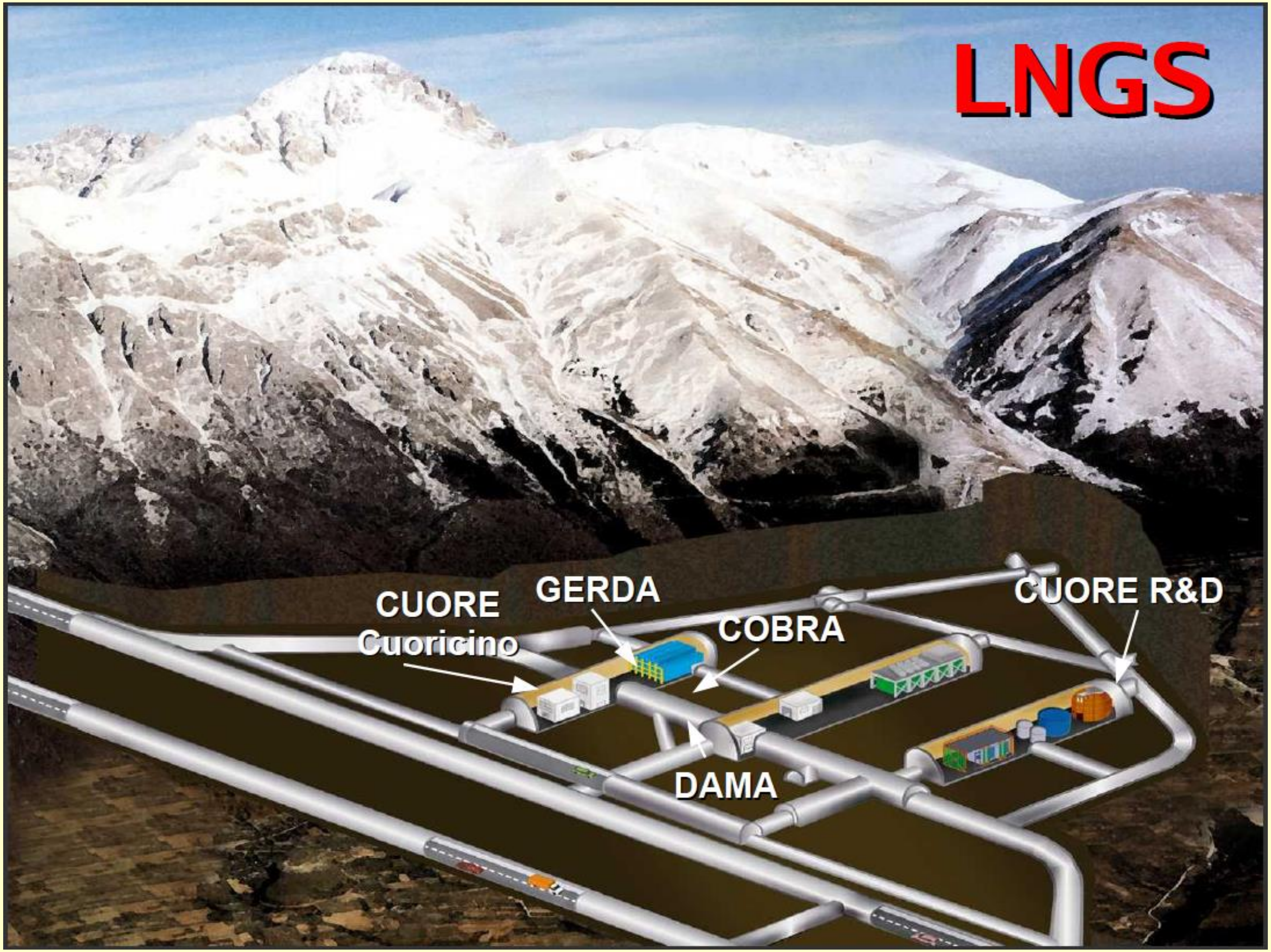


RARE events

One of the main problem => **background**

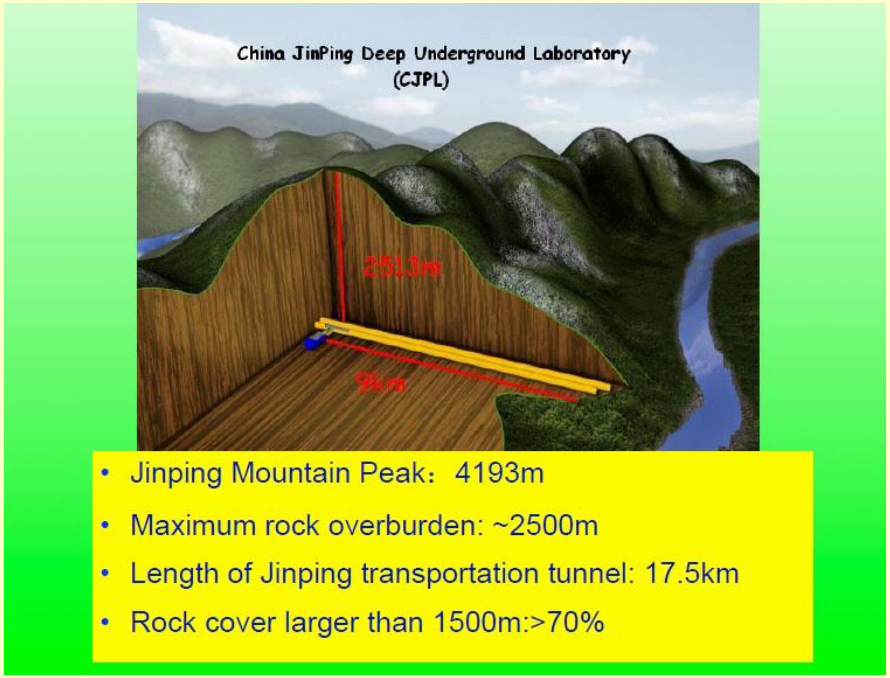
Cosmic rays => underground physics

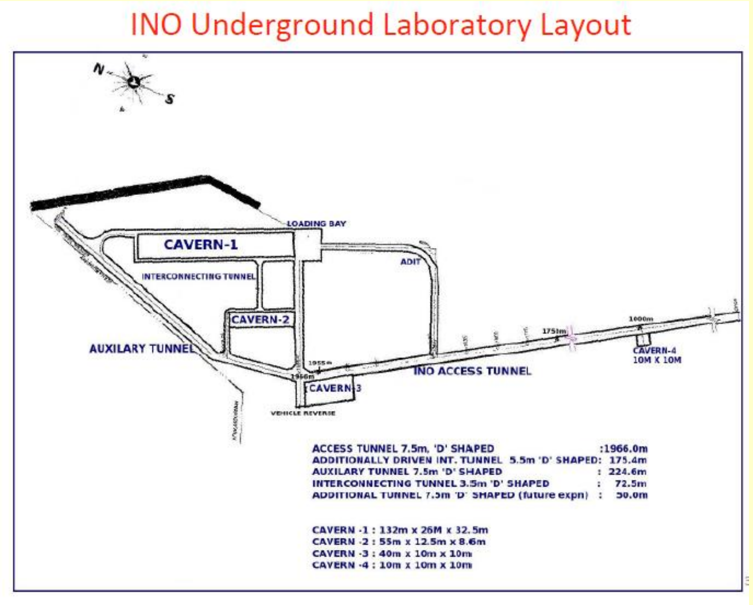



LNGS
CUORE
Cuoricino
GERDA
COBRA
CUORE R&D
DAMA

The Jin Ping Laboratory (700 m.w.e)

