Realization of a four-dimensional atomic Hall system

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Outline

1. 2D Hall effect in atomic gases
   - State of the art
   - Hall systems with a large synthetic dimension
2. Realization of a four-dimensional atomic Hall system
   - State of the art: 4D Hall physics with a 2D charge pump
   - Description of our system
   - Adiabatic Hall response
   - Velocity distribution and edge modes
   - Cyclotron orbits
   - Reconstructing the second Chern number
   - Direct observation of a 4D Hall non-linear response

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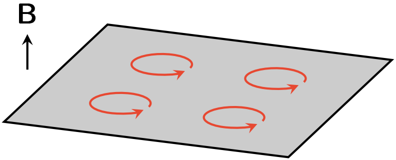
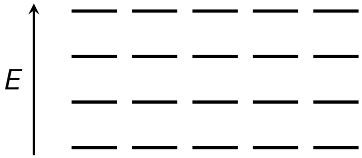
Topological systems

The archetype of topological systems: a **2D quantum Hall insulator**



classical orbits



Landau quantum levels
flat bands & non-trivial topology

A whole zoo of topological systems (topological insulators, superconductors) depending on discrete symmetry class and dimension

Altland, & Zirnbauer, PRB 1997

Topological systems in dimensions $D > 3$ accessible in engineered systems based on **synthetic dimensions**.

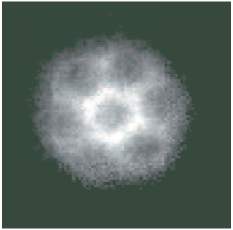
This talk: realization of a 4D quantum Hall system

Simulating an orbital magnetic field with ultracold atoms

Mimicking the Aharonov-Bohm geometrical phase

Rotation

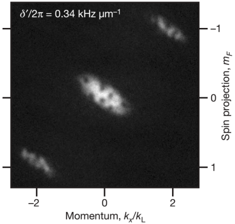
Sagnac phase



Madison et al, PRL 1999

Light dressing

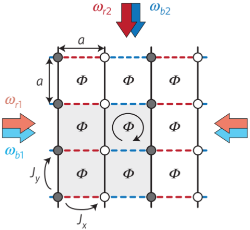
Berry phase



Lin et al, Nature 2009

Shaken lattices

Peierls phase

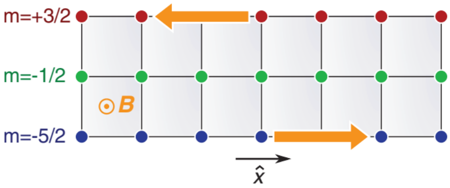


Aidelsburger, PRL 2013

Jotzu et al, Nature 2014

A new tool: synthetic dimensions

Encoding a dimension in a spin degree of freedom.
Magnetic projection m (with $-J \leq m \leq J$) acts as a coordinate.



first realizations with 3 states

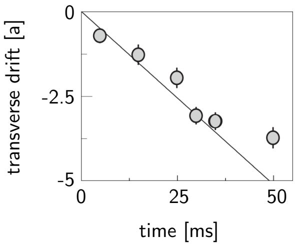
Mancini et al, Science (2015) and Stuhl et al, Science (2015)

Assets of this method

- simple realization of the magnetic field:
  light-induced spin transitions
- sharp edges

Probing quantum Hall physics in atomic systems

- Quantization of transverse response in large & smooth atomic ensembles



Aidelsburger et al, Nature Phys. (2015)

Probing quantum Hall physics in atomic systems

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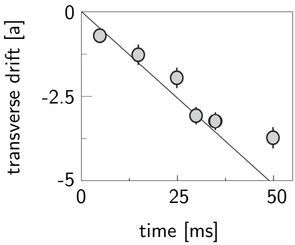
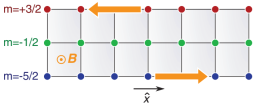
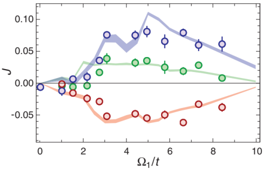


Aidelsburger et al, Nature Phys. (2015)

- Chiral edge modes in very small samples (no notion of a bulk)





Mancini et al, Science (2015)
Stuhl et al, Science (2015)

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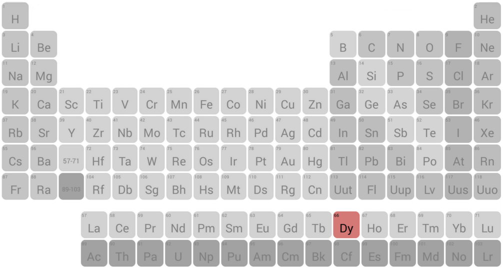


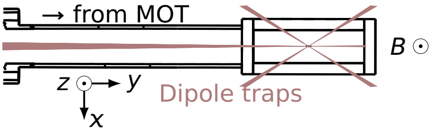
Filled $6s^2$ shell

Submerged open $4f^{10}$ shell

Electronic spin $J = 8$

Magnetic projection states m $(-J \leq m \leq J)$ encode a synthetic dimension with $2J + 1 = 17$ sites.

