Physics of composite fermions in the fractional quantum Hall effect

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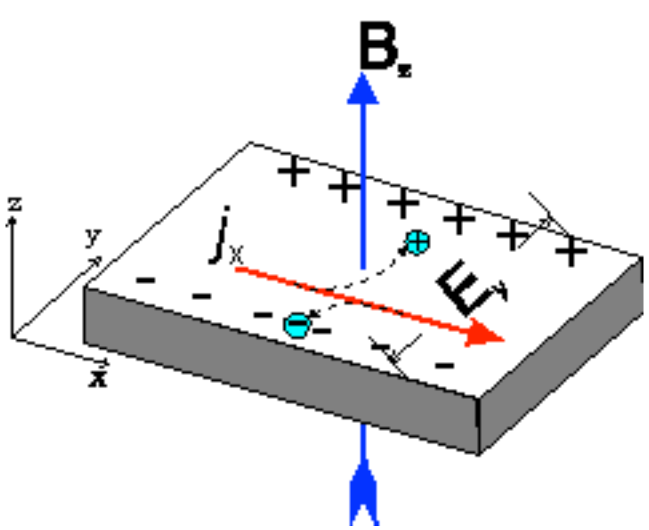
Plan

- Fractional quantum Hall effect
- Theories of the composite fermion: HLR and Dirac
- Bosonized composite Fermi surface
- Structure factor from composite fermions

