Spin-Locality and Star-Product Functions in Higher-Spin Theory

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Space-Time and Particles

Maximally symmetric space-times are characterized by their symmetry group S.

Anti-de Sitter space: $AdS_d = O(d-1,2)/O(d-1,1)$, $S = O(d-1,2)$

Minkowski contraction $M^4 = ISO(1,3)/SO(1,3)$, $S = ISO(1,3)$

Particles are associated with UIRs S**-modules** Wigner 1939

realized by spaces of solutions to S**-symmetric field equations**.

Simplest example: Klein-Gordon equation for spin-zero particle of mass m

$$(\Box + m^2)\Phi(x) = 0$$

Particles differ by their masses (lowest energies) and spins associated with weights E_0 **and** $\vec{s}$ **of** $O(2) \times O(d-1) \subset O(2, d-1)$, **respectively.**

Respective modules are denoted $D(E_0, \vec{s})$.

Massless Particles and Gauge Symmetries

In the special case of massless fields energy weight E_0 takes a spin-dependent minimal value compatible with unitarity

$$E_0 = E_0(\vec{s})\,, \qquad E_0(s) = s+1 \quad (d=4)$$

In this case $D(E_0, \vec{s})$ is indecomposable. The unitary module is its quotient $D'(E_0, \vec{s}) = D(E_0, \vec{s})/\tilde{D}(\tilde{E}_0, \tilde{\vec{s}})$

In physics factorization is realized by gauge symmetry becoming one of the fundamental physical principles.

Spin-s massless fields with $s = 1$ or 2 mediate electromagnetic and gravitational interactions.

Fronsdal Fields

Fronsdal fields 1978

All $m = 0$ HS fields are gauge fields

$\phi_{n_1 \ldots n_s}$ is a rank-s symmetric tensor obeying $\phi^k{}_k{}^m{}_{m n_5 \ldots n_s} = 0$

Gauge transformation:

$$\delta \phi_{n_1 \ldots n_s} = \partial_{(n_1} \varepsilon_{n_2 \ldots n_s)} \,, \qquad \varepsilon^m{}_{m n_3 \ldots n_{s-1}} = 0$$

The challenge is to find a nonlinear deformation of Fronsdal field equations that respects a nonlinear deformation of the gauge symmetry

In 60-70th it was argued (Weinberg, Coleman-Mandula) that HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

Green light: AdS background with $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \to 0$ is singular

Non-Locality of HS Gauge Theory

HS interactions contain higher derivatives:

A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984)

HS symmetries Fradkin, MV 1986 are infinite dimensional

Infinite towers of spins imply infinite towers of derivatives.

How (non)local is HS gauge theory?

HS symmetries do not commute with space-time symmetries

$$[T^n, T^{HS}] = T^{HS}, \qquad [T^{nm}, T^{HS}] = T^{HS}$$

Riemann geometry is not appropriate for HS theory

The mildest possibility: each vertex with fields of definite spins is local.

All vertices we have found so far up to the quintic order are spin-local:

local in the spinor space.

The worst option: HS theory is essentially nonlocal Sleight, Taronna 2017

Locality and Non-Locality

Equations of motion in perturbatively local field theory

$$E(\partial,\phi)=0\,,\qquad E(\partial,\phi)=\sum_{k=0,l=1}^{\infty}a^{n_1\ldots n_k}_{a_1\ldots a_l}\partial_{n_1}\ldots\partial_{n_k}\phi^{a_1}\ldots\phi^{a_l}$$

have a finite number of non-zero coefficients $a^{n_1\ldots n_k}_{a_1\ldots a_l}$ at any order l.

In non-local field theory this is not demanded.

Theories like HS theory involve infinite towers of fields: for instance Fronsdal fields of all spins. Hence a_i may take an infinite number of values. It makes sense to distinguish between the following cases

local: finite number of derivatives at any order

spin-local: finite number of derivatives at any order for any finite subset of fields

non-local: infinite number of derivatives at some order for some finite subset of fields.

Field Redefinitions

A local theory remains local under perturbatively local field redefinitions

$$\delta\phi^b = \sum_{k=0,l=1}^{\infty} a^{b n_1 \dots n_k}_{a_1 \dots a_l} \partial_{n_1} \dots \partial_{n_k} \phi^{a_1} \dots \phi^{a_l}$$

with a finite number of non-zero coefficients at any given order.