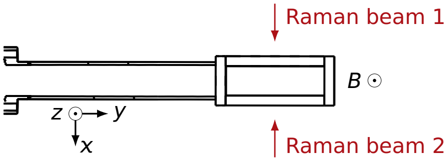
Realization of a quantum Hall ribbon: initial state

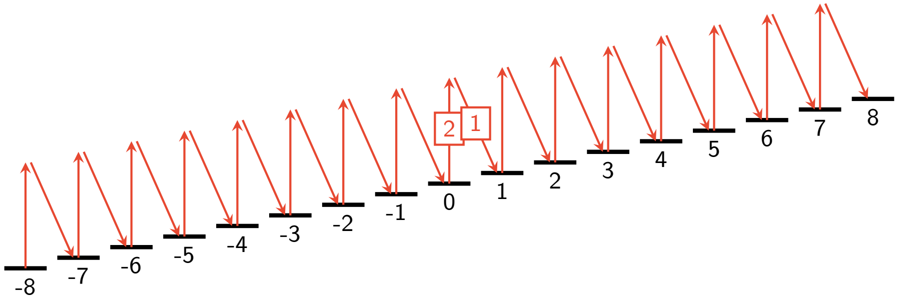


10^5 atoms held in optical tweezers, cooled down to $T = 0.5\,\mu\mathrm{K}$.

Magnetic field along z splits the m levels.

Realization of a quantum Hall ribbon: spin dynamics

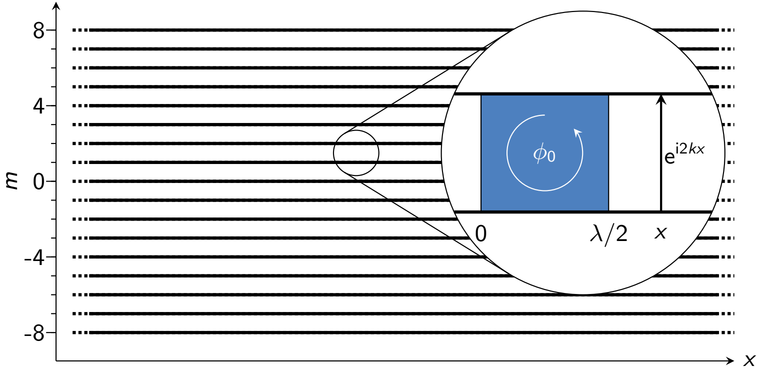




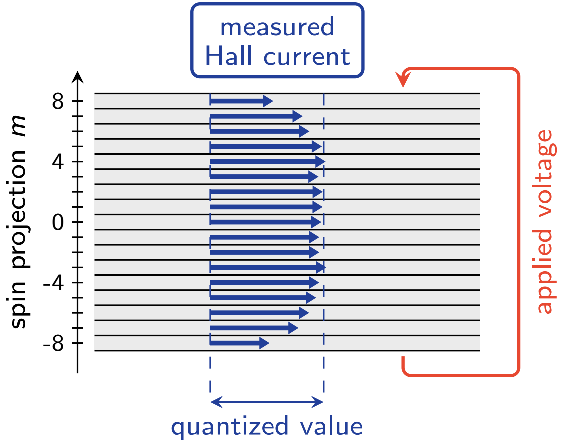
Transitions $m \to m+1$ together with momentum kick $Mv \to Mv - 2\hbar k$
$\Rightarrow$ conservation of momentum $p = Mv + 2\hbar km$.

Realization of a quantum Hall ribbon: effective B field

- Spin transitions inherit the complex amplitude $\propto e^{i2kx}$ from laser interference.
- Equivalent to Peierls phases (Aharonov-Bohm phases on discrete lattice).



Measuring a quantized Hall response



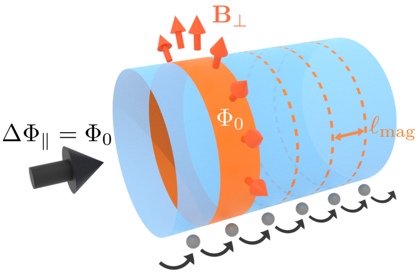
Homogeneous and quantized Hall response in the bulk

T. Chalopin et al, Nat. Phys. (2020)

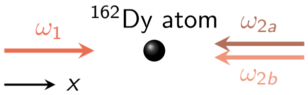
Realization of a quantum Hall cylinder

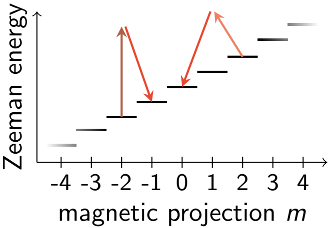
Motivations for a periodic synthetic dimension

- no edge effect
- cylinder geometry for Laughlin's charge pump experiment



Emergence of a cyclic synthetic dimension



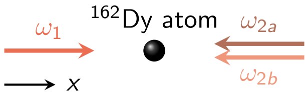


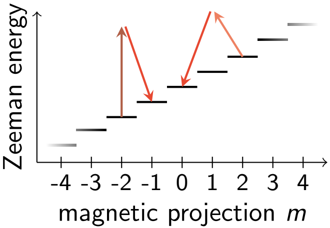
Combination of two Raman transitions

- $m \to m+1$ together with $Mv \to Mv - 2\hbar k$
- $m \to m-2$ together with $Mv \to Mv - 2\hbar k$

We lose the conservation of momentum $p = Mv + 2\hbar k m$.