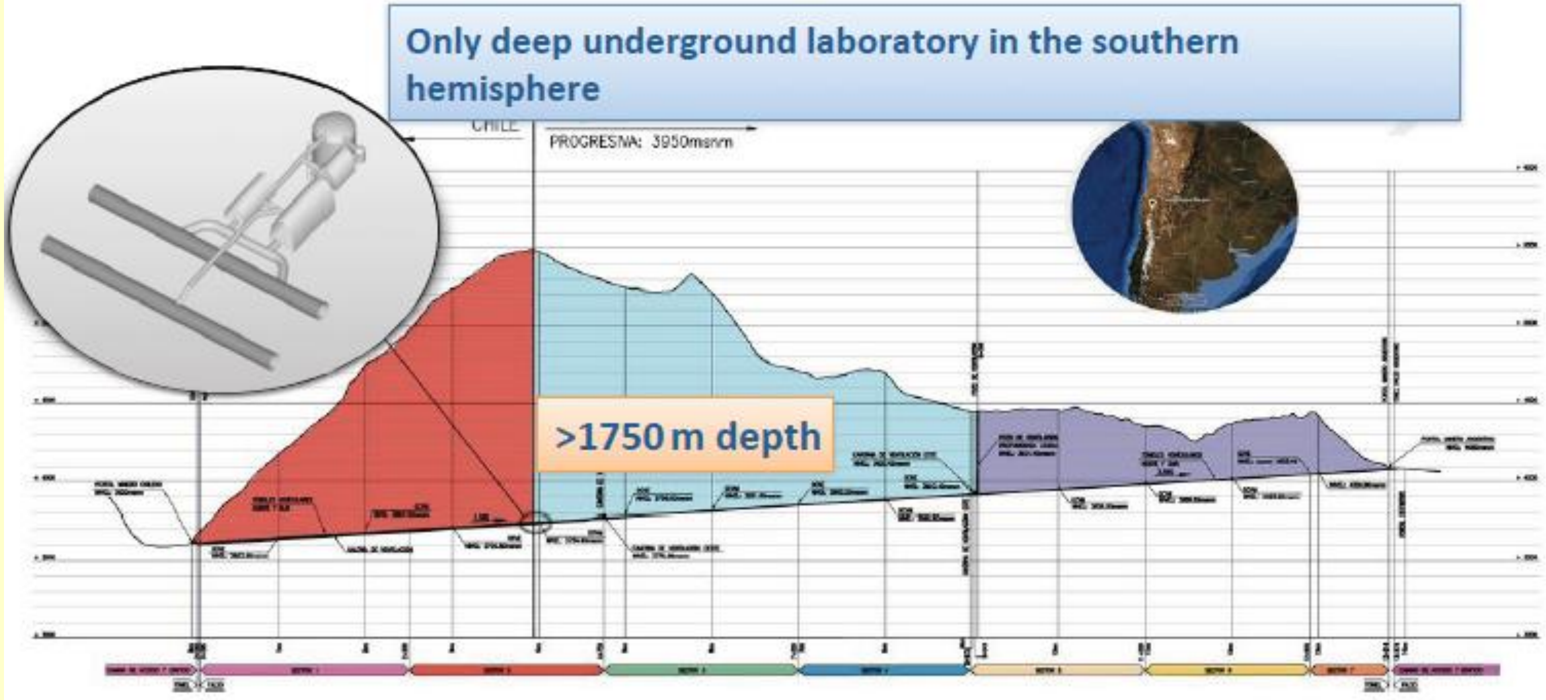
INO



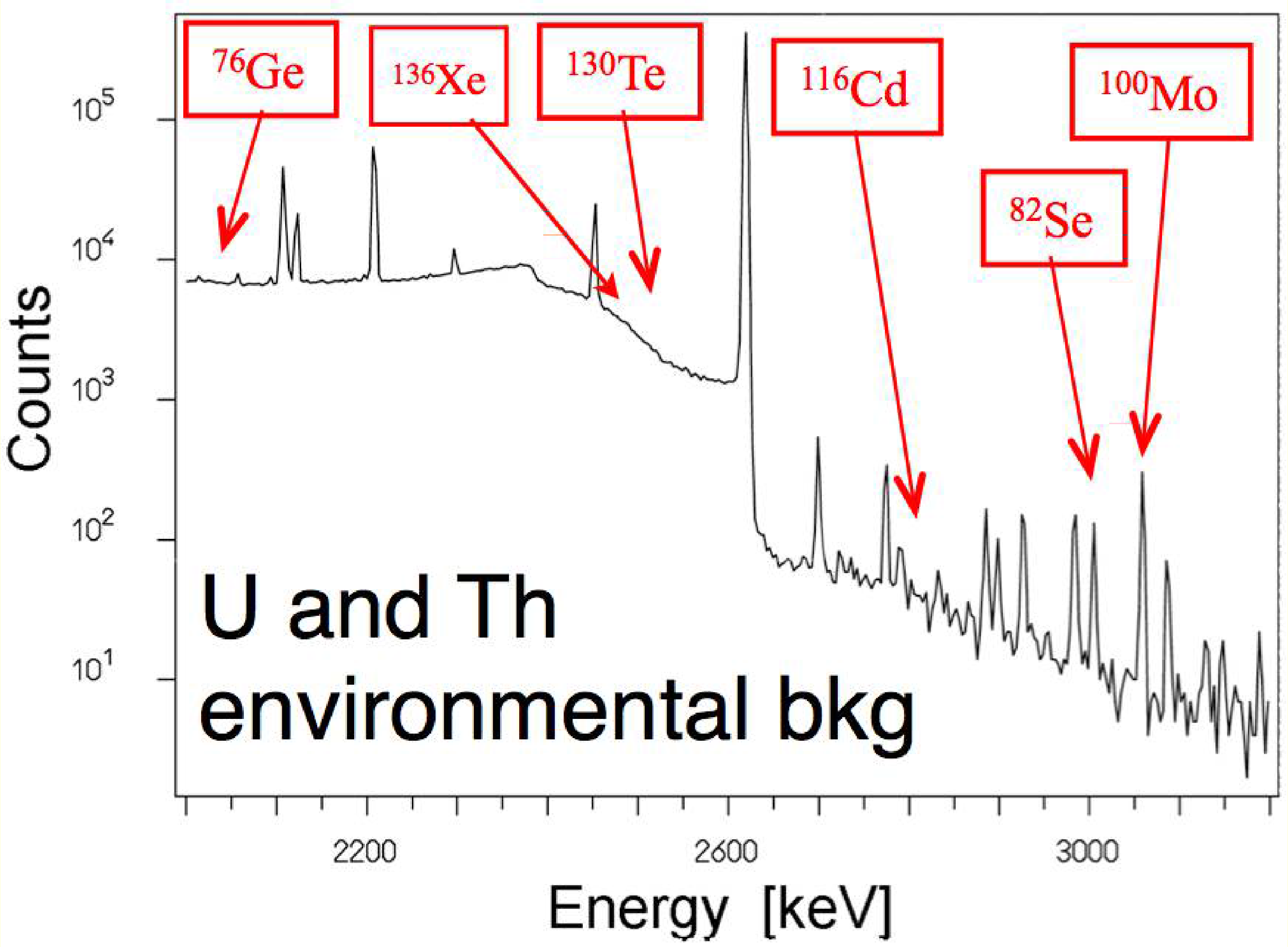


ANDES
The Agua Negra deep underground laboratory

- Agua Negra tunnel between Argentina and Chile, linking MERCOSUR to Asia
- Possible laboratory location as deep (or deeper) than Modane
- Construction planned 2012-2018 (tunnel opening)
- Horizontal access, size of ~4 000 m^2 and ~65 000 m^3 in 5 halls and pits



Radioactivity



CUORICINO

The second mystery of Ettore Majorana

Teoria simmetrica dell'elettrone e del positrone

NOTA DI ETTORE MAJORANA

"Il Nuovo Cimento", vol. 14, 1937, pp. 171-184.

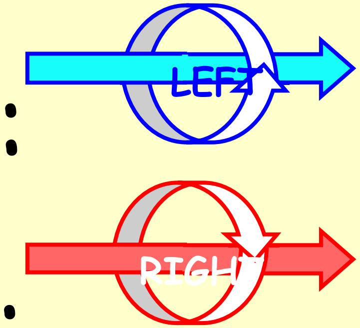
Sunto. — Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di DIRAC ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; né a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di "antiparticelle" corrispondenti ai "vuoti" di energia negativa.

In 1937, Majorana theoretically showed that the conclusions of the theory of β decay remain unchanged under the assumption of the existence of only onetype of the neutrino having no antiparticle (i.e., $\nu \equiv \bar{\nu}$).

Dirac or Majorana neutrino?

ν:

$\bar{\nu}$:



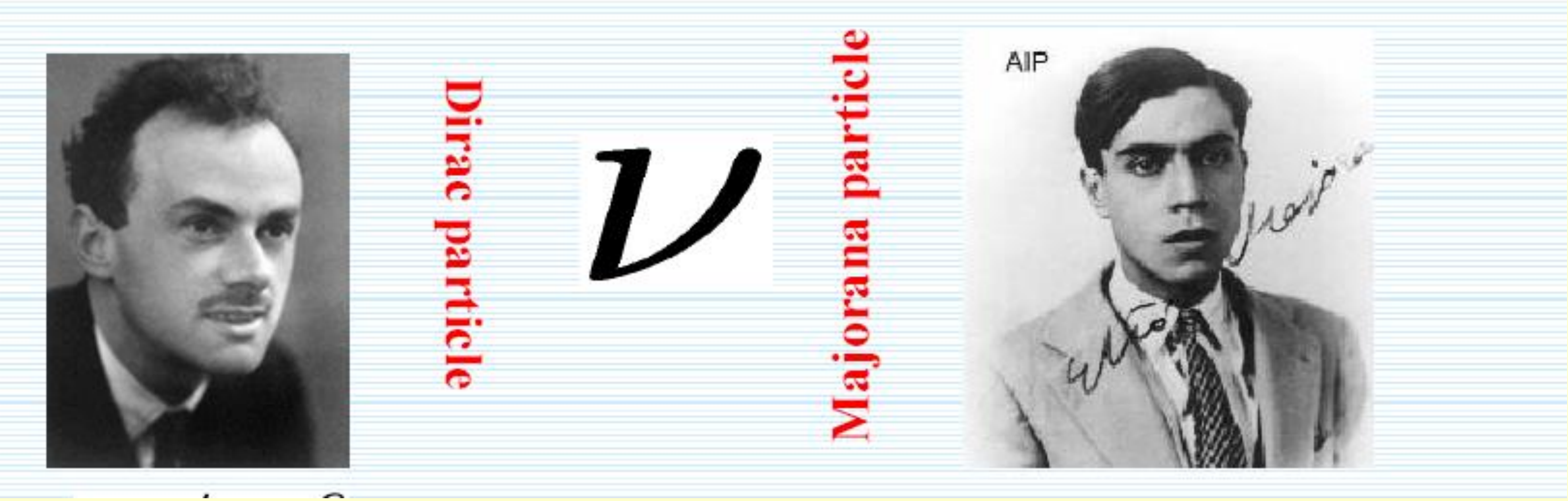
$\nu \neq \bar{\nu}$

$\nu = \bar{\nu}$

→ →

<= =>

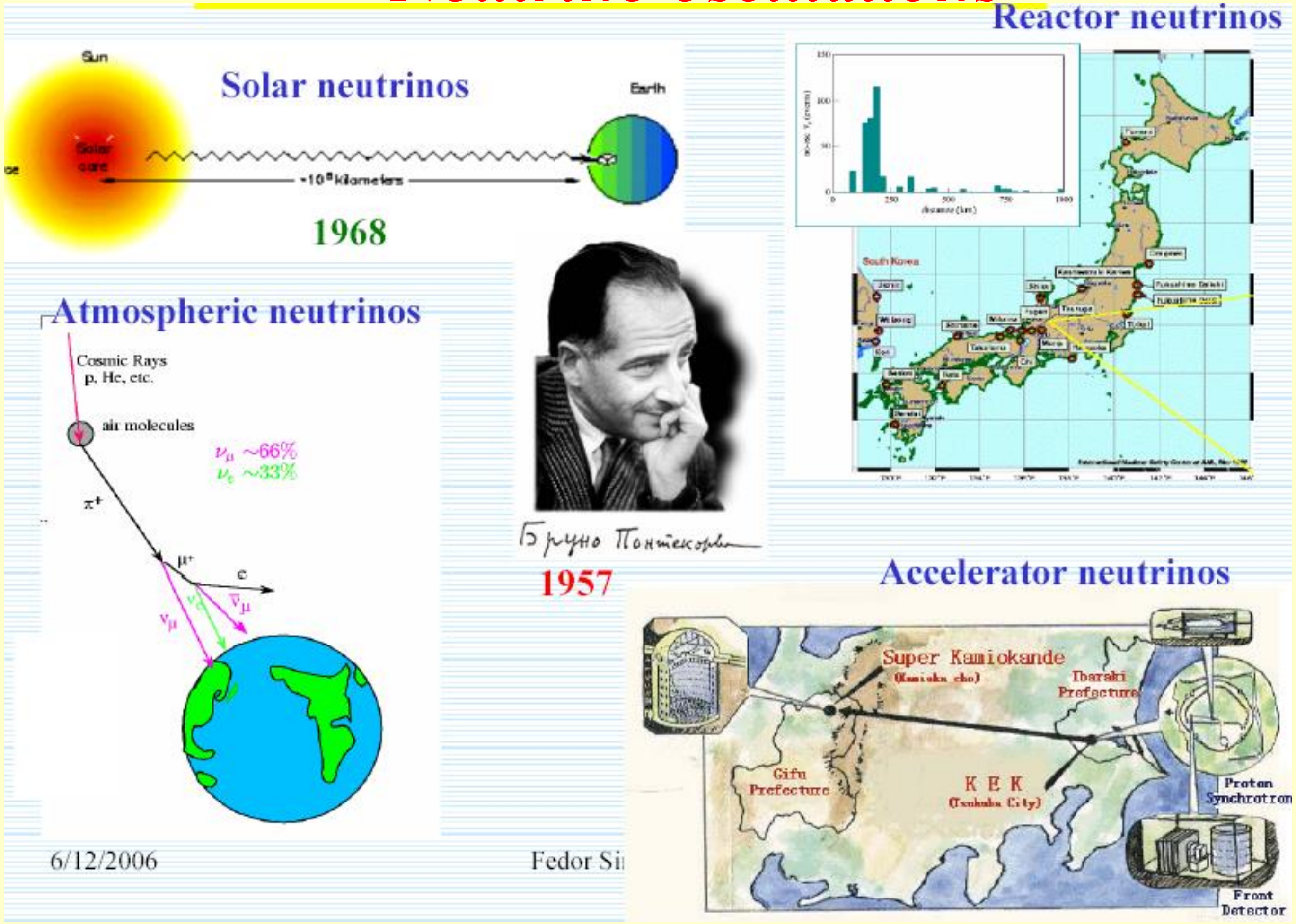
Majorana
=>1937



Chi l'ha visto?