Application of a nonlocal field redefinition makes it seemingly non-local.

Given non-locally looking field theory, the essential question is whether or not it admits a choice of variables making it local or spin-local.

One of the central problems in the HS gauge theory is to find an appropriate setup making it (spin-)local.

In this talk it will be sketched how this problem is reformulated in terms of star-product functions leading to the proper setup.

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

Unfolded dynamics: multidimensional generalization

$$\frac{\partial}{\partial t} \to \mathsf{d}\,, \qquad q^i(t) \to W^\Omega(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p} W^\Omega_{n_1\ldots n_p}(x)$$

$$\mathsf{d}\mathbf{W}^\Omega(\mathbf{x}) = \mathbf{G}^\Omega(\mathbf{W}(\mathbf{x}))\,, \qquad \mathsf{d} = \mathbf{dx^n}\partial_\mathbf{n} \qquad \text{MV} \quad 1988$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Phi_1\ldots\Phi_n} W^{\Phi_1} \wedge \ldots \wedge W^{\Phi_n}$$

Covariant first-order differential equations

$d > 1$: **Compatibility conditions**

$$G^\Phi(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Phi} \equiv 0$$

L_∞, A_∞, Q-**manifolds, etc** 1988, 2005

Q–Manifold Interpretation

$$\mathsf{d}f(W) = Q(f(W))$$

with

$$Q := G^A(W)\frac{\partial}{\partial W^A}\,, \qquad Q^2 = 0$$

being a homological vector field in the target space of W

Space-Time as Vacuum Solution

Let ω^α be a set of one-forms:

$$G^\alpha(\omega) = -f^\alpha_{\beta\gamma}\omega^\beta \wedge \omega^\gamma \qquad \textbf{space-time}$$

Consistency: Jacobi identity for a Lie algebra s

Unfolded equations: flatness condition

$$\mathrm{d}\omega^\alpha + f^\alpha_{\beta\gamma}\omega^\beta \wedge \omega^\gamma = 0$$

Free Fields as s-Modules

Let W^α contain p-forms $\mathcal{C}^i$ (e.g. 0-forms) and G^i be linear in ω and C

$$G^i(\omega, \mathcal{C}) = -\omega^\alpha (T_\alpha)^i{}_j \wedge \mathcal{C}^j \,.$$

Compatibility condition implies that $(T_\alpha)^i{}_j$ form some representation T of s, acting in a carrier space V of $\mathcal{C}^i$. The unfolded equation is

$$D_\omega \mathcal{C} = 0$$

$D_\omega \equiv \mathsf{d} + \omega$: covariant derivative in the s-module V.

The covariant constancy equation : linear equations in a chosen s-symmetric background described by the flat connection ω : $(D_\omega)^2 = 0$.

s: global symmetry

$$\delta \mathcal{C}^i(x) = \varepsilon^\alpha(x)(T_\alpha)^i{}_j \mathcal{C}^j(x)\,, \qquad D_\omega \varepsilon = 0$$

Properties

- General applicability
- Manifest (HS) gauge invariance
- Invariance under diffeomorphisms
- Clear group-theoretical interpretation of fields and equations in terms of modules and Chevalley-Eilenberg (Hochschild in HS theory) cohomology of a symmetry algebra s

  Background fields: flat connection of s

  Fields: s-modules

  Equations: covariant constancy conditions
- Local degrees of freedom are in 0-forms $C^i(x_0)$ at any $x = x_0$

  (as $q(t_0)$) infinite-dimensional module dual to the space of single-particle states: $C^i(x_0)$ moduli of solutions
- Independence of ambient space-time

  Geometry is encoded by $G^\Omega(W)$

Unfolding and holographic duality

Unfolding unifies various dualities including holographic duality

Extension of space-time without changing dynamics by letting the exterior derivative d and differential forms W live in a larger space

$$\mathsf{d} = dX^n \frac{\partial}{\partial X^n} \to \tilde{\mathsf{d}} = dX^n \frac{\partial}{\partial X^n} + d\widehat{X}^{\hat{n}} \frac{\partial}{\partial \widehat{X}^{\hat{n}}}\,, \qquad dX^n W_n \to dX^n W_n + d\widehat{X}^{\hat{n}} \widehat{W}_{\hat{n}}\,,$$

$\widehat{X}^{\hat{n}}$ are additional coordinates

$$\tilde{\mathsf{d}} W^{\Omega}(X, \hat{X}) = G^{\Omega}(W(X, \hat{X}))$$

Two unfolded systems in different space-times are equivalent (dual) if they have the same unfolded form. Given unfolded system generates a class of holographically dual theories in different dimensions.