Dung Xuan Nguyen and DTS, to be published

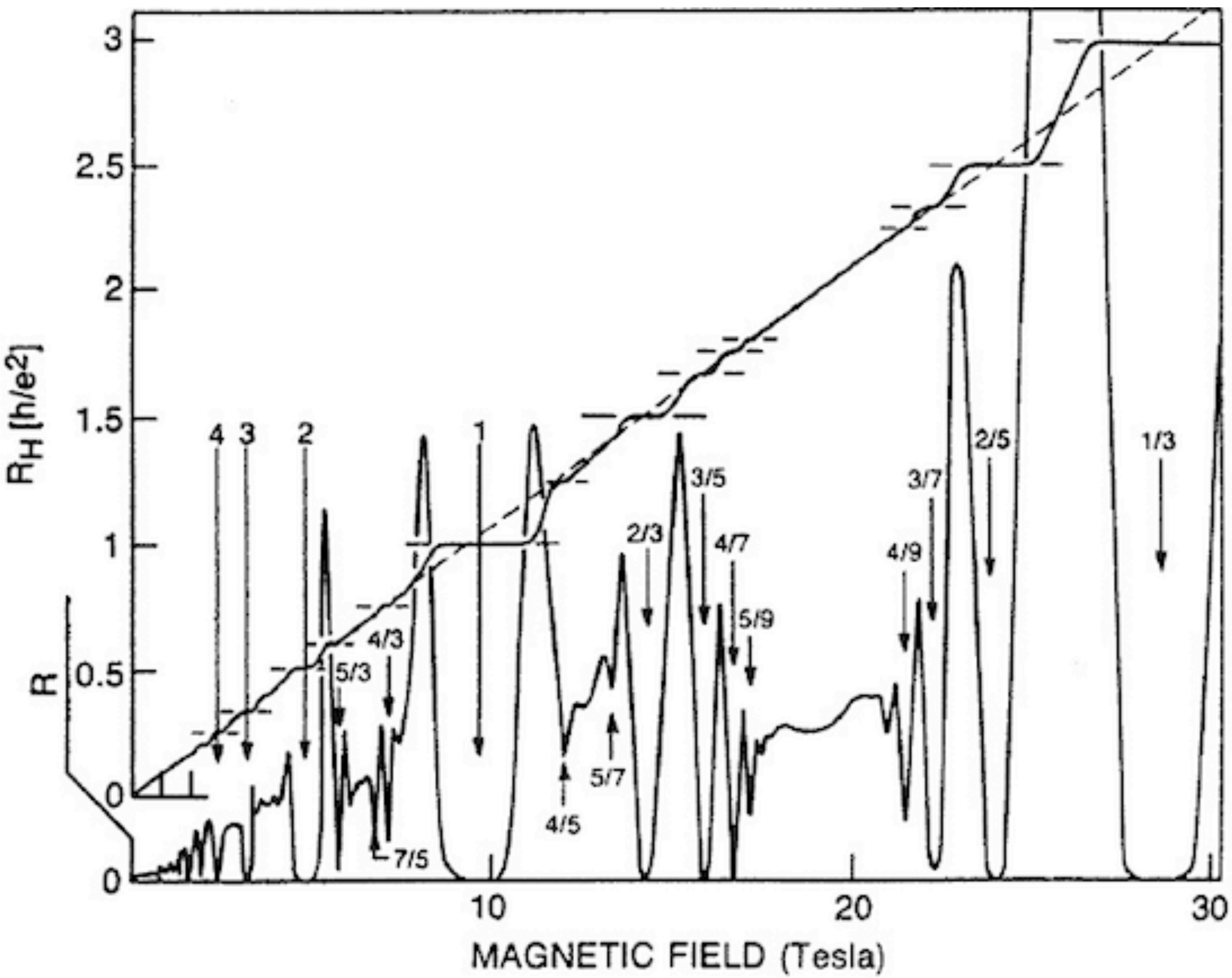
Quantum Hall Effect

2D electrons in a magnetic field



$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y$$

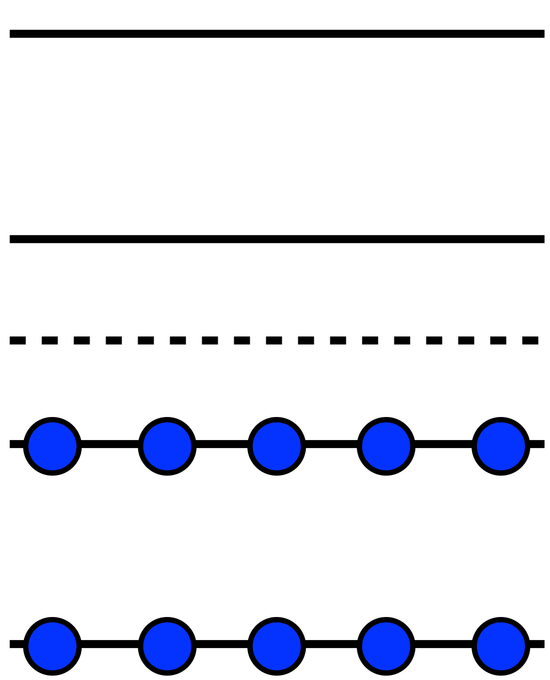
$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$



R_H [h/e²]
R
MAGNETIC FIELD (Tesla)
0
0.5
1
1.5
2
2.5
3
0
10
20
30
4
3
2
1
5/3
4/3
7/5
4/5
5/7
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3