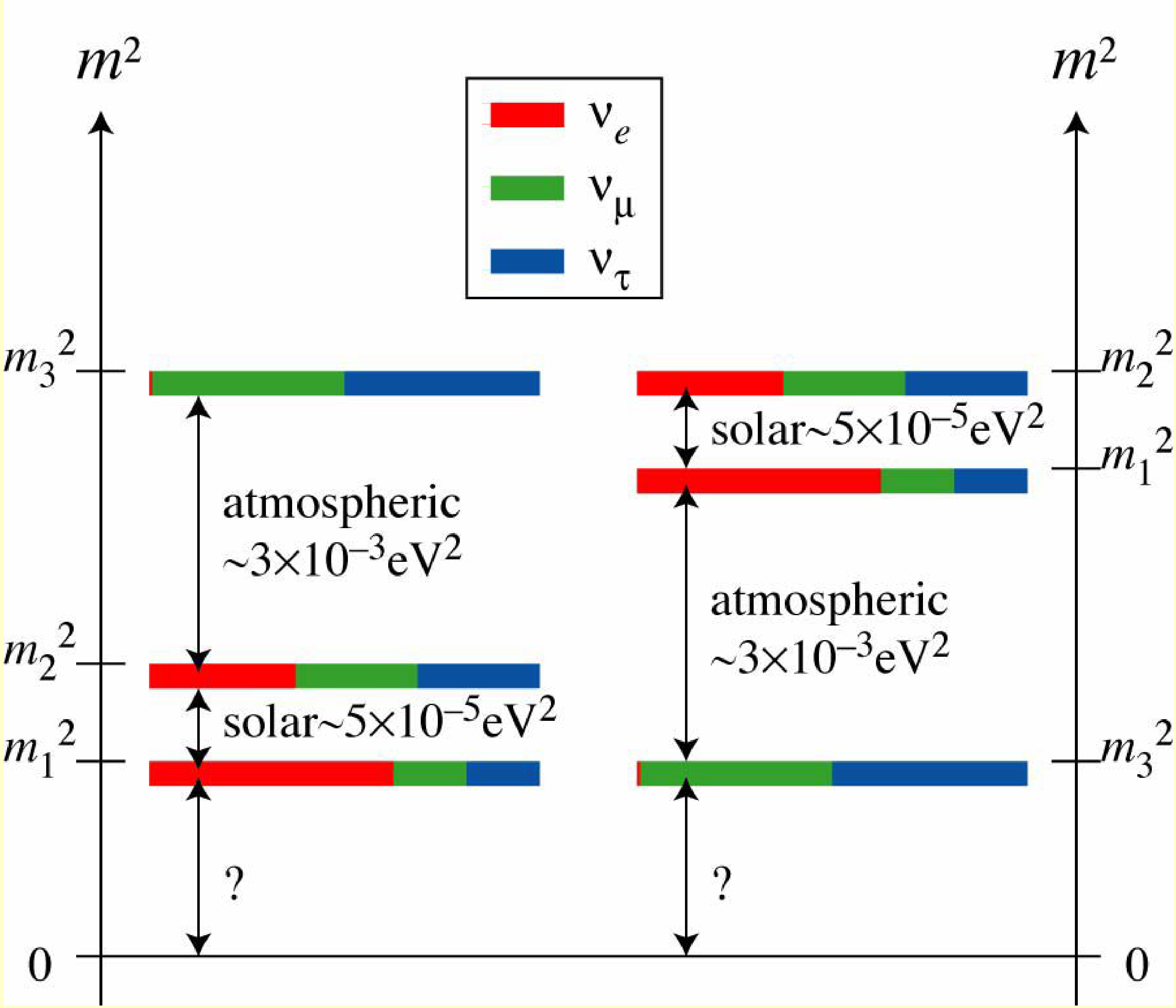
Ettore Majorana, ordinario di fisica teorica all'Università di Napoli, è misteriosamente scomparso dagli ultimi di marzo. Di anni 31, alto metri 1,70, snello, con capelli neri, occhi scuri, una lunga cicatrice sul dorso di una mano. Chi ne sapesse qualcosa è pregato di scrivere al R. P. E. Marianecci, Viale Regina Margherita 66 - Roma.

Neutrino oscillations



Sterile neutrinos

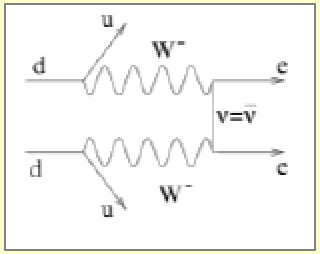
LSND,Miniboon,Gallex,reactors)



m^2
m^2
ν_e
ν_μ
ν_τ
$m_3{}^2$
$m_2{}^2$
$m_1{}^2$
atmospheric
$\sim 3\times10^{-3}\text{eV}^2$
solar$\sim 5\times10^{-5}\text{eV}^2$
?
0
$m_2{}^2$
$m_1{}^2$
$m_3{}^2$
solar$\sim 5\times10^{-5}\text{eV}^2$
atmospheric
$\sim 3\times10^{-3}\text{eV}^2$
?
0

0ν-DBD and neutrino physics

how **0ν-DBD** is connected to neutrino mixing matrix and masses in case of process induced by mass mechanism



neutrinoless Double Beta Decay **rate**

Phase space

Nuclear matrix elements

Effective **Majorana mass**

$$1/\tau = G(Q,Z)\,|M_{nucl}|^2 \langle M_{\beta\beta}\rangle^2$$

what the experimentalists try to measure

what the nuclear theorists try to calculate

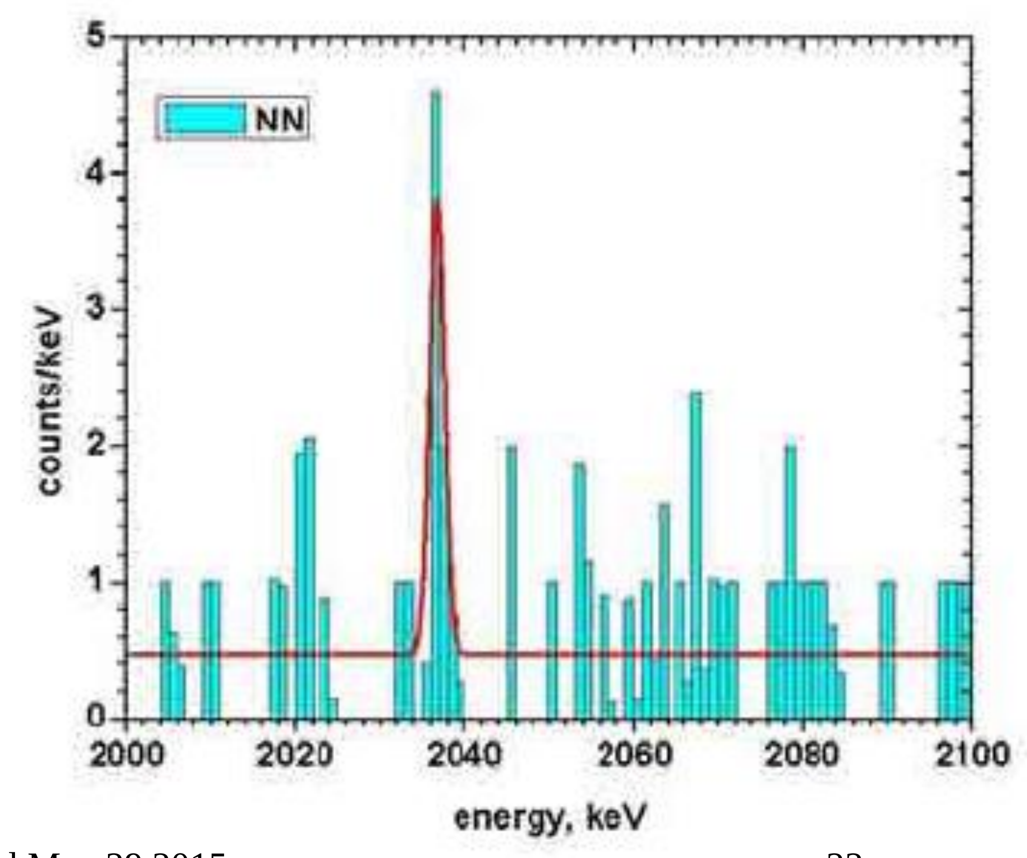
parameter containing the neutrino physics

$$\langle M_{\beta\beta}\rangle = \left| |U_{e1}|^2 M_1 + e^{i\alpha_1} |U_{e2}|^2 M_2 + e^{i\alpha_2} |U_{e3}|^2 M_3 \right|$$