Useful applications:

$sp(8)$-invariant formulation of $4d$ massless equations 2001

derivation of superfield formulations of SUSY models (Misuna, MV (2013))

HS holography 2012,2015

$2 \times 2 = 4$: Spinor Language for $4d$ Models

$$x^n = \sigma^n_{\alpha\dot\beta} x^{\alpha\dot\beta} \quad n = 0, 1, 2, 3, \quad \alpha = 1, 2, \dot\alpha = 1, 2$$

$$x^n x^m \eta_{nm} = \det |x^{\alpha\dot\alpha}| = x^{\alpha\dot\alpha} x^{\beta\dot\beta} \varepsilon_{\alpha\beta} \varepsilon_{\dot\alpha\dot\beta} \qquad sl_2(\mathbb{C}) \sim o(3,1)$$

$Sp(4)$ **symmetric space-time** AdS_4 **as vacuum geometry**

$$\mathbf{R}_{\alpha\beta} := \mathbf{d}\omega_{\alpha\beta} + \omega_{\alpha\gamma}\omega_\beta{}^\gamma - \mathbf{H}_{\alpha\beta} = \mathbf{0}\,, \qquad \mathbf{R}_{\alpha\dot\beta} := \mathbf{d} + \omega_{\alpha\gamma}\mathbf{h}^\gamma{}_{\dot\beta} + \overline{\omega}_{\dot\beta\dot\delta}\mathbf{h}_\alpha{}^{\dot\delta} = \mathbf{0}$$

$$\mathbf{H}^{\alpha\beta} := \mathbf{h}^{\alpha\dot\alpha} \wedge \mathbf{h}^\beta{}_{\dot\alpha}\,, \qquad \overline{\mathbf{H}}^{\dot\alpha\dot\beta} := \mathbf{h}^{\alpha\dot\alpha} \wedge \mathbf{h}_\alpha{}^{\dot\beta}$$

$4d$ Massless Fields

Infinite set of integer spins 1988

$\omega(y,\bar y \mid x)\,, \quad C(y,\bar y \mid x) \quad f(y,\bar y) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} f_{\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m} y^{\alpha_1}\dots y^{\alpha_n}\bar y^{\dot\alpha_1}\dots\bar y^{\dot\alpha_m}$

$$\omega(\mu y,\mu\bar y|x) = \mu^{2(s-1)}\omega(y,\bar y|x)\,, \qquad C(\mu y,\mu^{-1}\bar y|x) = \mu^{\pm 2s}C(y,\bar y|x)$$

ω: **finite number of components (derivatives) for definite spin**

C: **infinite number of components (derivatives) for definite spin**

Fronsdal fields:

$$\omega(\mu y,\mu^{-1}\bar y|x) = \omega(y,\bar y|x)\,, \qquad C(0,0|x)$$

All other components of ω and C are derivatives of the Fronsdal fields

Free Field Unfolded Massless Equations

The full unfolded system for free massless bosonic fields is

$$\star \qquad R_1(y,\overline{y}\mid x) = \frac{i}{4}\left(\eta\overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\overline{y}^{\dot\alpha}\partial\overline{y}^{\dot\beta}}\, C(0,\overline{y}\mid x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\, C(y,0\mid x)\right)$$

$$\star\star \qquad \tilde{D}_0 C(y,\overline{y}\mid x) = 0$$

$$R_1(y,\bar{y}\mid x) := D_0^{ad}\omega(y,\bar{y}\mid x) \qquad D_0^{ad} = D^L - h^{\alpha\dot\beta}\left(y_\alpha\frac{\partial}{\partial\bar{y}^{\dot\beta}} + \frac{\partial}{\partial y^\alpha}\bar{y}_{\dot\beta}\right)$$

$$\tilde{D}_0 = D^L + h^{\alpha\dot\beta}\left(y_\alpha\bar{y}_{\dot\beta} + \frac{\partial^2}{\partial \mathbf{y}^\alpha\partial\bar{\mathbf{y}}^{\dot\beta}}\right) \qquad D^L = \mathsf{d}_x - \left(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta} + \bar\omega^{\dot\alpha\dot\beta}\bar{y}_{\dot\alpha}\frac{\partial}{\partial\bar{y}^{\dot\beta}}\right)$$