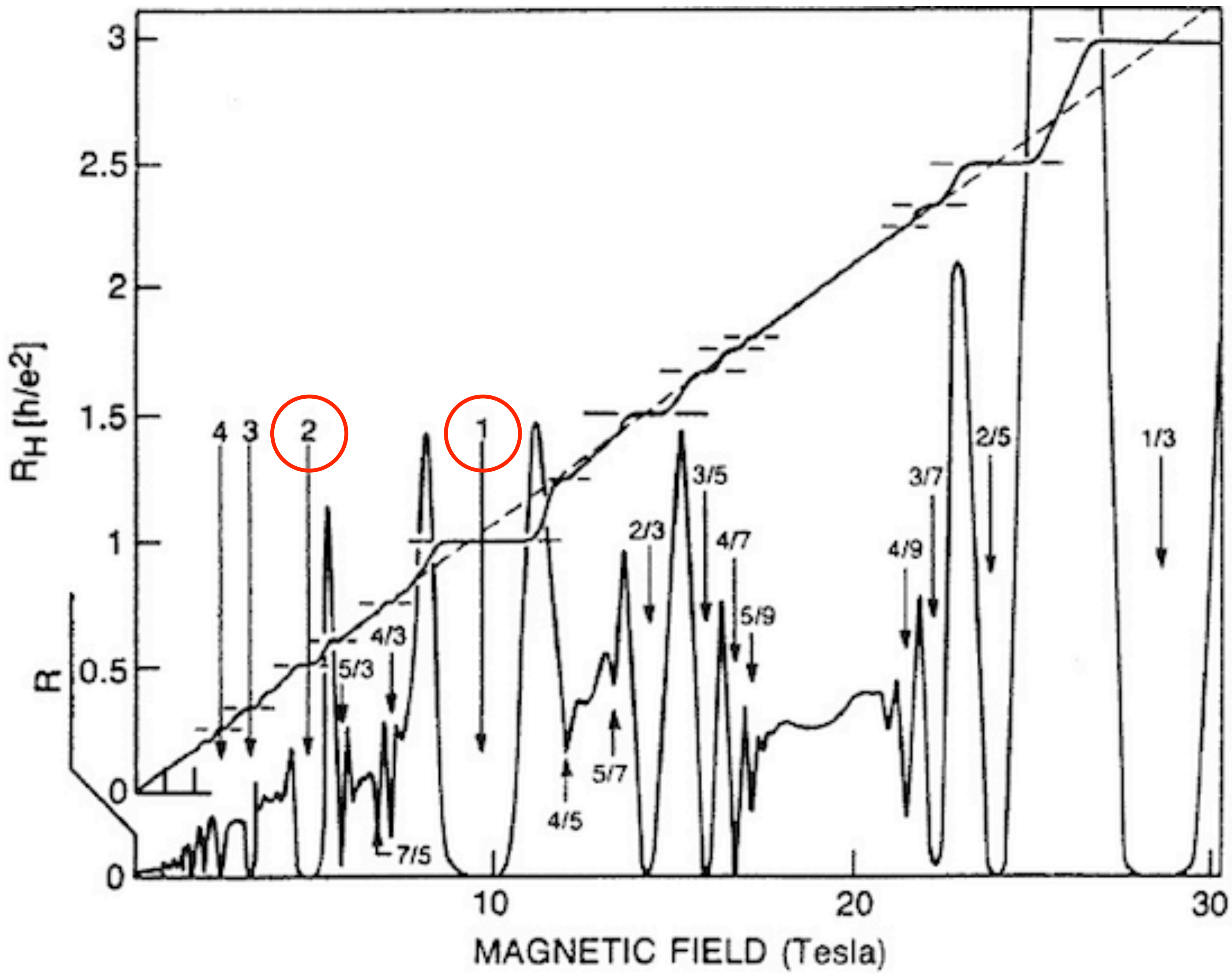
Integer quantum Hall effect

- Electrons completely fill n Landau levels
- gapped ground state
- interaction can be treated perturbatively



$$\Delta = \frac{B}{m}$$

degeneracy $\frac{BA}{2\pi}$

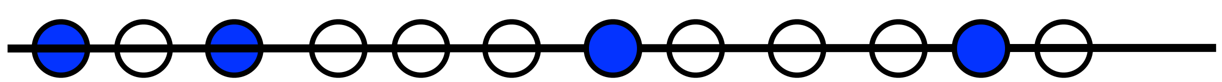


R_H [h/e²]
R
3
2.5
2
1.5
1
0.5
0
0
10
20
30
MAGNETIC FIELD (Tesla)
4
3
2
1
4/3
5/3
7/5
2/3
3/5
4/7
5/9
4/5
5/7
4/9
3/7
2/5
1/3