Emergence of a cyclic synthetic dimension





Both transitions satisfy

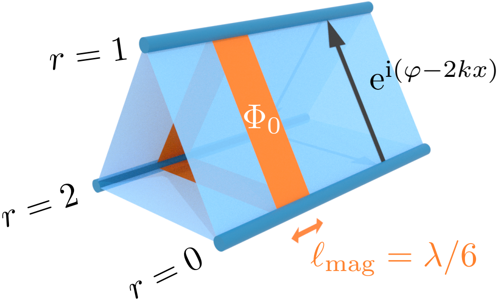
$$r \to r + 1 \text{ together with } Mv \to Mv - 2\hbar k$$

for the cyclic dimension

$$r = m \pmod 3.$$

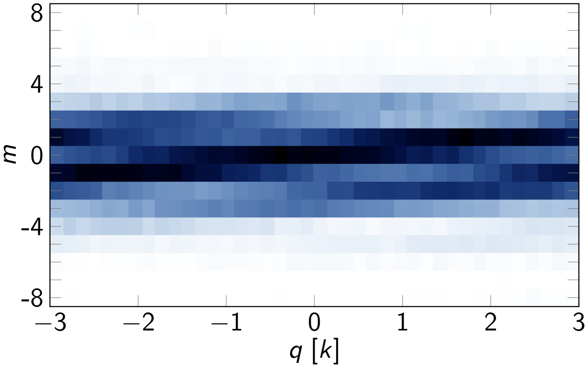
Conservation of the quasi-momentum $q = Mv + 2\hbar k r \pmod{6\hbar k}$

Emergence of a Hall cylinder



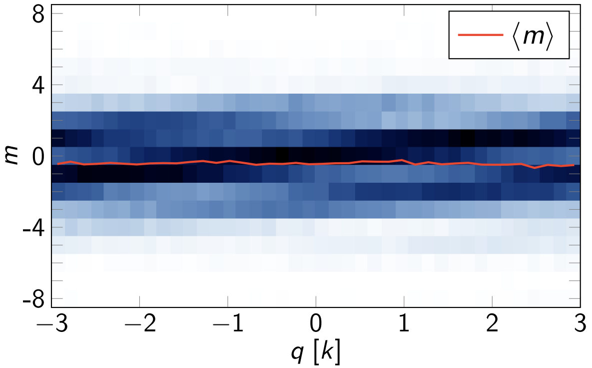
Topological charge pump in a Bloch oscillation

Evolution of spin projection probabilities



A. Fabre et al, Phys. Rev. Lett. (2022)

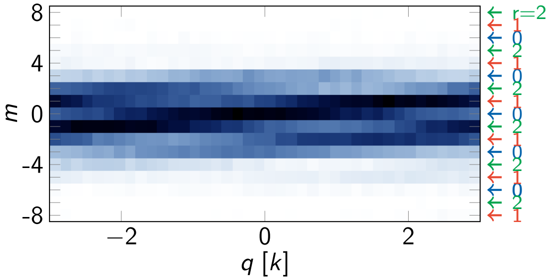
Topological charge pump in a Bloch oscillation



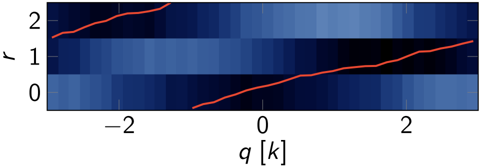
No drift of the mean $\langle m \rangle$

A. Fabre et al, Phys. Rev. Lett. (2022)

Topological charge pump in a Bloch oscillation



regrouping to infer the r-projection probabilities



Quantized increase $\Delta r = 3$ for each Bloch oscillation cycle.