Bracking news => 0νββ in ^{76}Ge

$$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25}\ \mathrm{y}.$$

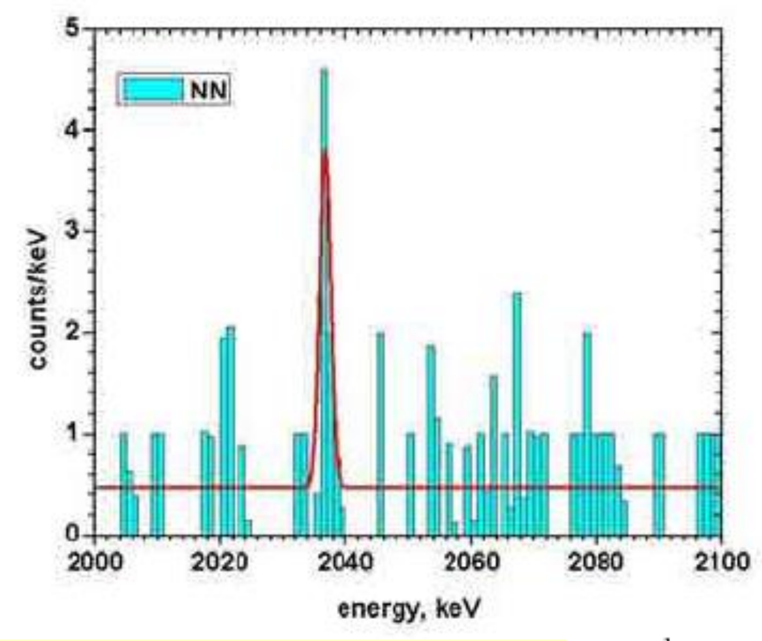
$\langle m_\nu \rangle \sim 0.34$eV



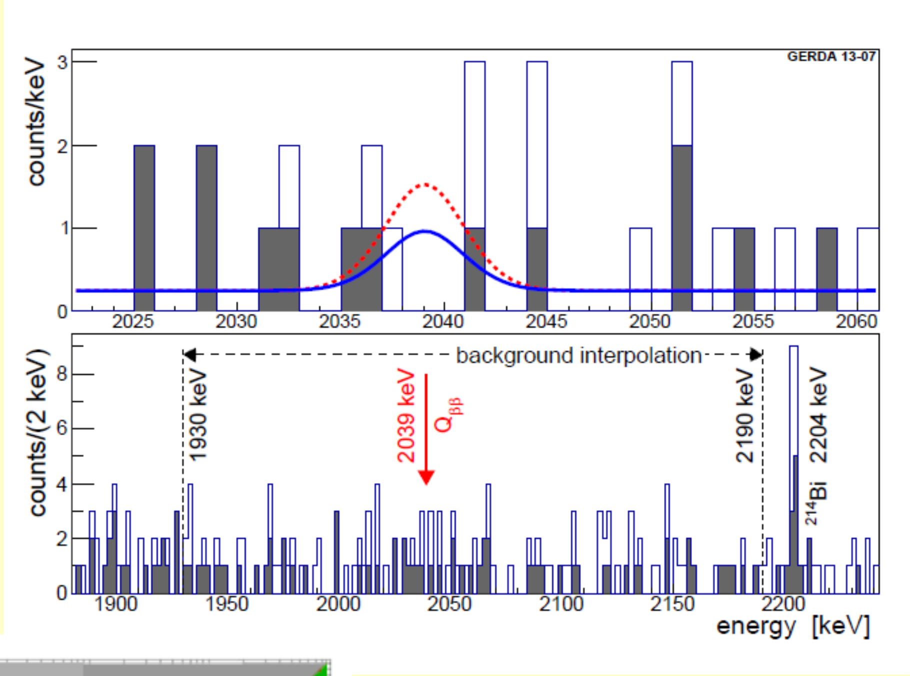
Present results on neutrinoess DBD

Isotope	Technique	$\tau^{0\nu}_{1/2}$ (y)	$<m_{\beta\beta}>$ eV	NEW
^{48}Ca	CaF_2 scint	$>1.4\times10^{22}$	<7-45	28 ± 180
^{76}Ge (HM)	Ge diode	$>1.9\times10^{25}$	<(0.3-1.27)	1.4 ± 6
^{76}Ge (IGEX)	Ge diode	$>1.6\times10^{25}$	<(0.33-1.35)	1.6 ± 6.3
^{76}Ge (Klapdor 2004)	Ge diode	1.2×10^{25}	.38	1.8
^{76}Ge (Klapdor 2006)	Ge diode	2.2×10^{25}	.28	1.3
^{76}Ge (GERDA I)	Ge diode	$>2.1\times10^{25}$	<(.29-1.1)	.1.4 ± 6.7
^{76}Ge (GERDA+HM+IGEX)	Ge diode	$>3\times10^{25}$	<(.25-.98)	1.3 ± 4.3
^{82}Se	Foil&track	$>.6\times10^{24}$	<(0.89-2.)	5 ± 10
^{96}Zr	Foil&track	$>9.2\times10^{21}$	<(7.2-19.5)	37 ± 100
^{100}Mo	Foil&track	$>1.1\times10^{24}$	<(0.31-.79)	1.6 ± 4.2
^{116}Cd	Scintillator	$>1.7\times10^{23}$	<1.7	9.2
^{128}Te	Geochem	$>7.7\times10^{24}$	<(1.1-1.35)	6.3 ± 77
^{130}Te	Bolometer	$>2.8\times10^{24}$	<(0.3-.7)	1.7± 4
^{136}Xe	EXO	$>1.6\times10^{25}$	<140-380	± 180
^{136}Xe	Kamland Zen	$>1.9\times10^{25}$	<128-349	.84± 2.3
^{136}Xe	EXO+Kamzen		<120-250	.72 ± 2.1
^{150}Nd	Foil TPC	$>1.8\times10^{22}$		.71 ± 1.5

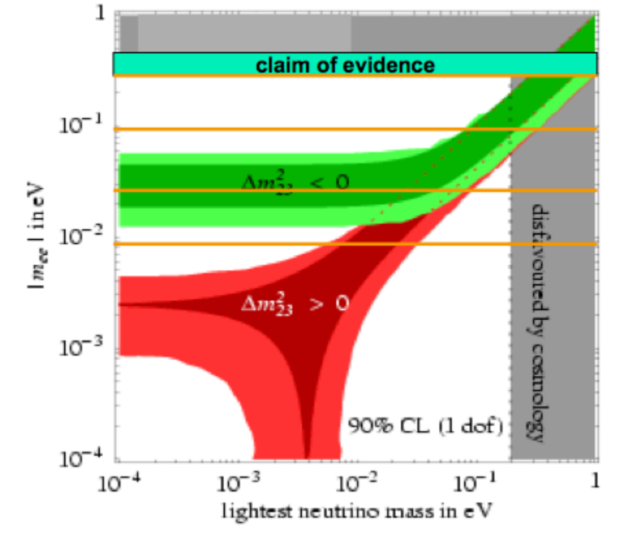
BUT => GERDA



NN
counts/keV
energy, keV



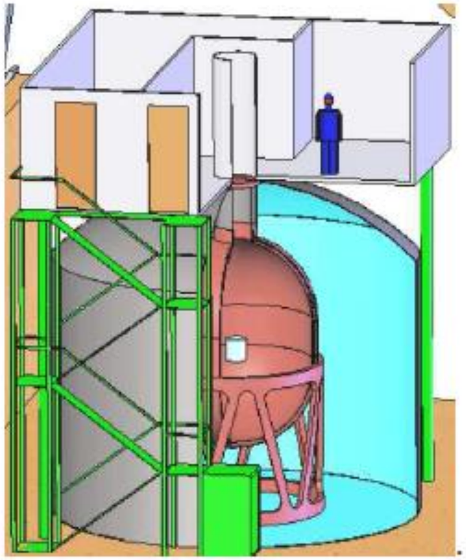
GERDA 13-07
counts/keV
counts/(2 keV)
background interpolation
1930 keV
2039 keV
$Q_{\beta\beta}$
2190 keV
2204 keV
^{214}Bi
energy [keV]

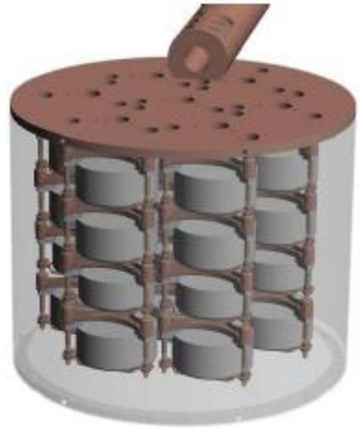


claim of evidence
$\Delta m^2_{23} < 0$
$\Delta m^2_{23} > 0$
disfavoured by cosmology
90% CL (1 dof)
$|m_{ee}|$ in eV
lightest neutrino mass in eV

Future experiments on DBD

Experiment	Isotope	Mass	Technique	Present Status	Location
AMoRE[89,90]	^{100}Mo	50 kg	$CaMoO_4$ scint. bolometer crystals	Development	Yangyang
CANDLES[91]	^{48}Ca	0.35 kg	CaF_2 scint. crystals	Prototype	Kamioka
CARVEL[92]	^{48}Ca	1 ton	CaF_2 scint. crystals	Development	Solotvina
COBRA[93]	^{116}Cd	183 kg	^{enr}Cd CZT semicond. det.	Prototype	Gran Sasso
CUORE-0[69]	^{130}Te	11 kg	TeO_2 bolometers	Construction - 2012	Gran Sasso
CUORE[69]	^{130}Te	203 kg	TeO_2 bolometers	Construction - 2013	Gran Sasso
DCBA[94]	^{150}Ne	20 kg	^{enr}Nd foils and tracking	Development	Kamioka
EXO-200[57]	^{136}Xe	160 kg	Liq. ^{enr}Xe TPC/scint.	Operating - 2011	WIPP
EXO[70]	^{136}Xe	1-10 t	Liq. ^{enr}Xe TPC/scint.	Proposal	SURF
GERDA[71]	^{76}Ge	≈35 kg	^{enr}Ge semicond. det.	Operating - 2011	Gran Sasso
GSO[95]	^{160}Gd	2 ton	Gd_2SiO_5:Ce crys. scint. in liq. scint.	Development	
KamLAND-Zen[96]	^{136}Xe	400 kg	^{enr}Xe disolved in liq. scint.	Operating - 2011	Kamioka
LUCIFER[97,98]	^{82}Se	18 kg	ZnSe scint. bolometer crystals	Development	Gran Sasso
MAJORANA[77,78,79]	^{76}Ge	26 kg	^{enr}Ge semicond. det.	Construction - 2013	SURF
MOON [99]	^{100}Mo	1 t	^{enr}Mofoils/scint.	Development	
SuperNEMO-Dem[87]	^{82}Se	7 kg	^{enr}Se foils/tracking	Construction - 2014	Fréjus
SuperNEMO[87]	^{82}Se	100 kg	^{enr}Se foils/tracking	Proposal - 2019	Fréjus
NEXT [82,83]	^{136}Xe	100 kg	gas TPC	Development - 2014	Canfranc
SNO+[84,85]	^{150}Nd	55 kg	Nd loaded liq. scint.	Construction - 2013	SNOLab





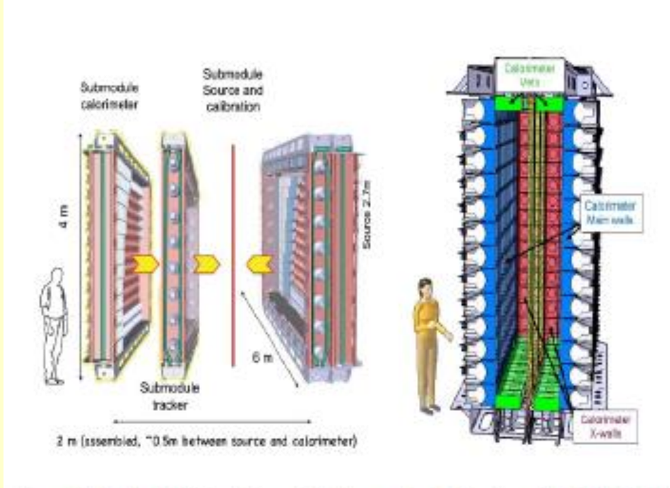
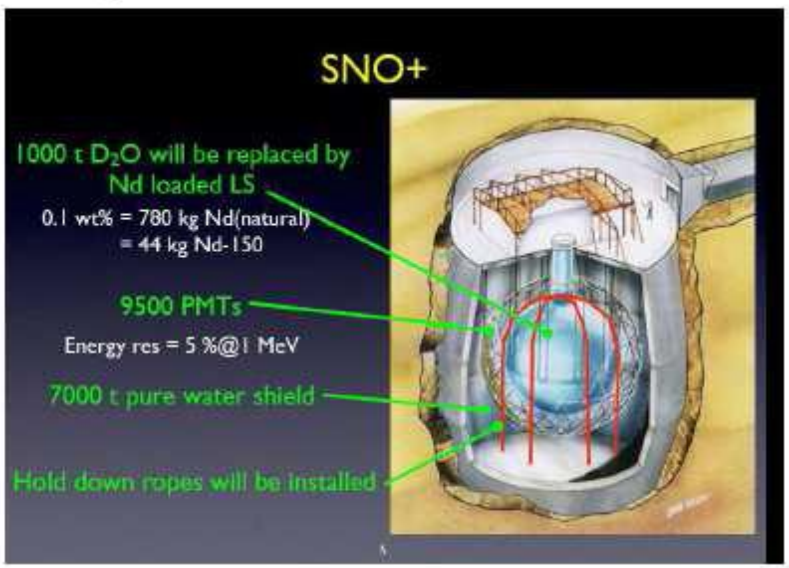
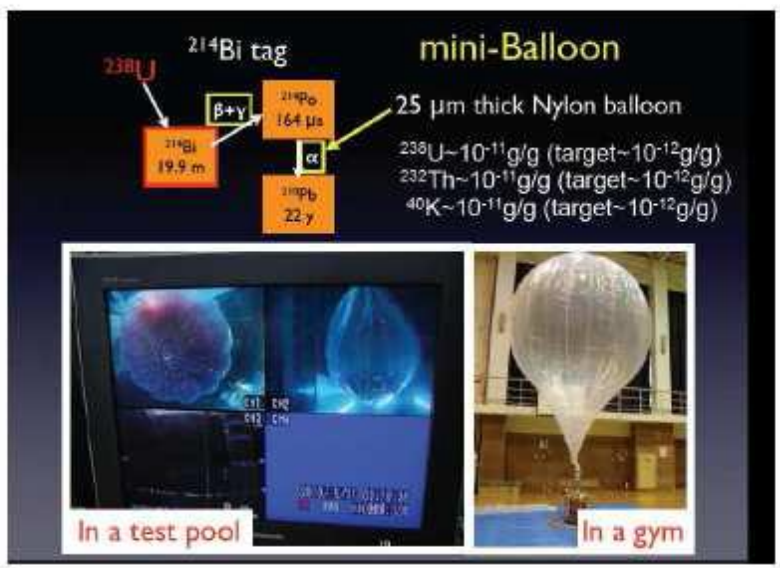
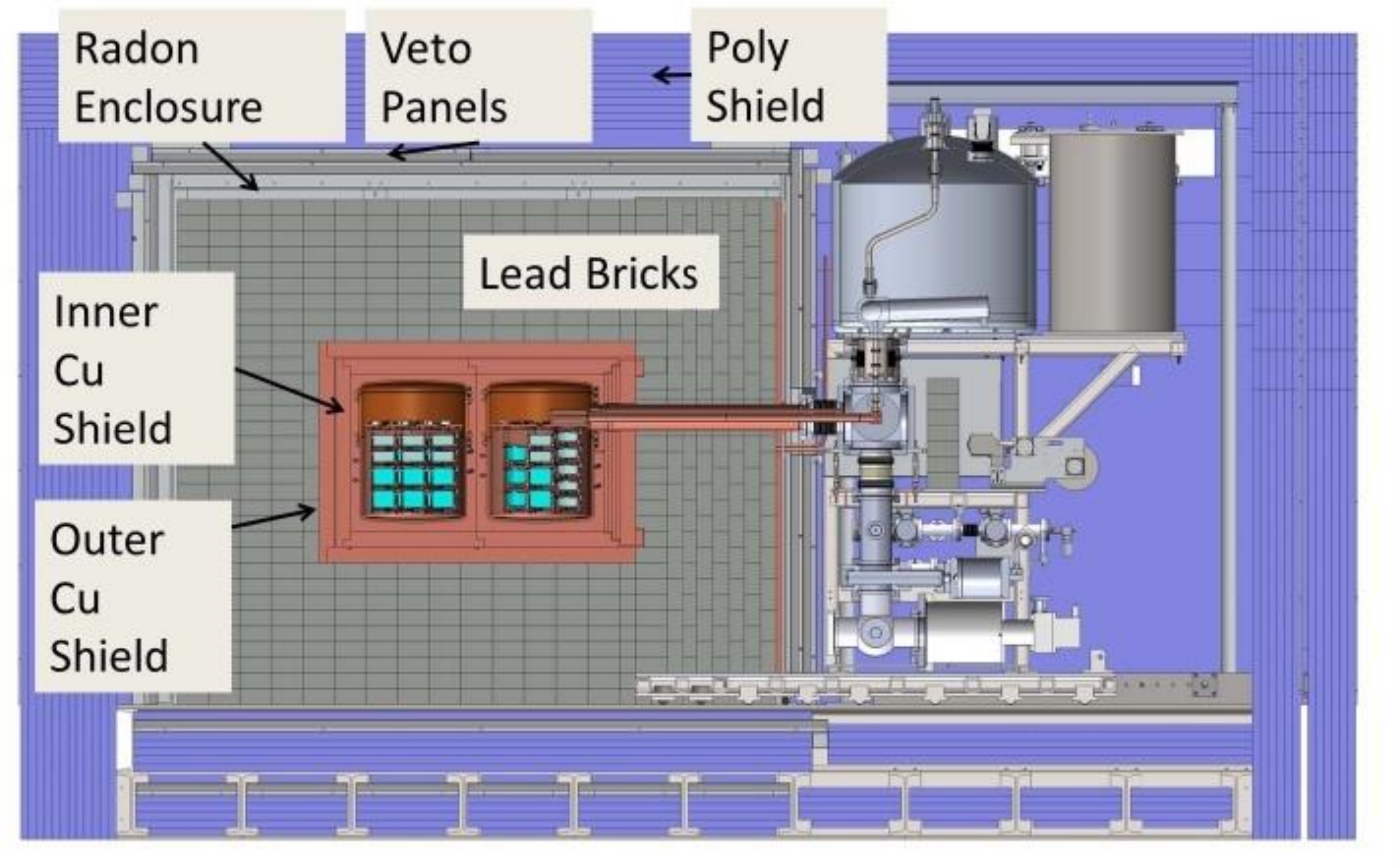