Fractional QHE

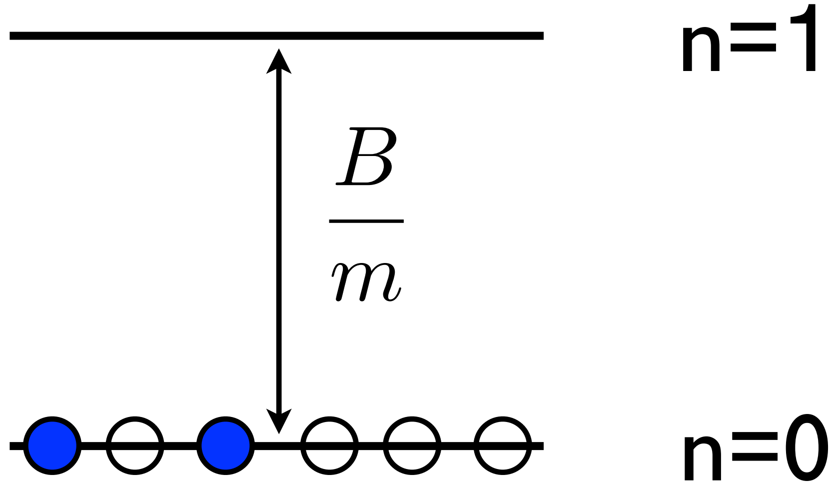
Electrons fill a fraction ν of a Landau level
Interactions cannot be treated perturbatively

FQHE exists in the limit of a single Landau level
for example, the lowest Landau level



Lowest Landau level limit

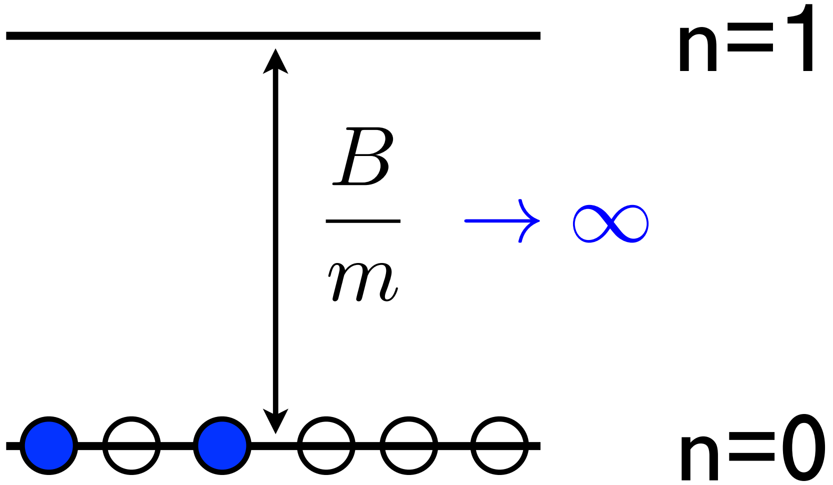
$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$