A. Fabre et al, Phys. Rev. Lett. (2022)

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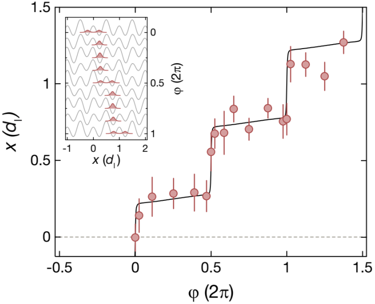
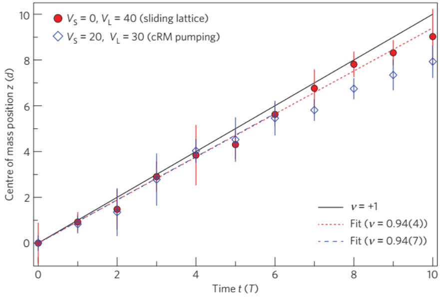
adiabatic & time-periodic deformation
of a quantum lattice system
↓
quantized charge pump

Thouless, PRB 1983

Realizations in cold atomic systems



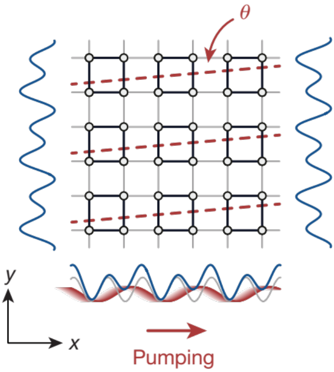


Lohse et al, Nature Phys. 2016

Nakajima et al, Nature Phys. 2016

4D Hall physics with a 2D charge pump

A two-dimensional (super-)lattice system



Cyclic deformation of the superlattice drives quantized charge pump described by a second Chern number.

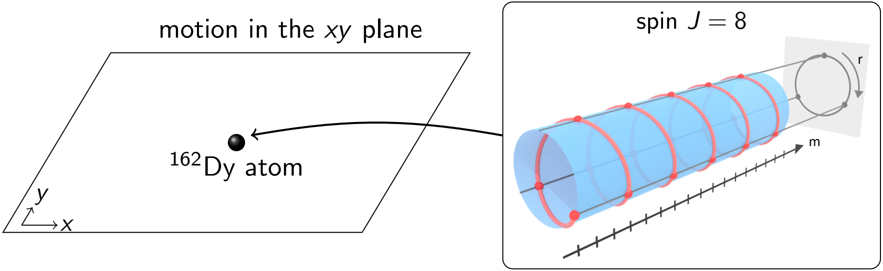
Lohse et al, Nature 2018

Outline

Definition of the four dimensions

Implementation inspired from previous proposals

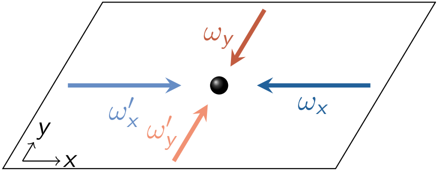
Kraus et al, Phys. Rev. Lett. 2013
Price et al, Phys. Rev. Lett. 2015

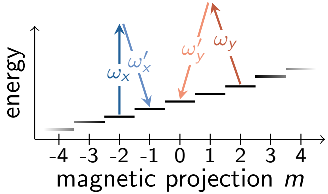


Dynamics occurs on a four-dimensional space of coordinates (x, y, m, r)

- x and y are genuine spatial dimensions
- m and r are encoded in the spin and restricted to $2J + 1 = 17$ sites on a cylinder

Fabre et al, PRA 2022





Two types of Raman transitions

transition	Δm	Δr	$M\Delta\mathbf{v}$
x	1	1	$-2\hbar k\hat{\mathbf{x}}$
y	-2	1	$-2\hbar k\hat{\mathbf{y}}$

$\Rightarrow$ non-trivial cycles $m \xrightarrow{x} m+1 \xrightarrow{x} m+2 \xrightarrow{y} m$ imparting a velocity kick

$$\mathbf{K} = 2k(2\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

$\Rightarrow$ Conservation of the quasi-momentum

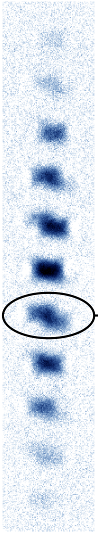
$$\mathbf{P} = M\mathbf{v} + 2km\hat{\mathbf{x}} \pmod{\mathbf{K}}$$

Experimental sequence

1. Ultracold gas of 10^5 atoms
at $T = 0.4\,\mu\mathrm{K}$
polarized in $m = -J$

2. Switch on Raman couplings
(off resonant)

3. Ramp detunings adiabatically
in frame moving with lattice
interference: **inertial force**
$\Rightarrow$ control of quasi-momentum $\mathbf{P}$



4. Measurement of velocity
and spin distributions
after time-of-flight

$\updownarrow$ separation of m levels
with a B field gradient

$\rightarrow$ xy velocity distribution
for each m level

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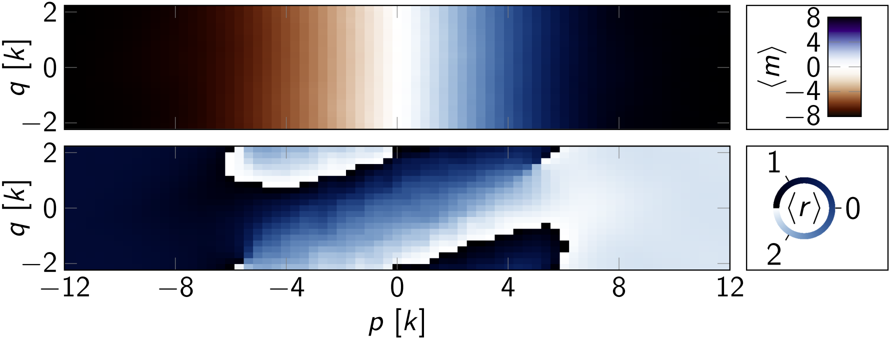
Adiabatic spin pumping