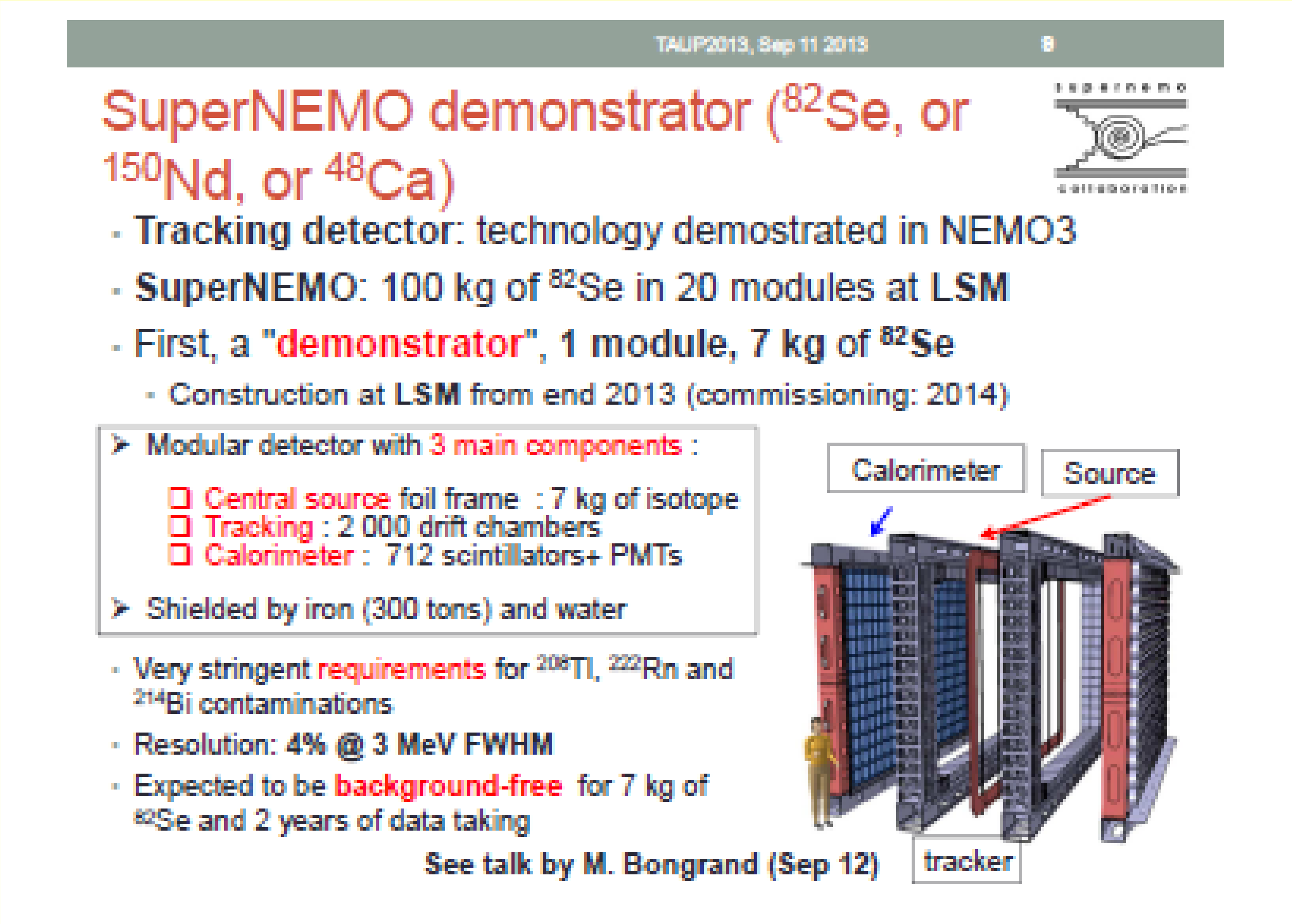
Figure 9: The SuperNEMO single module (picture from X. Sarasin, arXiv:1210.7666v1)

GERDA **Majorana** **Supernemo**

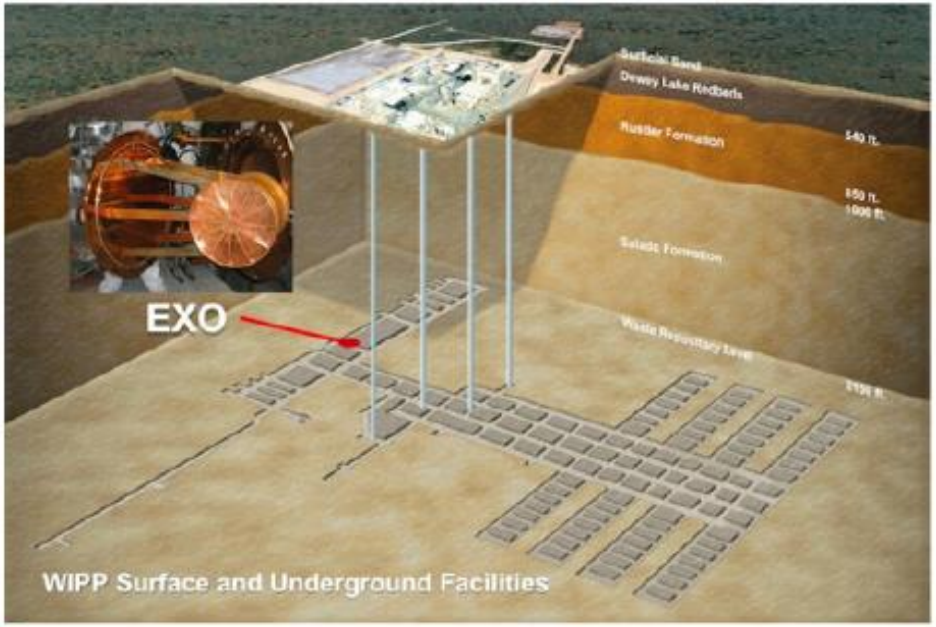


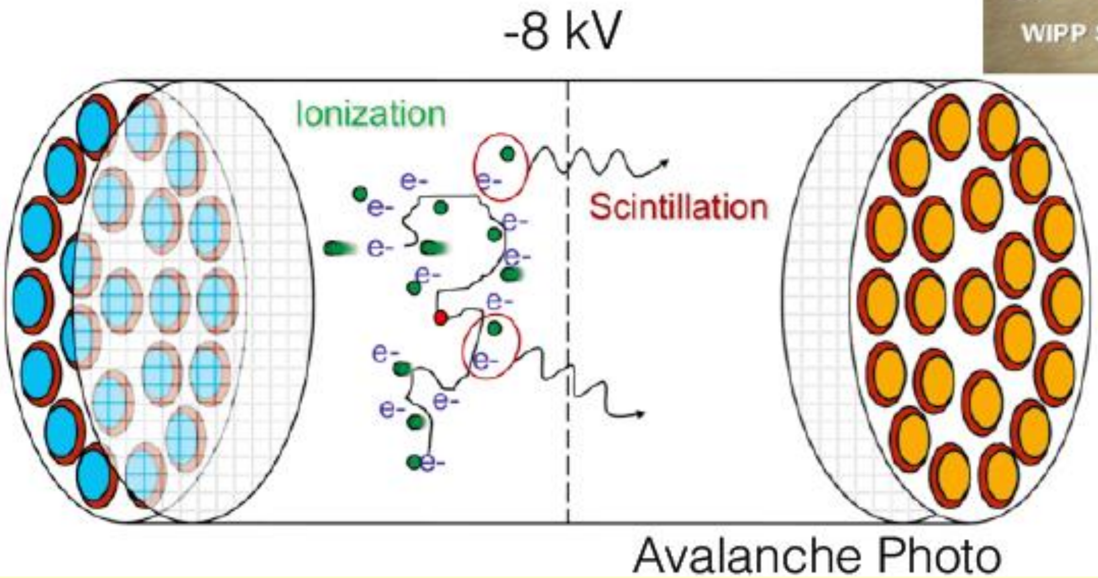



Radon
Enclosure
Veto
Panels
Poly
Shield
Lead Bricks
Inner
Cu
Shield
Outer
Cu
Shield