Lowest Landau level limit

$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

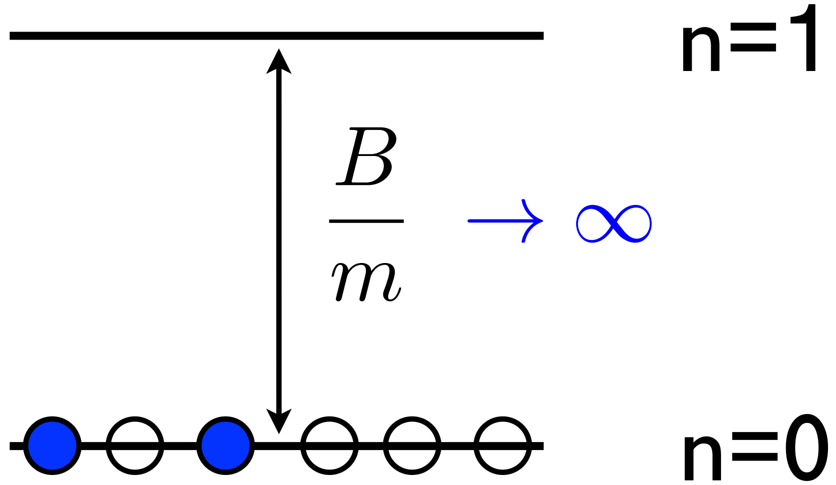
$m \to 0$



Lowest Landau level limit

$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

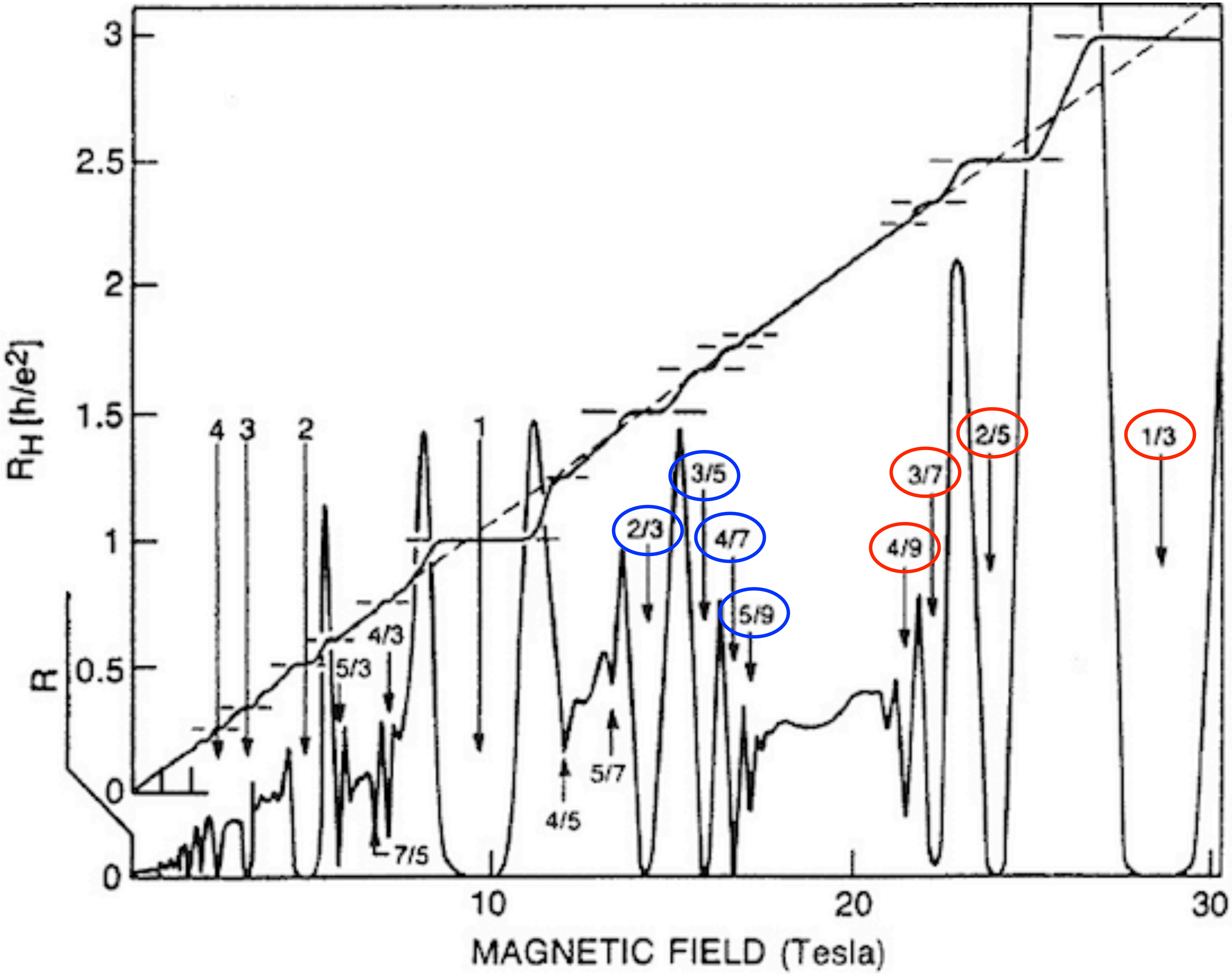
$m \to 0$



$$H = P_{\mathrm{LLL}} \sum_{a,b} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