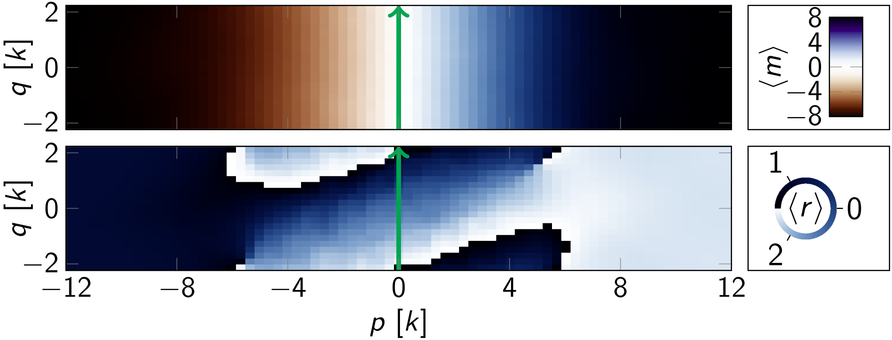
We decompose the quasi-momentum as

$$\mathbf{P} = p\hat{\mathbf{X}} + q\hat{\mathbf{Y}},$$

with $\hat{\mathbf{Y}} \parallel \mathbf{K}$, p arbitrary and $|q| < K/2$.



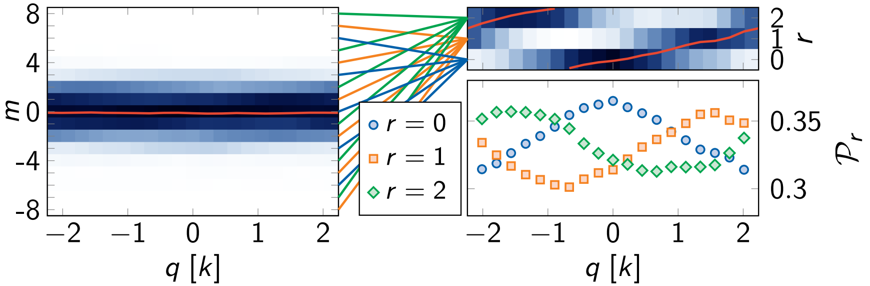
Geometrical pumping along r



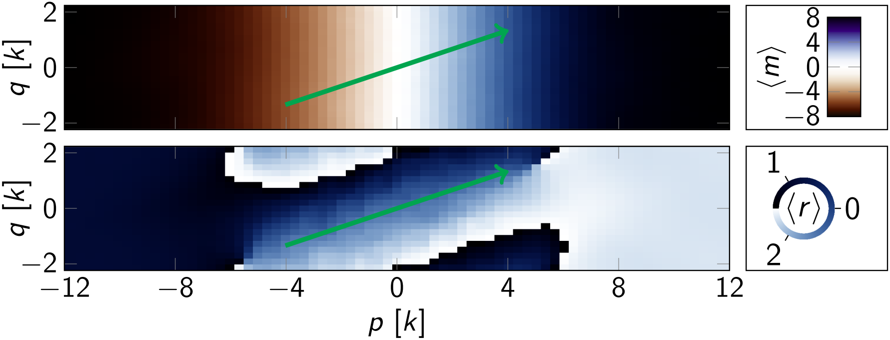
Geometrical pumping along r

transition	Δm	Δr	$M\Delta\mathbf{v}$
x	1	1	$-2\hbar k\hat{\mathbf{x}}$
y	-2	1	$-2\hbar k\hat{\mathbf{y}}$

When pushing the system along $2\hat{\mathbf{x}}+\hat{\mathbf{y}}$, the velocity increase is compensated by Raman transitions, such that $\langle m\rangle$ is kept constant on average while $\langle r\rangle$ increases.



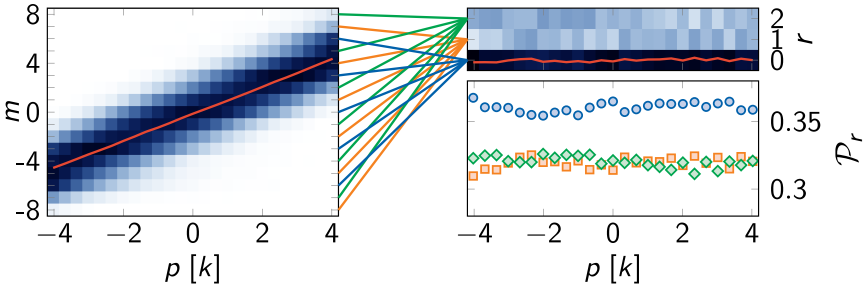
Geometrical pumping along m



Geometrical pumping along m

transition	Δm	Δr	$M\Delta\mathbf{v}$
x	1	1	$-2\hbar k\hat{\mathbf{x}}$
y	-2	1	$-2\hbar k\hat{\mathbf{y}}$