TAUP2013, Sep 11 2013
8
SuperNEMO demonstrator (^{82}Se, or ^{150}Nd, or ^{48}Ca)
Tracking detector: technology demostrated in NEMO3
SuperNEMO: 100 kg of ^{82}Se in 20 modules at LSM
First, a "demonstrator", 1 module, 7 kg of ^{82}Se
Construction at LSM from end 2013 (commissioning: 2014)
Modular detector with 3 main components :
Central source foil frame : 7 kg of isotope
Tracking : 2 000 drift chambers
Calorimeter : 712 scintillators+ PMTs
Shielded by iron (300 tons) and water
Very stringent requirements for ^{208}Tl, ^{222}Rn and ^{214}Bi contaminations
Resolution: 4% @ 3 MeV FWHM
Expected to be background-free for 7 kg of ^{82}Se and 2 years of data taking
See talk by M. Bongrand (Sep 12)
Calorimeter
Source
tracker

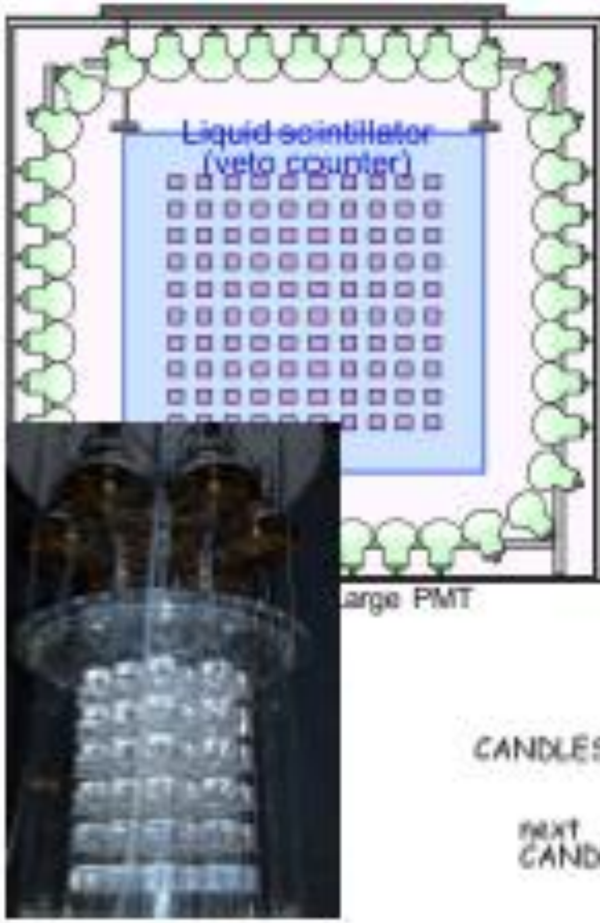
- Liquid Xe Time Projection Chamber (TPC)
- Enriched ^{136}Xe to 80.6%
- Q-value 2458 keV



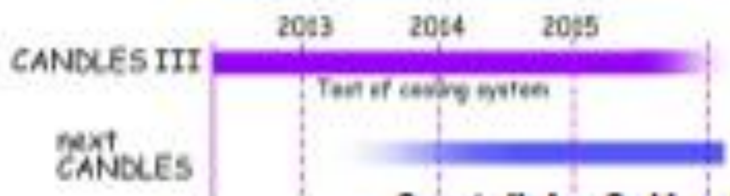


- Located at Waste Isolation Pilot Plant (WIPP) in Carlsbad, NM, USA
- 1585 meters water equivalent

CANDLES (^{48}Ca)

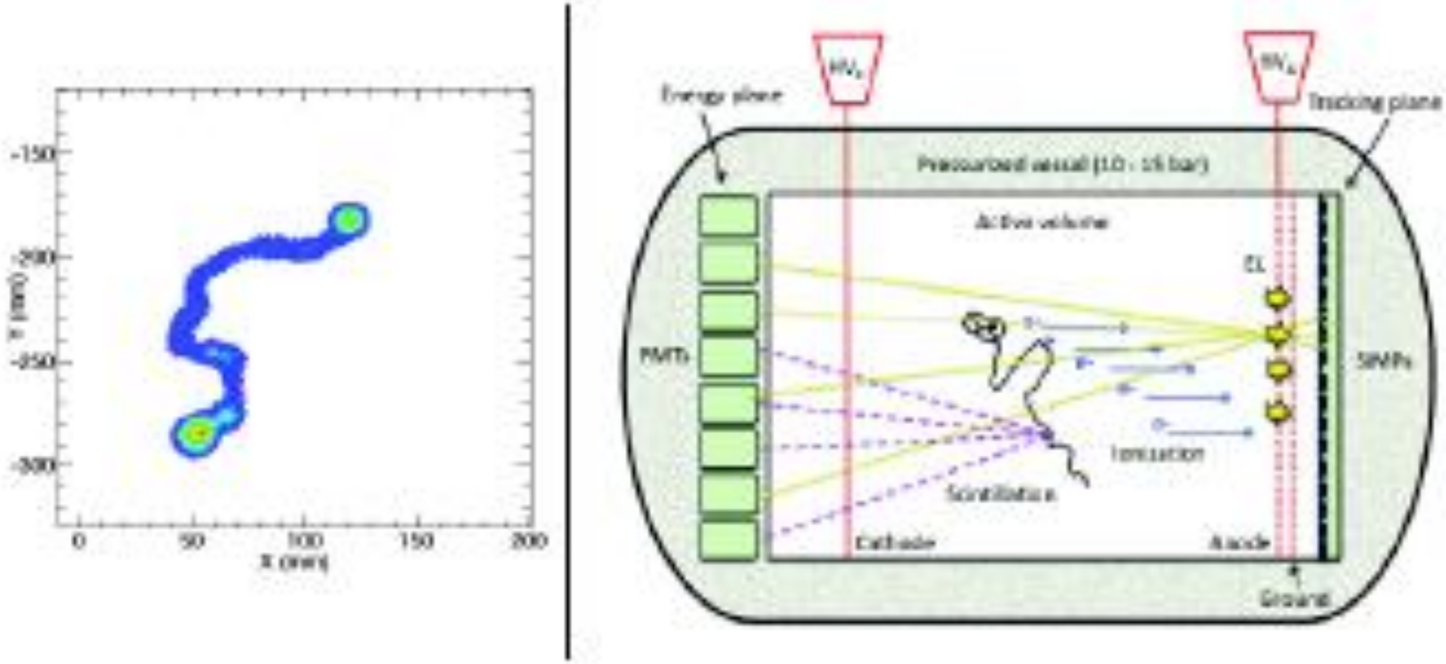


- **CANDLES System**
  - For ^{48}Ca ($Q_{\beta\beta}$=4.3MeV, but 0.187%)
  - CaF_2 detector and 4π active shield
  - Enriched ^{48}Ca
- **CANDLES III** at Kamioka Lab.
  - 96 CaF_2 (305 kg, 0.187% ^{48}Ca) + liquid scintillator
  - **Data taking** and background studies from Spring 2013
  - Sensitivity of 0.5 eV
- R&D for ^{48}Ca **enrichment**
  - CANDLES IV & V
  - Not funded yet



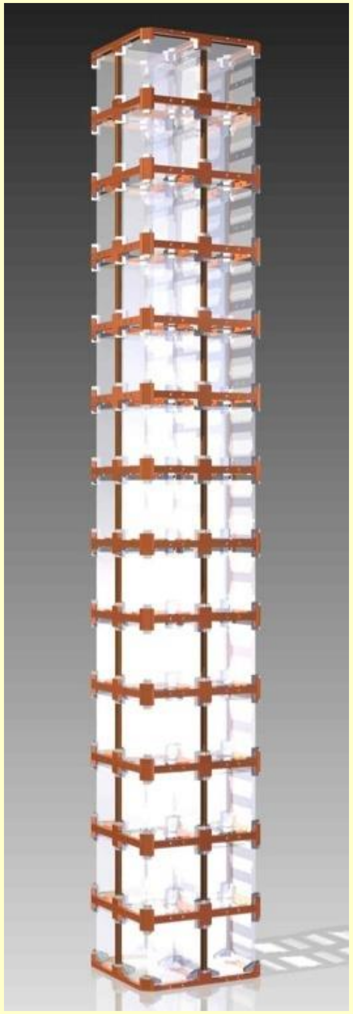
See talk by S. Umehara (Sep 12)

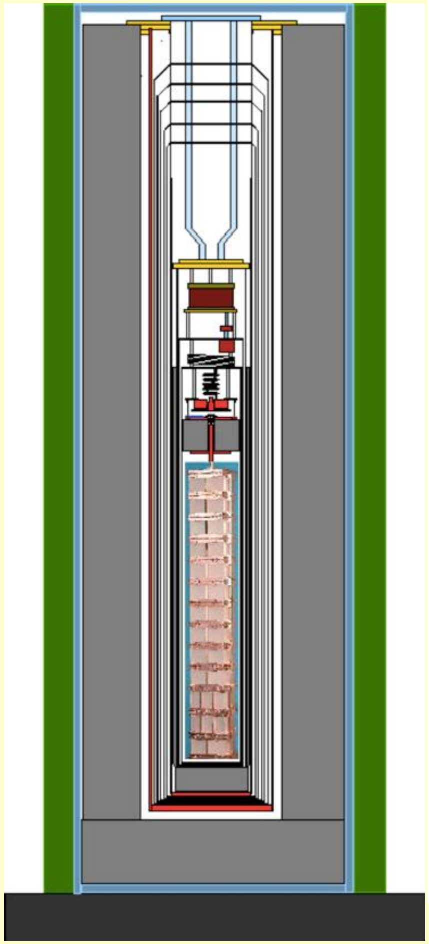
NEXT

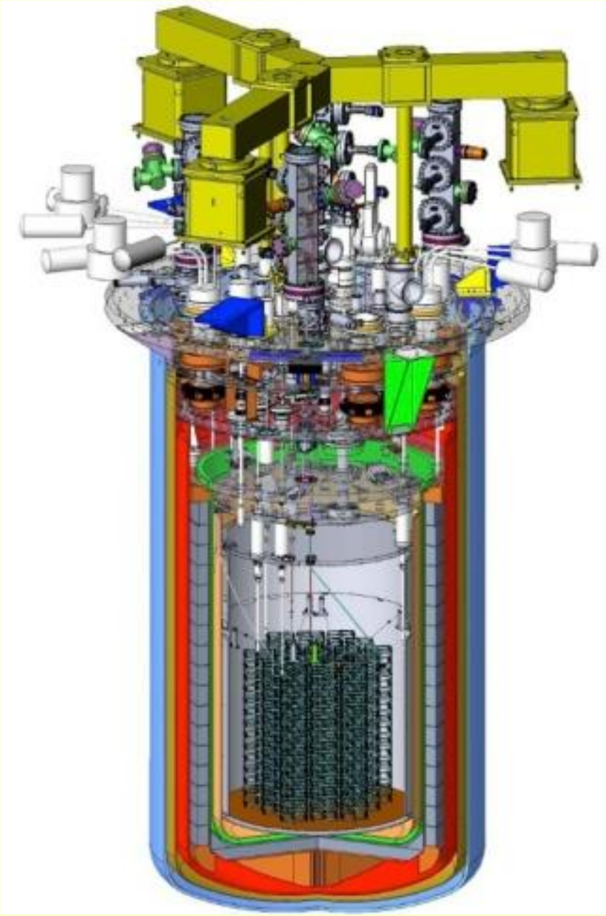


FWHM	12.5 keV
MASS	90 kg of ^{136}Xe (100 kg of enriched Xe)
BKG	
rejection	event topology (predicted background rejection ratios are of $\sim 2\times10^{-7}$, detection efficiency of 25%)
goal	8×10^{-4} counts/(keV·kg·yr)(evaluation done on the basis of the background budget and rejection factors)
$F^{0\nu}_{90\%C.L.}$	1.6×10^{26} yr in 5 years