Projection to
lowest Landau level

Jain's sequences



energy gap at $\nu = \dfrac{n+1}{2n+1}$ $\nu = \dfrac{n}{2n+1}$

Composite fermion

- Halperin-Lee-Read 1993: low-energy quasiparticle of half-filled Landau level: a “composite fermion”
- “attaching 2 flux quanta to an electrons”
- End result: an effective field theory of the composite fermion

HLR field theory

$$\mathcal{L} = i\psi^\dagger(\partial_0 - iA_0 + ia_0)\psi - \frac{1}{2m}|(\partial_i - iA_i + ia_i)\psi|^2 + \frac{1}{2}\frac{1}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

$$b = \nabla \times a = 2 \times 2\pi\psi^\dagger\psi$$ "flux attachment"

mean field: $$B_{\text{eff}} = B - b = B - 4\pi n$$

$$\nu = \frac{1}{2}$$ $$B_{\text{eff}} = 0$$

In half filled Landau level CFs form a Fermi surface

away from half filling: CFs in a magnetic field

Particle-hole symmetr

- One problem with HLR theory: lack of particle-hole symmetry
- Solving this problem: the Dirac composite fermion theory
- Modification to the HLR theory: CF has a Berry phase of π around the Fermi disk
- Composite fermion: particle-vortex dual of the electron

Particle-vortex duality

original fermion	composite fermion
magnetic field	density
density	magnetic field

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda + \frac{1}{8\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

$$\rho = \frac{\delta S}{\delta A_0} = \frac{B-b}{4\pi}$$

$$\frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle\bar{\psi}\gamma^0\psi\rangle = \frac{B}{4\pi}$$

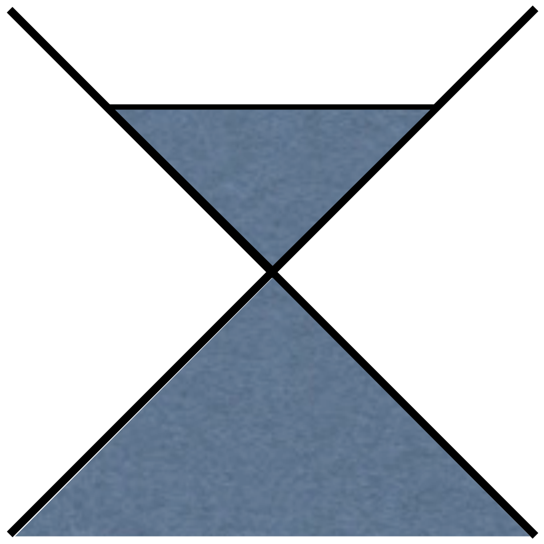
Mapping from electrons to CFs

electron

$$\nu = \frac{1}{2}, \quad B \neq 0$$

half-filled Landau level

composite fermion



$\rho \neq 0$ $\quad b = 0$

Fermi liquid of CFs

Deviation from half filling → CF in b field

Is there difference between HLR and Dirac theories?

- CFs have different densities in the 2 theories
  - HLR: equal to number of electrons
  - DCF: equal to 1/2 number of fluxes
- Can we distinguish the two theories based on the number of composite fermions?
- Problem: CF has a Fermi surface only when $\nu=1/2$, where the values in 2 theories are the same
- Can the density of CFs be defined away from $\nu=1/2$?
- Static structure factor