$\star\star$ **implies that higher-order terms in** y **and** $\bar{y}$ **describe higher-derivative descendants of the primary HS fields**

Space-Time Versus Fibers

Perturbative unfolded equations

$$\mathsf{d}_x C = \sigma_- C + \text{lower-derivative and nonlinear terms}$$

$$\sigma_- := \mathbf{h}^{\alpha\dot{\beta}} \frac{\partial^2}{\partial \mathbf{y}^\alpha \partial \bar{\mathbf{y}}^{\dot{\beta}}}\,, \qquad \sigma_-^2 = 0\,.$$

σ_- **is the substitute of space-time differential in the unfolded dynamics formalism with respect to which spin-locality has to be defined in general unfolded system.**

Pattern of unfolded dynamics like dynamical (Fronsdal) fields, gauge symmetries, field equations etc are encoded by the σ_- **cohomology**

For instance the right-hand-sides of Fronsdal equations, *i.e.* **currents, are associated with** $H_2(\sigma_-)$

HS Vertices

The problem: consistent nonlinear corrections 1988 in the local frame

$$\mathsf{d}_x\omega = -\omega * \omega + \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \dots\,,$$

$$\mathsf{d}_x C = -[\omega, C]_* + \Upsilon(\omega, C, C) + \dots$$

$$(f*g)(Y) = \int dSdT \,\exp iS_A T^A f(Y+S)g(Y+T)\,, \qquad Y^A = (y^\alpha, \bar{y}^{\dot\alpha})$$

$\Upsilon(\omega,\omega,C)$ **Central On-Mass-Shell Theorem (1988)**

$\Upsilon(\omega,C,C)$ **zero-form sector corrections** Didenko, Gelfond, Korybut, MV 2016-2018

$\Upsilon(\omega,\omega,C,C)$ Didenko, Gelfond, Korybut, MV 2019 **has the structure**

$$\Upsilon(\omega,\omega,C,C) = \Upsilon^{\eta\eta} + \Upsilon^{\bar\eta\bar\eta} + \Upsilon^{\eta\bar\eta}\,,$$

where η is an arbitrary complex parameter of the $d=4$ HS theory.

$$\Upsilon^{\eta\eta}(\omega,\omega,C,C) = \Upsilon^{\eta\eta}_{\omega\omega CC} + \Upsilon^{\eta\eta}_{\omega C\omega C} + \Upsilon^{\eta\eta}_{C\omega\omega C} + \Upsilon^{\eta\eta}_{C\omega C\omega} + \Upsilon^{\eta\eta}_{CC\omega\omega} + \Upsilon^{\eta\eta}_{\omega CC\omega}\,,$$

$$\Upsilon^{\bar\eta\bar\eta}(\omega,\omega,C,C) = \Upsilon^{\bar\eta\bar\eta}_{\omega\omega CC} + \Upsilon^{\bar\eta\bar\eta}_{\omega C\omega C} + \Upsilon^{\bar\eta\bar\eta}_{C\omega\omega C} + \Upsilon^{\bar\eta\bar\eta}_{C\omega C\omega} + \Upsilon^{\bar\eta\bar\eta}_{CC\omega\omega} + \Upsilon^{\bar\eta\bar\eta}_{\omega CC\omega}\,,$$

$$\Upsilon^{\eta\bar\eta}(\omega,\omega,C,C) = \Upsilon^{\eta\bar\eta}_{\omega\omega CC} + \Upsilon^{\eta\bar\eta}_{\omega C\omega C} + \Upsilon^{\eta\bar\eta}_{C\omega\omega C} + \Upsilon^{\eta\bar\eta}_{C\omega C\omega} + \Upsilon^{\eta\bar\eta}_{CC\omega\omega} + \Upsilon^{\eta\bar\eta}_{\omega CC\omega}$$