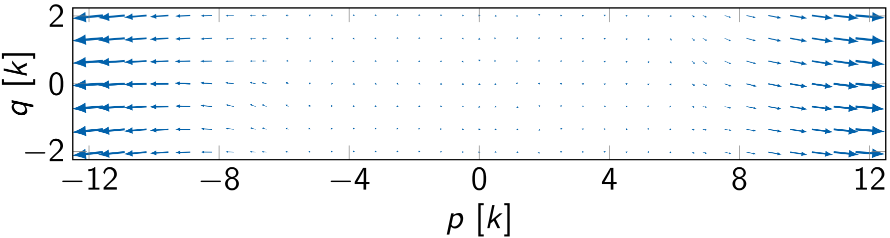
When pushing the system along $\hat{\mathbf{x}} - \hat{\mathbf{y}}$, the velocity increase is compensated by Raman transitions, such that $\langle r\rangle$ is kept constant on average while $\langle m\rangle$ increases.



Outline

Velocity distribution and edge modes

We extract for each momentum state the mean atom velocity.



Arrow length $\propto ||\mathbf{v}||$, with max length $\equiv 5v_r$

- velocity remains very small in the bulk
- on the edges $m = \pm J$, ballistic motion along $\pm\hat{\mathbf{X}}$, frozen motion along other directions

Outline

Rotations in four-dimensional space and cyclotron motion

Parametrization of rotations

- **Dimension 2**: center and angle
- **Dimension 3**: axis and angle
- **Dimension 4**: 2 invariant orthogonal planes, and two rotation angles (one for each plane)

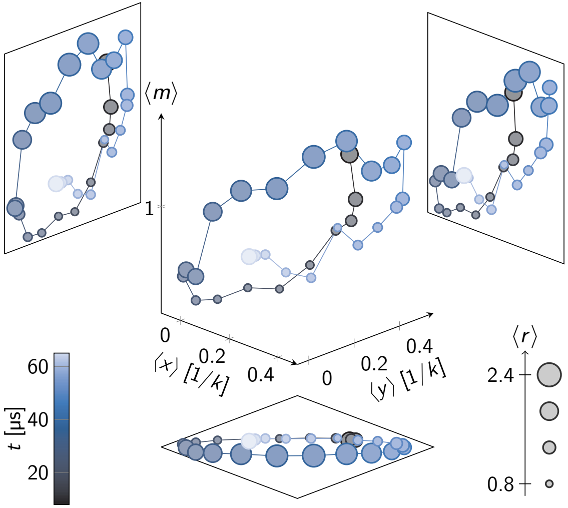
$\Rightarrow$ charged-particle motion in a magnetic field involves two frequencies.

For our system

- Raman process x leads to a rotation in the plane $(\hat{\mathbf{x}}, \hat{\mathbf{m}} + \hat{\mathbf{r}})$ of frequency ω_x
- Raman process y leads to a rotation in the plane $(\hat{\mathbf{y}}, -2\hat{\mathbf{m}} + \hat{\mathbf{r}})$ of frequency ω_y

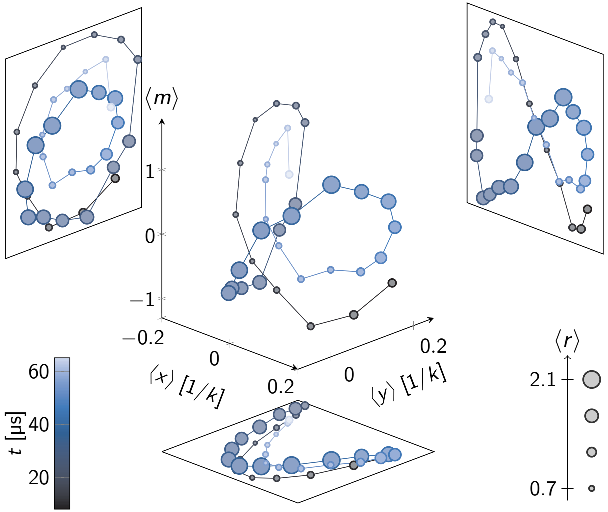
Cyclotron orbit for $\omega_y/\omega_x = 1$

We kick the atoms to drive a cyclotron motion of the center of mass.