Static structure factor

- Equal time density-density correlation function

$$s(q) = \frac{1}{\rho_0} \int d\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}} \langle \rho(t,\mathbf{x}) \rho(t,\mathbf{0}) \rangle$$

if restricted to LLL states: "projected static structure factor"

$$\bar{s}(q) = s(q) - (1 - e^{-q^2/2})$$

For gapped states at small q $\quad \bar{s}(q) \sim q^4$

Can be read out from the wave function

Density correlator in HLR and DCF theories

$\langle \rho\rho \rangle_q$

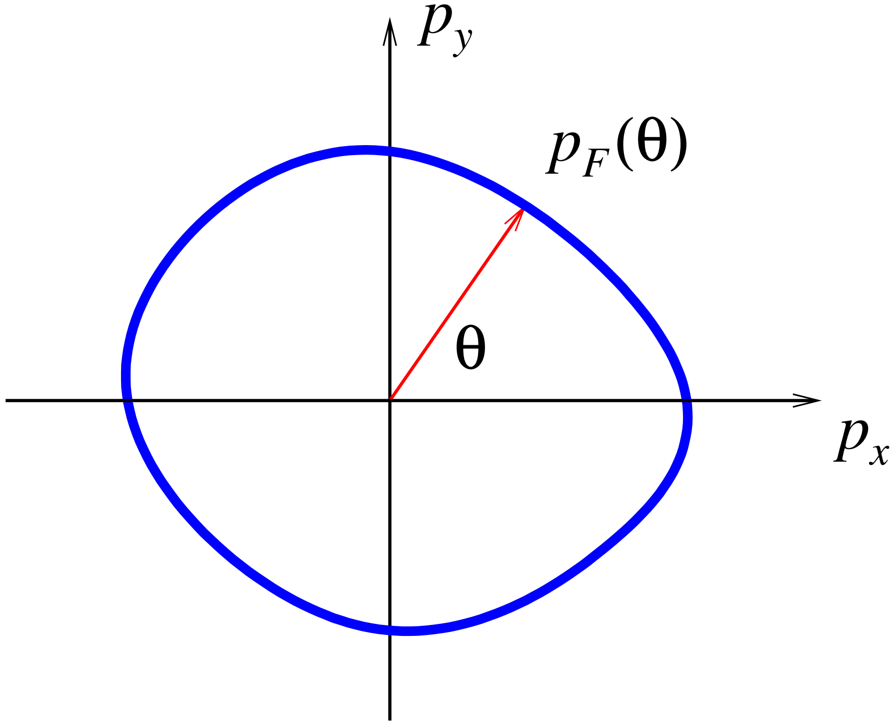
	$\nu = \frac{N}{2N+1}$	$\nu = \frac{N+1}{2N+1}$
HLR*	N^2	$(N+1)^2$
DCF	$N(N+1)$	$N(N+1)$

$$\times \frac{q^4}{16\pi(2N+1)}$$

*: in Galilean invariant MRPA (by Simon and Halperin)

- The result is actually very robust and depends only on the kinematics of the composite Fermi surface

Bosonic excitations



Low-energy, long-wavelength excitations: fluctuations of the shape of the Fermi surface

$$p_F(t, \mathbf{x}, \theta) = p_F^0 + \sum_{n=-\infty}^{\infty} u_n(t, \mathbf{x})\, e^{-in\theta}.$$

One scalar field per spin

u_0: density of composite fermions
$u_{\pm 1}$: momentum density
$u_{\pm 2}$: eccentricity of the Fermi disk

Algebra of shapes

Haldane

$$[u_n(\mathbf{x}),\, u_{-n}(\mathbf{x}')] = 2\pi b\, n \delta(\mathbf{x} - \mathbf{x}')$$

$$[u_n(\mathbf{x}),\, u_{-n-1}(\mathbf{x}')] = 2\pi \frac{\partial}{\partial z} \delta(\mathbf{x} - \mathbf{x}')$$

$$[u_n(\mathbf{x}),\, u_{-n+1}(\mathbf{x}')] = 2\pi \frac{\partial}{\partial \bar{z}} \delta(\mathbf{x} - \mathbf{x}')$$

To describe dynamics we need a Hamiltonian, e.g.