Spin-Locality in $4d$ HS Theory

Nonlinear corrections have the form

$$F(P^{ij},\bar{P}^{kl})C(Y_1)\dots C(Y_n)\,,\qquad P^{ij}:=\frac{\partial}{\partial y_i^{\alpha}}\frac{\partial}{\partial y_{j\alpha}}\,,\qquad \bar{P}^{ij}:=\frac{\partial}{\partial \bar{y}_i{}^{\dot\alpha}}\frac{\partial}{\partial \bar{y}_{j\dot\alpha}}$$

with some non-polynomial functions $F(P^{ij},\bar{P}^{kl})$

Spin-locality: polynomiality of $F(P^{ij},\bar{P}^{kl})$ in either P or $\bar{P}$

Projector on fixed spins relates degree in P^{ij} and $\bar{P}^{kl}$ to each other!

Potential nonlocality follows from the nonlocality of the star-product underlying nonlinear HS equations

$$(f\star g)(Z,Y)=\int dSdT\,\exp iS_AT^A\,f(Z+S,Y+S)g(Z-T,Y+T)$$

$$[Y_A,Y_B]_\star=-[Z_A,Z_B]_\star=2iC_{AB}$$

$Z-Y\,:\,Z+Y$ normal ordering

Via relation between space-time derivatives and $Y;Z$-derivatives non-locality of the star product induces space-time non-locality

Nonlinear System via Doubling of Spinors

Direct analysis of nonlinear deformation of the free unfolded equations is possible in the lower orders 1988 but quickly gets complicated: Hochschild cohomology for the HS algebra

(MV 1989, Skvortsov and Sharapov 2016-on).

The efficient trick MV 1992 reduces the problem to De Rham cohomology with respect to additional spinor variables in presence of Klein operators K (Cherednik algebra)

$$\omega(Y;K|x) \longrightarrow W(Z;Y;K|x)\,, \qquad C(Y;K|x) \longrightarrow B(Z;Y;K|x)$$

$Y^A = (y^\alpha, \bar{y}^{\dot\alpha})$, $Z^A = (z^\alpha, \bar{z}^{\dot\alpha})$

Some of the nonlinear HS equations determine the dependence on Z_A in terms of "initial data" $\omega(Y;K|x)$ and $C(Y;K|x)$

$S(Z;Y;K|x) = \theta^A S_A(Z;Y;K|x)$ is a connection along Z^A $(\theta^A \equiv dZ^A)$

Klein operators $K = (k, \bar{k})$ generate chirality automorphisms

$$kf(A) = f(\tilde{A})k\,, \qquad A = (a_\alpha\,, \bar{a}_{\dot\alpha}) : \qquad \tilde{A} = (-a_\alpha\,, \bar{a}_{\dot\alpha})$$

Nonlinear HS Equations

$$
\begin{cases}
\mathrm{d}W + W \star W = 0 \\
\mathrm{d}B + W \star B - B \star W = 0 \\
\mathrm{d}S + W \star S + S \star W = 0 \\
\mathbf{S} \star \mathbf{B} - \mathbf{B} \star \mathbf{S} = \mathbf{0} \\
\mathbf{S} \star \mathbf{S} = \mathbf{i}(\theta^{\mathbf{A}}\theta_{\mathbf{A}} + \eta\theta^{\alpha}\theta_{\alpha}\mathbf{B} \star \mathbf{k} \star \kappa + \bar{\eta}\bar{\theta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}\mathbf{B} \star \mathbf{k} \star \bar{\kappa})
\end{cases}
$$

1992

Inner Klein operators:

$$\kappa = \exp i z_\alpha y^\alpha\,, \qquad \bar{\kappa} = \exp i \bar{z}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}}\,, \qquad \kappa \star f = \tilde{f} \star \kappa\,, \qquad \kappa \star \kappa = 1$$

Dynamical content is located in the x-independent twistor sector

$\eta = \exp i\varphi$ **is a free phase parameter suggesting 3d bosonization.**

The non-zero curvature has the form of Z_2-Cherednik algebra

Knowing solution for any $B = const$ allows us to reconstruct the unfolded system and solutions for any fields $B(x)$ by the integrating flow Prokushkin, MV (1998)