Consistent with a planar circular orbit

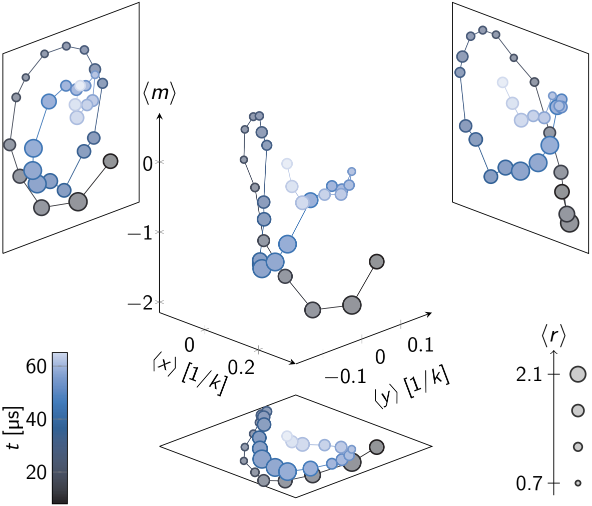
Cyclotron orbit for $\omega_y/\omega_x = 2$



Quasi-closed Lissajous orbit

Cyclotron orbit for $\omega_y/\omega_x = \varphi$

For a ratio close to the golden number φ, the orbit ceases to be closed.



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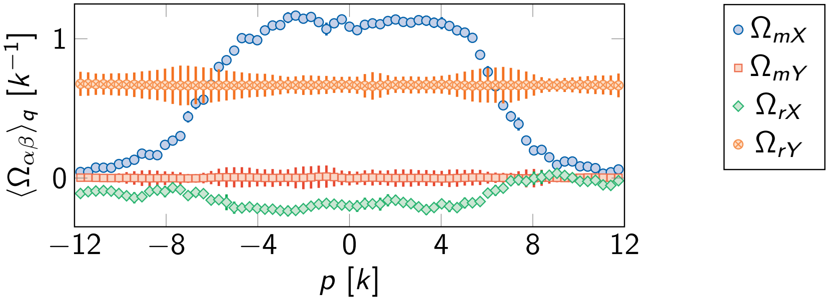
Berry curvatures

Variations of the mean velocity with momentum give access to Berry curvatures, e.g.

$$\Omega^{Ym} = -\frac{\sqrt{5}}{2k}\frac{\partial\langle v^Y\rangle}{\partial q}$$



Expected Berry curvature in the bulk

Spin hopping induced by the x Raman lasers occurs with a Peierls phase

$$\phi_x = -2kx = A_m \Delta m + A_r \Delta r,$$

with hopping ranges $\Delta m = \Delta r = 1$. Similar expressions for the y transitions, with $\Delta m = -2$ and $\Delta r = 1$.

We deduce the vector potential

$$\mathbf{A} = \frac{1}{3}(0, 0, \phi_x - \phi_y, 2\phi_x + \phi_y)_{x,y,m,r},$$

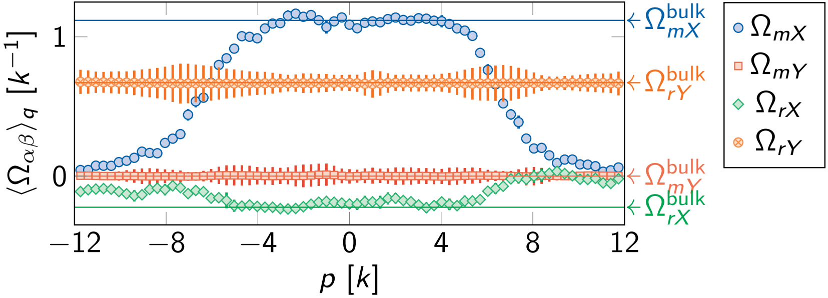
hence the magnetic field $B_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\mathbf{B} = \frac{2k}{3}\begin{pmatrix} 0 & 0 & -1 & -2 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{pmatrix}.$$

Expected Berry curvature in the bulk

In the bulk and in the absence of dispersion, the Berry curvature is uniform with

$$\boldsymbol{\Omega}_{\text{bulk}} = \mathbf{B}^{-1} = \frac{1}{2k}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 1 \\ -1 & 2 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}$$



Reconstructing the local second Chern marker

In uniform systems, the second Chern number is given by the band integral of Berry curvature products

$$\mathcal{C}_2 = \frac{1}{8\pi^2} \int \rho_2 \, \mathrm{d}^4 q,$$

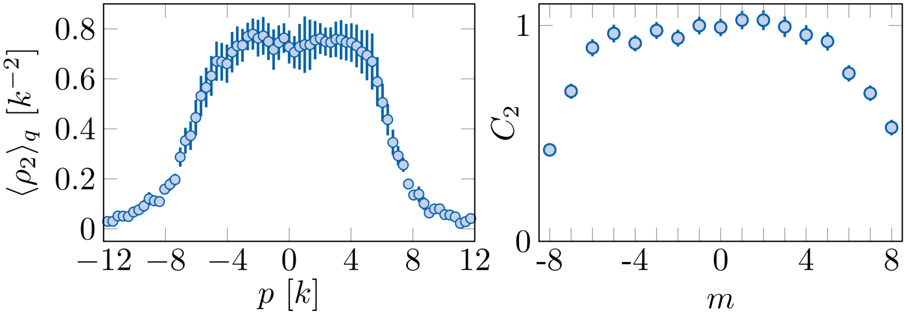
where $\rho_2 = \epsilon_{\mu\nu\delta\gamma} \Omega^{\mu\nu} \Omega^{\delta\gamma}$ is the second Chern character.
In our finite system with edge, we expect a quantized non-linear response *in the bulk only*.