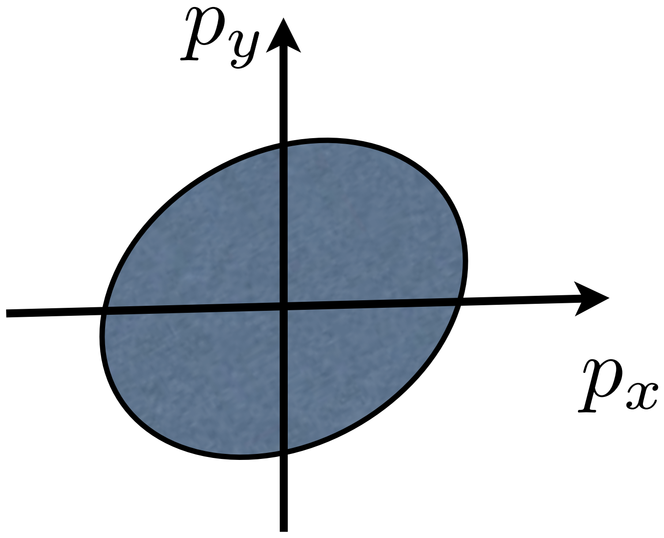
$$H = \frac{p_F^2}{4\pi\, m_*} \int d\mathbf{x} \sum_{n=-\infty}^{\infty} (1 + F_n) u_n(\mathbf{x}) u_{-n}(\mathbf{x})$$

$$\dot{u}_n = i[H,\, u_n]$$

Interpreting the result

- It turns out that in both the HLR and DCF theory one can relate the projected density with the dipole deformation of the composite Fermi surface

(requires electric dipole moment of CFs)



$$g_{ij}(x) \approx \delta_{ij} + h_{ij}$$
$$= \begin{pmatrix} 1+Q_1 & Q_2 \\ Q_2 & 1-Q_1 \end{pmatrix}$$

$$u_{\pm 2} \sim Q_1 \pm i Q_2$$

$$\delta\rho_{\rm el} = \frac{k_F^4}{16\pi}\frac{1}{Bb}\,\partial_i\partial_j h_{ij} \quad = \frac{k_F^4}{16\pi}\frac{1}{Bb}\,R[g_{ij}]$$

Haldane

- Computing the static structure factor becomes an algebraic exercise

$$[h_{\bar z\bar z}(\mathbf{x}),\ h_{zz}(\mathbf{y})] = \frac{4\pi b}{k_F^4}\delta(\mathbf{x}-\mathbf{y})$$

a $\qquad$ $a^\dagger$

$$h_{\bar z\bar z}|0\rangle = 0$$

$$\delta\rho_{\rm el} \sim \partial_z^2 h_{\bar z\bar z} + \partial_{\bar z}^2 h_{zz}$$

$$\langle\rho\rho\rangle_q = \frac{\pi}{4}\frac{\rho_{\rm CF}^2}{b}(q\ell_B)^4$$

Lessons

- The formula reproduces the results of more detailed (and rather cumbersome) calculations in the HLR and DCF theories; explains their difference
- also suggests that it is not easy to modify the HLR theory to get the correct result
  - the problems is in the kinematics, not dynamics
- In fact modifications of the HLR theory have been tried but I was told that none of the considered modifications give the correct structure factor while preserving Galilean invariance Chong Wang, B. Halperin 2017-2018

Outlook

- The static structure factor carries direct information about the density of the composite fermion
- Can give information about the Berry phase
- Technique can be applied to $\nu=1/4$, where the Berry phase is not fixed by symmetry
- Higher-harmonic deformations of Fermi surface? Physical implications
- Higher-spin description of FQHE and non-Fermi liquids?

$$\lim_{N\to\infty}\left[\bar{s}_4\left(\frac{N}{2nN+1}\right)-\bar{s}_4\left(\frac{N}{2nN-1}\right)\right]=\frac{\eta}{4}$$