Perturbative Analysis

Vacuum solution

$$B_0 = 0\,, \qquad S_0 = \theta^A Z_A\,, \qquad W_0 = \frac{1}{2} w^{AB}(x) Y_A Y_B$$

$$\mathsf{d} W_0 + W_0 \star W_0 = 0\,, \qquad w^{AB} : AdS_4$$

$$[\mathbf{S_0}\,,\mathbf{f}]_\star = -\mathbf{2id_Z f}\,, \qquad \mathsf{d}_{\mathbf{Z}} = \theta^{\mathbf{A}} \frac{\partial}{\partial \mathbf{Z^A}}$$

First-order fluctuations

$$B_1 = C(Y)\,, \qquad S = S_0 + S_1\,, \qquad W = W_0(Y) + W_1(Y) + W_0(Y) C(Y)$$

Order-n equations containing S have the form

$$\mathsf{d}_Z U_n(Z;Y|dZ) = V[U_{<n}](Z;Y|\theta) \qquad \mathsf{d}_Z V[U_{<n}](Z;Y|\theta) = 0$$

can be solved as

$$U_n(Z;Y|\theta) = \mathsf{d}^*_Z V[U_{<n}](Z;Y|\theta) + \mathbf{h(Y)} + \mathsf{d}_Z \epsilon(Z;Y|\theta)$$

$$\mathsf{d}^*_Z V(Z;Y|\theta) = (Z^A - Q^A)\frac{\partial}{\partial \theta^A} \int_0^1 \frac{dt}{t} V(tZ + (1-t)Q; Y|t\theta)$$

Interpretation

The contracting homotopy freedom encodes:

All possible gauge choices in d_z-exact forms $\mathsf{d}_z\epsilon(Z;Y|dZ)$

All possible choices of field variables in d_z cohomology $\mathbf{h(Y)}$

Any unfolded HS system is associated with one or another solution to the nonlinear HS system.

Unfolded equations that appear in the sector of d_Z cohomology automatically reproduce consistent HS vertices solving the Hochschild cohomology problem.

How to single out the proper (e.g., minimally nonlocal) frames?

Spin-local limit: $\beta \to -\infty$ with $Q_A = \beta \frac{\partial}{\partial Y^A}$

Didenko, Gelfond, Korybut, MV 1909.04876

Local vertices up to the quintic order!

Shifted Homotopy

Contracting homotopy $\Delta_{q,\beta}$

$$\Delta_{q,\beta} f(z,y,\theta) := \int \frac{d^2u d^2v}{(2\pi)^2} e^{(iv_\alpha u^\alpha)} \int_0^1 \frac{dt}{t} (z-u+q)^\alpha \frac{\partial}{\partial\theta^\alpha} f(tz+(1-t)(u-q), \beta v+y, t\theta)$$

Obeys resolution of identity

$$\{\mathsf{d}_Z\,, \Delta_a\} + h_a = Id\,.$$

with the cohomology projector

$$h_{q,\beta}(f(z,y,\theta)) = \int \frac{d^2u d^2v}{(2\pi)^2} \exp iv_\alpha u^\alpha f(u-q, \beta v + y, 0)$$

Spin-local limit: $\beta \to -\infty$.

Class $\mathcal{H}$

Typical functions

$$f(z,y,\theta)=\int d\tau\phi(\tau z,(1-\tau)y,\tau\theta,\tau)\exp[i\tau z_\alpha y^\alpha]$$

$$\phi(\tau_1 z,\tau_2 y,\tau_1\theta,\tau_1)=\frac{\tau_2}{\tau_1}\psi(\tau_1 z,\tau_2 y,\frac{\tau_1}{\tau_2}\theta,\tau_1)$$

Space of fields $\mathcal{H}$ 1502.02271

$$\mathcal{H}:=\oplus_{p=0}^{2}\mathcal{H}_p\,.$$

$\mathcal{H}_p$ is spanned by such p-forms in θ that

$$\lim_{\tau\to 0}\tau^{1-p+\varepsilon}\phi(w,u,\theta,\tau)=0\,,\qquad \lim_{\tau\to 1}(1-\tau)^{p-1+\varepsilon}\phi(w,u,\theta,\tau)=0\qquad \forall\varepsilon>0\,.$$