CUORICINO **CUORE0** **CUORE**

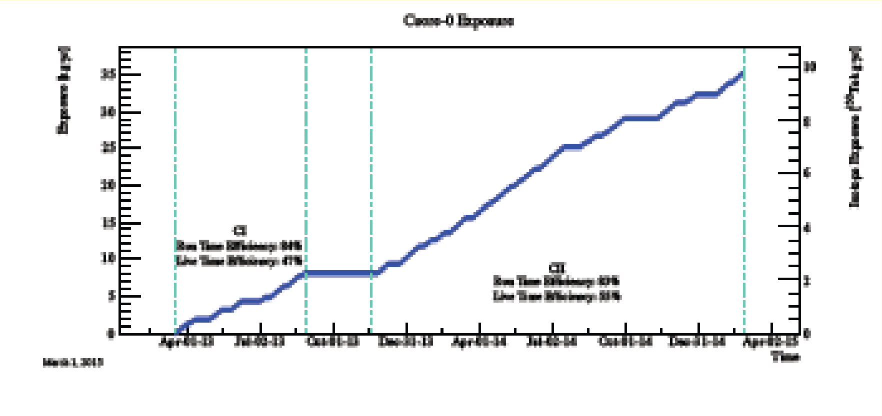


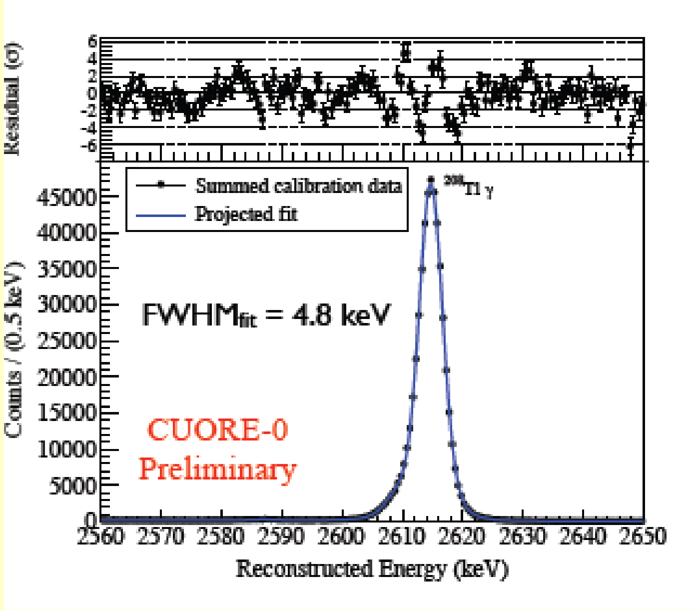


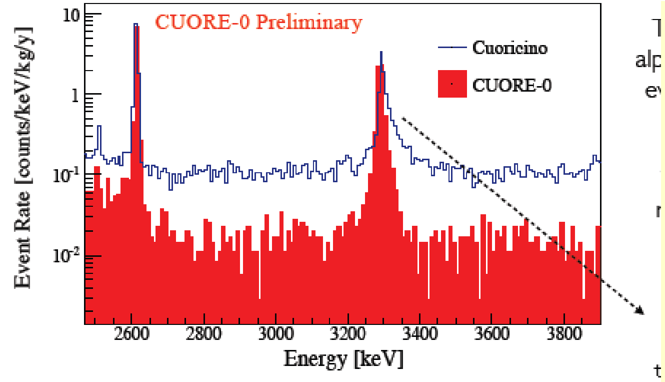


52 TeO_2 crystals, total mass = 39 kg TeO_2 = 10.9 kg ^{130}Te

New





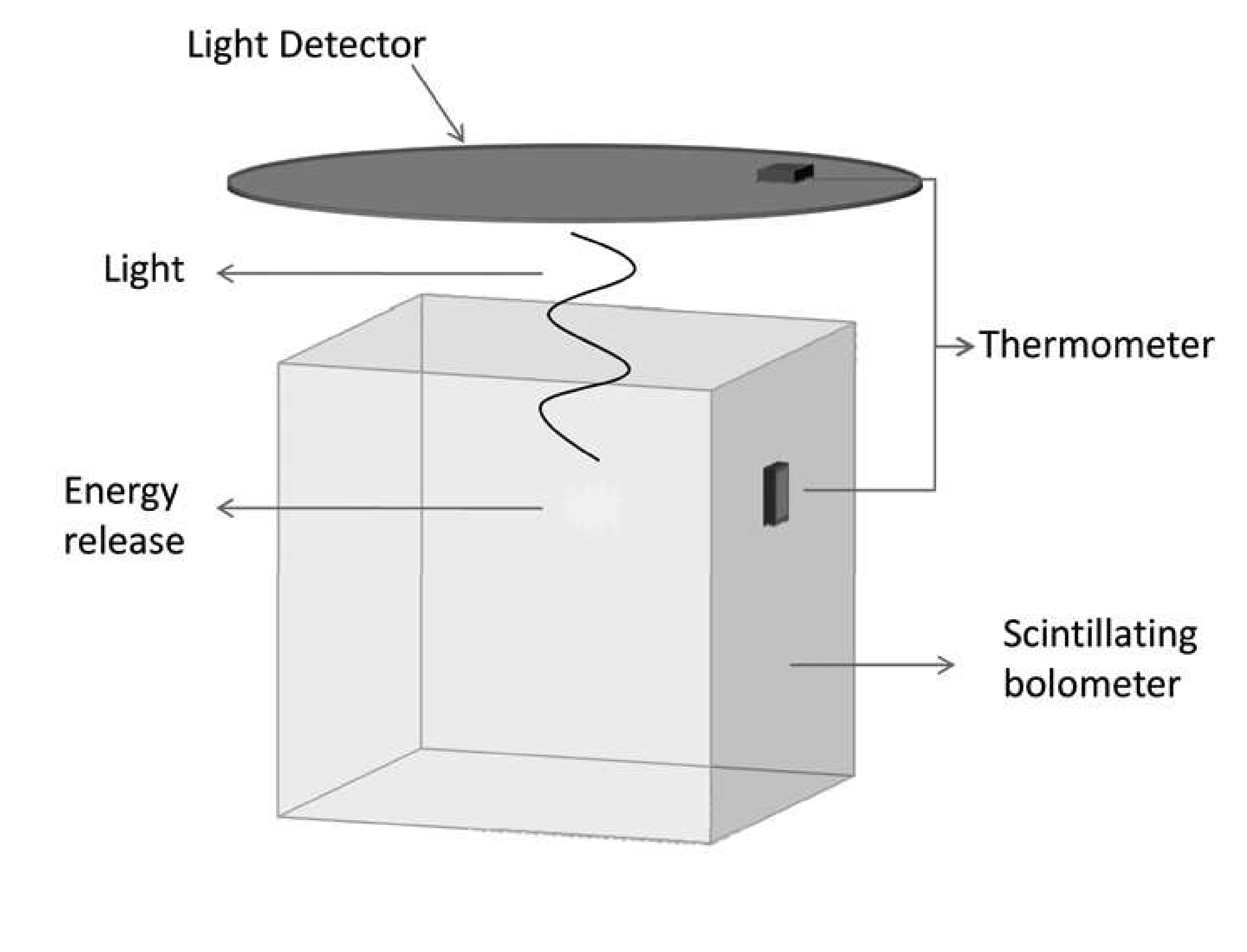


	2.7-3.9 MeV	eff [%]
CUORE-0	0.016 ± 0.001	81 ± 1
Cuoricino	0.110 ± 0.001	83 ± 1

CUORE0 $\Rightarrow T_{0\nu}^{1/2} > 2.7 \times 10^{24}$ **90% c.l.**

CUORE+CUORICINO $\Rightarrow T_{0n}^{1/2} > 4 \times 10^{24}$ **90% c.l.**

The future => Hybrid techniques



Now a **competitor** AMORE

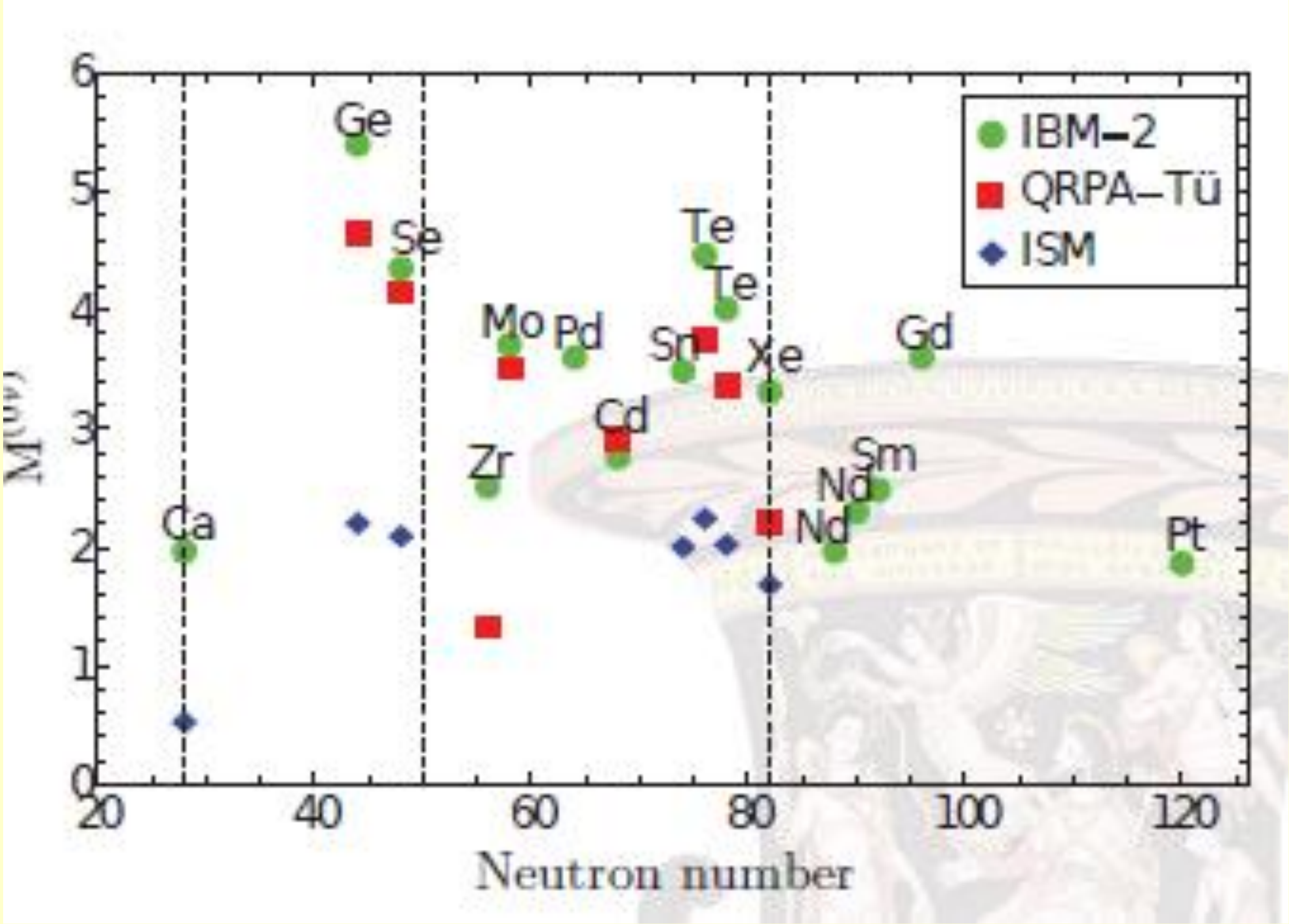
Advanced **Mo** based **R**are process **E**xperiment