We define a local second Chern marker

$$C_2(m) = \frac{1}{3} \int \rho_2(p, q) \Pi_m(p, q) \mathrm{d}p \mathrm{d}q$$

by weighting the second Chern character with the projection probability Π_m.

Quantized local second Chern marker in the bulk



The local second Chern marker is close to $\mathcal{C}_2 = 1$ in the bulk $-5 \leq m \leq 5$.

Outline

1. 2D Hall effect in atomic gases
   - State of the art
   - Hall systems with a large synthetic dimension
2. Realization of a four-dimensional atomic Hall system
   - State of the art: 4D Hall physics with a 2D charge pump
   - Description of our system
   - Adiabatic Hall response
   - Velocity distribution and edge modes
   - Cyclotron orbits
   - Reconstructing the second Chern number
   - Direct observation of a 4D Hall non-linear response

The second Chern number quantizes a non-linear response

We expect a quantized non-linear response to both perturbative electric field f_ν and magnetic field $b_{\alpha\beta}$

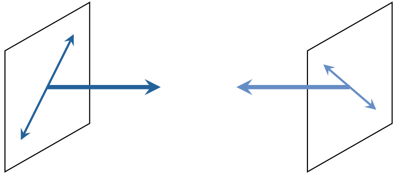
$$j^\mu_{\mathrm{NL}} = \frac{\mathcal{C}_2}{4\pi^2}\epsilon^{\mu\alpha\beta\nu} f_\nu b_{\alpha\beta}.$$

In other words, a magnetic perturbation $b_{\alpha\beta}$ induces a Hall conductivity $\propto \mathcal{C}_2 b_{\alpha\beta}$ in the orthogonal plane.

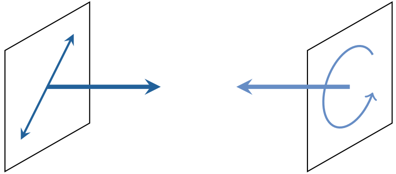
In our system, we implement a **magnetic perturbation b_{mr}** in the *mr* plane
$\Rightarrow$ appearance of a **Hall effect in the xy plane**

Implementation of the magnetic perturbation b_{mr}

We play with the polarizations of one x Raman beam.



linear polarizations $\Rightarrow$ spin hopping algebra J_+



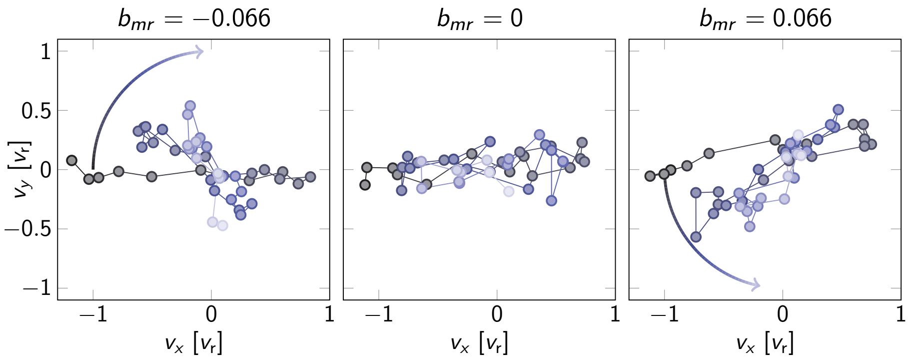
$$J_+ + \mathrm{i}\epsilon\{J_+, J_z\} \simeq J_+ \mathrm{e}^{\mathrm{i}m\epsilon}$$

r-hopping acquires a complex phase $\propto m \Rightarrow b_{mr}$ field

Foucault pendulum precession

We study the modification of cyclotron dynamics induced by the b_{mr} field.

We use $\omega_x = \omega_y$, i.e. isotropic harmonic trapping in the xy plane.



The measured precession rates match well the expected values, governed by the second Chern character ρ_2.

Outlook

Interacting many-body systems in a 4D quantum Hall structure.
Connection with quantum gravity and Yang-Mills theories?

Zhang & Hu, Science 2001
Barns-Graham et al, J. High Energ. Phys. 2018

Requirements:

- characterization of interactions between components of the spin $J = 8$
- control of the interaction range (spatial separation of m levels)

Extension to other high-dimensional topological systems

- Weyl semi-metals in 5D
- Quantum Hall systems in 6D

Lian & Zhang, Phys. Rev. B 2016
Petrides at al, Phys. Rev. B 2018
Lee at al, Phys. Rev. B 2018

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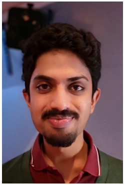

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Thank you for your attention!