Higher-Spin Star Product

$\mathcal{H}$ **forms an algebra with respect to the star product using** 1502.02271

$$
\begin{aligned}
f_1 \star f_2 \quad &= \int_0^1 d\tau_1 \int_0^1 d\tau_2 \int d^2 s d^2 t \,\mathsf{exp}\, i[\tau_1 \circ \tau_2 z_\alpha y^\alpha + s_\alpha t^\alpha] \\
&\times \phi_1(\tau_1((1-\tau_2)z - \tau_2 y + s), (1-\tau_1)((1-\tau_2)y - \tau_2 z + s), \tau_1\theta, \tau_1) \\
&\times \phi_2(\tau_2((1-\tau_1)z + \tau_1 y - t), (1-\tau_2)((1-\tau_1)y + \tau_1 z + t), \tau_2\theta, \tau_2)\,,
\end{aligned}
$$

$$
\tau_1 \circ \tau_2 = \tau_1(1-\tau_2) + \tau_2(1-\tau_1)\,.
$$

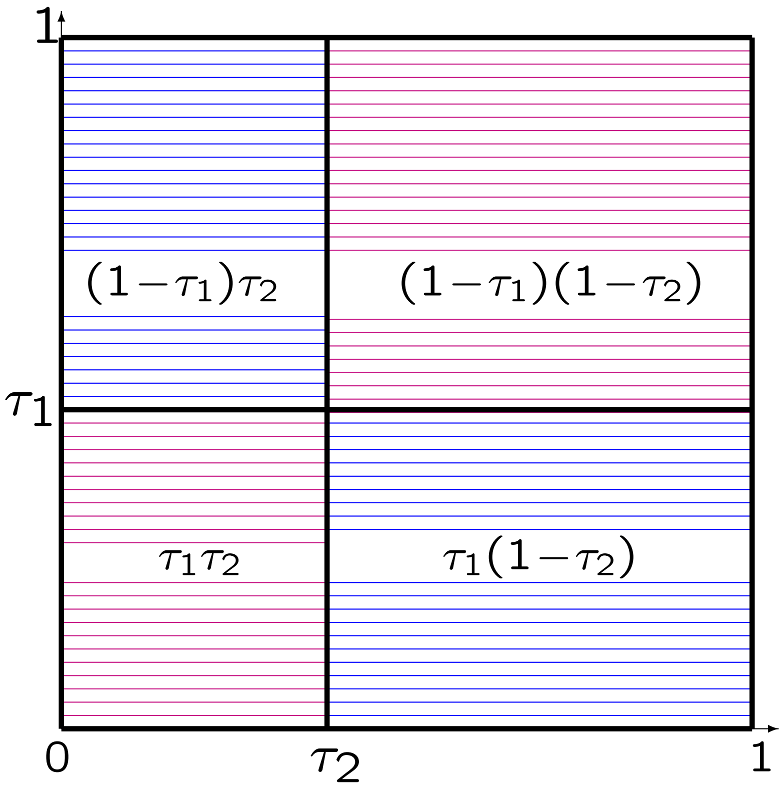
The product $\circ$ is commutative and associative.

Analysis is based on the elementary facts that

$$
\int_0^1 d\tau_1 \int_0^1 d\tau_2 \delta(\tau - \tau_1 \circ \tau_2) = -\frac{1}{2}\log((1-2\tau)^2),
$$

and that a part is smaller than the entire

Magic Square



$$1 - \tau_1 \circ \tau_2 = \tau_1\tau_2 + (1-\tau_2)(1-\tau_1) \leq 1$$

$$\tau_1 \circ \tau_2 = \tau_1(1-\tau_2) + (1-\tau_1)\tau_2 \leq 1$$

$\mathcal{H}^{0+}$ and $\mathcal{H}^{+0}$

$\mathcal{H}^{0+}$ and $\mathcal{H}^{+0}$ have polynomially softer behaviour either at $\tau \to 1$ or $\tau \to 0$, respectively:

$$f \in \mathcal{H}_p^{0+}: \quad \exists\, \varepsilon > 0: \quad \lim_{\tau \to 1} (1-\tau)^{p-1-\varepsilon} \phi(w,u,\theta,\tau) = 0$$

$$f \in \mathcal{H}_p^{+0}: \quad \exists \varepsilon > 0\;: \quad \lim_{\tau \to 0} \tau^{1-p-\varepsilon} \phi(w,u,\theta,\tau) = 0$$