$^{40}Ca^{100}MoO_4$ cryogenic scintillation detector as a tool for Mo-100 DBD search

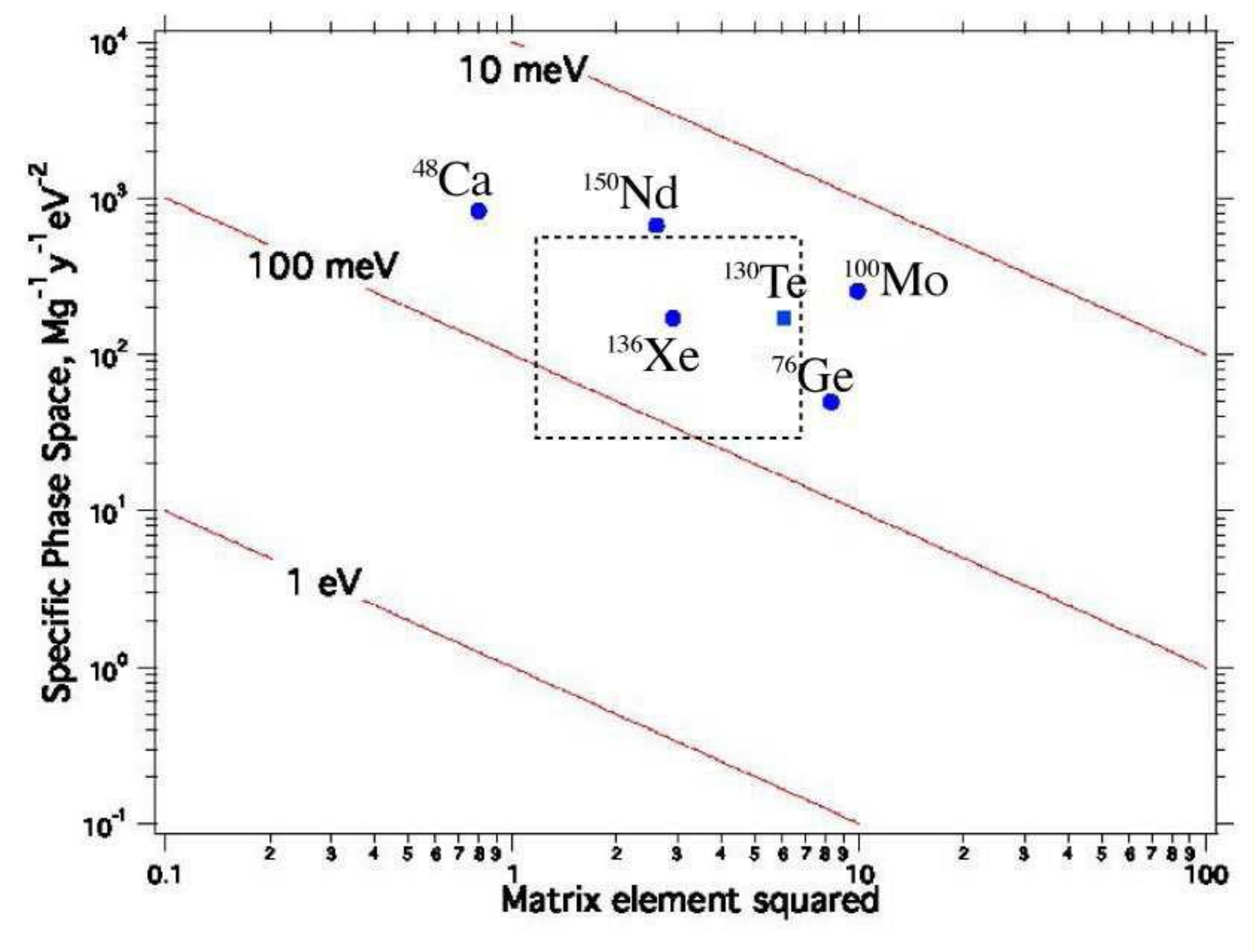
"Detector = Source": ε ~ 80 - 90% efficiency;

- High content of working isotope (Mo) in compound: 50% (stoichiometry ratio);
- Good energy resolution and high light yield for scintillation mode;
- High energy resolution for phonon mode (comparable with the resolution for HPGe detectors): no $2\nu\beta\beta$ of Mo–100 background;
- α/β- pulse shape discrimination (α/β-ratio = 0,20)
- Scalability of the experiment (increasing of mass by break-in "crystal by crystal");
- Chochralsky technology of the production → High purity, including radioactive isotopes
- Mo-100 production at **hundred kg scale**: centrifuges

The past


IBM−2
QRPA−Tü
ISM
Ge
Se
Te
Te
Mo
Pd
Sn
Xe
Gd
Cd
Zr
Sm
Nd
Nd
Ca
Pt
Neutron number

Robertson



10 meV
100 meV
1 eV
48Ca
150Nd
130Te
100Mo
136Xe
76Ge
Specific Phase Space, Mg-1 y-1 eV-2
Matrix element squared
10^4
10^3
10^2
10^1
10^0
10^-1
0.1
1
10
100

Bracking news unpleasant

F.Iachello => S.Dell'Oro and F.Vissani

A.Faessler et al. J.Phys. G35 (2008) 075104

Quenching g_A from 2ν ββ decay and one ν β decay

$$t_i^{1/2} \sim \mathscr{M}_i^{-2} = g_A^{-4} \mathscr{M}_{0\nu,i}^{-2}$$

A conservative treatment of the uncertainties should consider at least three cases: *

$$g_A = \begin{cases} g_{\text{nucleon}} & = 1.269 \\ g_{\text{quark}} & = 1 \\ g_{\text{phen.}} & = 1.269 \cdot A^{-0.18} \end{cases}$$

The first strike by Brutus

Experiment	Isotope	$t^{1/2}$ (90% C. L.) [10^{25} yr]	Lower bound for $m_{\beta\beta}$ [eV]		
			g_{nucleon}	g_{quark}	$g_{\text{phen.}}$
IGEX, [9]	^{76}Ge	1.57	0.31 ± 0.03	0.49 ± 0.05	1.44 ± 0.16
HEIDELBERG-MOSCOW, [10]	^{76}Ge	1.9	0.28 ± 0.03	0.44 ± 0.05	1.31 ± 0.14
GERDA-I, [11]	^{76}Ge	2.1	0.26 ± 0.03	0.42 ± 0.05	1.25 ± 0.14
KamLAND-Zen-I, [12]	^{136}Xe	1.9	0.18 ± 0.02	0.29 ± 0.03	1.06 ± 0.12
KamLAND-Zen-II, [13]	^{136}Xe	1.3	0.22 ± 0.02	0.35 ± 0.04	1.28 ± 0.14
EXO-200, [14]	^{136}Xe	1.1	0.24 ± 0.03	0.38 ± 0.04	1.39 ± 0.15
Combined Ge, [11]	^{76}Ge	3.0	0.22 ± 0.02	0.35 ± 0.04	1.05 ± 0.11
Combined Xe	^{136}Xe	2.6	0.15 ± 0.02	0.25 ± 0.03	0.91 ± 0.10
Combined Ge+Xe	^{76}Ge/^{136}Xe		0.15 ± 0.01	0.24 ± 0.02	0.81 ± 0.07
CUORE, [15]	^{130}Te	9.5	0.07 ± 0.01	0.11 ± 0.01	0.39 ± 0.04
GERDA-II, [16]	^{76}Ge	15	0.10 ± 0.01	0.16 ± 0.02	0.47 ± 0.05
SuperNEMO, [17]	^{82}Se	10	0.07 ± 0.01	0.12 ± 0.01	0.36 ± 0.04

The final strike by Brutus

The contribution of light Majorana neutrinos to neutrinoless double beta decay and cosmology

Stefano Dell'Oro,[1,*] Simone Marcocci,[1,†] Matteo Viel,[2,3,‡] and Francesco Vissani[4,1,§]
[1]*INFN, Gran Sasso Science Institute, Viale F. Crispi 7, 67100 L'Aquila, Italy*
[2]*INAF, Osservatorio Astronomico di Trieste, Via G.B. Tiepolo 11, 34131 Trieste, Italy*
[3]*INFN, Sezione di Trieste, Via Valerio 2, 34127 Trieste, Italy*
[4]*INFN, Laboratori Nazionali del Gran Sasso, Via G. Acitelli 22, 67100 Assergi (AQ), Italy*
(Dated: May 15, 2015)

- **Impressive progress by cosmology on neutrino mass.**
- **Recent data by Planck**
- **A detailed analysis of cosmological results possibly favoring the direct hierarchy (Palanque Delabrouille) $\Sigma m_\nu < 0.17$ eV or**

$$\Sigma < 84\,\text{meV} \qquad (1\sigma\ \text{C.L.})$$
$$\Sigma < 146\,\text{meV} \qquad (2\sigma\ \text{C.L.})$$
$$\Sigma < 208\,\text{meV} \qquad (3\sigma\ \text{C.L.})$$

Amicus Plato, sed magis amica veritas
Platus is a friend.but truth even more

Majorana returns

Frank Wilczek

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of 'Majorana fermions' — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

Majorana => can a ½ spin particle be its own antiparticle?
So far application to neutrino
Conventional Cooper excitons are bosons and not Majorana particles
In superconductors Cooper pairs, being boson like particle, form condensates respecting Pauli principle
How to produce excitons which are Majorana particles
Cooper pairs can be added to holes producing spin ½ exitons.
Superconductors can contain magnetic wires acting as quasiparticles
Form magnetic flux tubes (vortices)

PRL 114, 017001 (2015) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2015

Experimental Detection of a Majorana Mode in the core of a Magnetic Vortex inside a Topological Insulator-Superconductor $Bi_2Te_3/NbSe_2$ Heterostructure

Jin-Peng Xu,[1] Mei-Xiao Wang,[1] Zhi Long Liu,[1] Jian-Feng Ge,[1] Xiaojun Yang,[2] Canhua Liu,[1,5,*] Zhu An Xu,[2,5] Dandan Guan,[1] Chun Lei Gao,[1] Dong Qian,[1] Ying Liu,[1,3,5] Qiang-Hua Wang,[4,5] Fu-Chun Zhang,[2,5] Qi-Kun Xue,[6] and Jin-Feng Jia[1,5,†]

Bi_2Te_3 on a superconductor $NbSe_2$ In normal superconductors Scanning with a tunneling microscope and sepctrocope one can find explicity a single Majorana mode.

Common opinion:

Majorana theory can be as impoortnt in solid state like it is In elementary particle physics

Conclusions

Massive detectors (now expensive) of **unique sophistication**

Reduction of **background** which could hide **neutrinoless ββ events** has reached an **unprecedented sensitivity**

The **quenching** of g_A reduces the **matrix** would ref+duce sensitivity oon neutrino mass limit by a factor **4 to 5**. If so no present hope to reach the **inverse hierarchy** region

The validity to extend to **neutrinoless** DBD the g_A obtained from **two neutrino** and **single beta decay** should be justified

The **sophistication** reached and to be reached in serches for DBD could have **appplications** in **other fields of science**

Aplication to **solid state physics** of the **Maiorana theory** could bring to results as important as in **solid state** physics

There are more things in Earth and Heaven , Polonius, that can be dreamt of by your Philosophy Hamlet