$\mathcal{H}^{0+}$ forms a subalgebra of $\mathcal{H}$ conjectured to be related to local field redefinitions. MV 2015

Two-sided ideal of $\mathcal{H}$

$$\mathcal{I} := \mathcal{H}^{0+} \cap \mathcal{H}^{+0}$$

These star-product functional classes are most relevant to the analysis of HS dynamics in terms of limiting contracting homotopy

Factorization Lemma

In the limit $\beta \to -\infty$ cohomology projector gives zero on $\mathcal{H}^{+0}$:

$$h'_{q\,,-\infty}(\mathcal{H}^{+0}) = 0$$

Typical integrals that appear in the limit $\beta \to -\infty$ have the form

$$\lim_{\beta \to -\infty} \int_0^1 \mathrm{d}\tau \frac{\beta(\beta\tau)^m}{(1-\beta\tau)^{2+n}}\,, \qquad n \geq m$$

Giving a finite result after the change of variables $\tau \to \tau' = -\beta\tau$. However, if there is an additional factor of τ^{ε} this is equivalent to the appearance of the factor of $(-\beta)^{-\varepsilon}$ that sends the final result to zero in the limit $\beta \to -\infty$.

This implies that all terms from $\mathcal{H}^{+0}$ that appear on the r.h.s. of the field equations

$$dW + W * W = 0\,, \qquad dB + W * B - B * W = 0$$

disappear in the limit $\beta \to -\infty$. These are the terms with higher derivatives between different factors of C.

β-Deformed Star Product

$\beta-$deformed star product

$$O_\beta f(z,y) = \int \frac{\mathrm{d}u\mathrm{d}v}{(2\pi)^2} f((1-\beta)z + v, y + \beta u)\exp(iu_\alpha v^\alpha)$$

It can be checked that

$$O_\beta \triangle_{q,\,\beta} = \triangle_{q,\,0} O_\beta$$

Shifted contracting homotopy in the standard HS star product is equivalent to conventional homotopy in the $\beta-$deformed star product.

$\beta-$deformed star product is ill defined in the limit $\beta \to -\infty$.

However, its application to computation of HS vertices is well defined since the final result belongs to the d_Z- cohomology free from Z-dependence generating divergencies in the limit $\beta \to -\infty$.

$\beta \to 1$ **limit:** De Filippi, Iazeolla, Sundell 1905.06325

Conclusion

The shifted homotopy scheme is proposed leading to spin-local HS vertices derived from the nonlinear equations.

A class of new local vertices is found up to the quintic order.

Didenko, Gelfond, Korybut, MV 1909.04876

Classes of star-product functions are identified appropriate for spin-locality Gelfond, MV 1910.00487

Indications that HS gauge theory is spin-local in higher orders

Space-time interpretation of spin-locality is given: HS currents should be considered as independent s-modules

Main problem on the agenda:

spontaneous breaking of HS symmetries in the Coxeter HS models

Coxeter HS Equations

Unfolded equations for 1804.06520 **$\mathcal{C}$-HS theories remain the same except**

$$iS \star S = dZ^{An} dZ_{An} + \sum_i \sum_{v \in \mathcal{R}_i} \eta_i B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v$$

κ_v **are generators of $\mathcal{C}$ acting trivially on all elements except for** dZ_n^A

$$\kappa_v \star dZ_A^n = R_v{}^n{}_m dZ_A^m \star \kappa_v \,.$$

η_i **is a coupling constant on the conjugacy class $\mathcal{R}_i$ of $\mathcal{C}$.**

In the important case of the Coxeter group B_p

$$iS \star S = dZ^{An} dZ_{An} + \sum_{v \in \mathcal{R}_1} \eta_1 B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v + \sum_{v \in \mathcal{R}_2} \eta_2 B \frac{dZ_n^A v^n dZ_{Am} v^m}{(v,v)} \star \kappa_v$$

with arbitrary η_1 and η_2 responsible for the

HS and stringy/tensorial features, respectively

$\eta_2 \neq 0$ **for** $p \geq 2$

The framed construction leads to a proper massless spectrum.

Jacobi for Cherednik imply consistency of field equations.