Higher-spin gauge theory, matrix models & the quantum structure of space-time

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how to formulate quantum theory of spacetime & gravity?

guidelines:

- simple, constructive
- gauge theory (Minkowski signature!)
- finite dof per volume (Planck scale)
  $\rightarrow$ underlying d.o.f. non-geometric
- space-time & gravity should emerge from fundamental d.o.f.
- good UV properties (cf. string theory)

Matrix Models (of Yang-Mills type)

$S = Tr([Y^\mu, Y^\nu][Y_\mu, Y_\nu] + ...)$ provide such models!

- simple
- describe dynamical (noncomm.) spaces, gauge theory

  $Y^a \to U^{-1} Y^a U$
- well suited for quantization: $\int dY\, e^{-S[Y]}$

  IKKT model: protected from UV/IR mixing (maximal SUSY)

  cf. critical string
- how to understand gravity?

summary & outline:

- the IKKT matrix model & "matrix (fuzzy) geometry"
- *4D covariant quantum spaces*: fuzzy H_n^4
  truncated tower of higher-spin modes M. Sperling, HS 1806.05907
- projection $\rightarrow$ cosmological space-time $\mathcal{M}_n^{3,1}$
  M. Sperling, HS 1901.03522
- ($\mathfrak{hs}$-) volume-preserving diffeos
  from higher-dim. symplectomorphisms
- no ghosts HS 1910.00839
- linearized Schwarzschild HS 1905.07255
- nonlinear regime: torsion as a source for Einstein tensor
  (dark matter ?) HS 2002.02742

The IKKT model

IKKT or IIB model

$$S[Y,\Psi] = -Tr\Big([Y^a,Y^b][Y^{a'},Y^{b'}]\eta_{aa'}\eta_{bb'} \;+ m^2 Y^a Y_a \;+ \bar\Psi\gamma_a[Y^a,\Psi]\Big)$$

$$Y^a = Y^{a\dagger} \in Mat(N,\mathbb{C}), \qquad a = 0,...,9, \qquad N \to \infty$$

gauge invariance $Y^a \to UY^aU^{-1}$, $SO(9,1)$, ~~SUSY~~

Ishibashi, Kawai, Kitazawa, Tsuchiya hep-th/9612115

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, N large
- equations of motion:
  $\Box Y^a + m^2 Y^a = 0, \qquad \Box \equiv \eta_{ab}[Y^a,[Y^b,.]]$
- quantization: $Z = \int dYd\Psi\, e^{iS[Y,\Psi]}$, (SUSY !)

how to get physics from matrix models?

- no a priori space-time, geometry
- solutions X^μ $\rightarrow$ space(time)

  cf. branes, generically non-commutative
- fluctuations $X^\mu + \mathcal{A}^\mu$ $\rightarrow$ gauge theory

  dynamical geometry $\rightarrow$ gravity ?! (not holographic !)
- $\int dX$ = path integral, including geometry

numerical studies possible & underway

evidence for emergent 3+1D expanding space-time

Nishimura, Tsuchiya 1904.05919, Kim, Nishimura, Tsuchiya arXiv:1108.1540 ff examples of "matrix geometries":

1) Moyal-Weyl quantum plane $\mathbb{R}^{3,1}_\theta$:

$$[X^\mu, X^\nu] = i\theta^{\mu\nu}\,\mathbf{1}$$

admits translations $X^\mu \to X^\mu + c^\mu \mathbf{1}$, rotation invariance broken

fluctuations $X^\mu + \mathcal{A}^\mu$ in IKKT $\to$ NC $\mathcal{N} = 4$ SYM

2) fuzzy 2-sphere S^2_N

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \qquad [X_i, X_j] = i\epsilon_{ijk}X_k$$

fully covariant under $SO(3)$ (Hoppe, Madore)

NC ("fuzzy") space = quantized symplectic manifold $\subset \mathbb{R}^n$

map $\mathcal{Q}: \quad \mathcal{C}^\infty(\mathcal{M}) \to \mathrm{End}(\mathcal{H}), \qquad \dim\mathcal{H} \sim \mathrm{Vol}(\mathcal{M})$

4D covariant quantum spaces

- in 4D: Poisson tensor $\{x^\mu, x^\nu\} = \theta^{\mu\nu}$ breaks Lorentz-invar.
- avoided on covariant quantum spaces

  example: fuzzy S^4_N

  Grosse-Klimcik-Presnajder; Castelino-Lee-Taylor; Medina-o'Connor; Ramgoolam; Kimura; Abe; Karabail-Nair; Zhang-Hu 2001 (QHE); HS
- noncompact H^4_n Hasebe 1207.1968 , M. Sperling, HS 1806.05907
- projection of H^4_n $\to$ cosmological space-time $\mathcal{M}_n^{3,1}$

  HS, 1710.11495, 1709.10480, M. Sperling, HS 1901.03522, ff.

covariance $\to$ higher-spin gravity from matrix model(s)

introductory review: HS arXiv:1911.03162

Euclidean fuzzy hyperboloid H_n^4 (=$EAdS_n^4$)

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) \ .$$

$$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$$

choose "short" discrete unitary irreps $\mathcal{H}_n$ ("minireps", doubletons)

<u>special properties:</u>

- irreps under $\mathfrak{so}(4,1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, ...\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n+1$-dim.

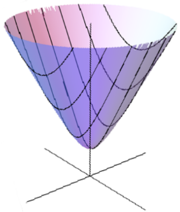
fuzzy hyperboloid H_n^4

5 hermitian generators

$$X^a \;:= r\mathcal{M}^{a5}, \qquad a = 0, ..., 4$$

satisfy

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbf{1}, \qquad R^2 = r^2(n^2-4)$$

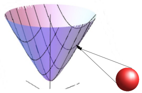


hyperboloid $H^4 \subset \mathbb{R}^{1,4}$, covariant under $SO(4,1)$

noncommutative $[X^a, X^b] = ir^2\mathcal{M}^{ab} =: i\Theta^{ab}$

claim:

$H_n^4 \quad = \quad$ quantized $\mathbb{C}P^{1,2} = S^2$ -bundle $\{\theta^{\mu\nu}$ selfdual$\}$ over H^4



can be seen from oscillator construction:
4 bosonic oscillators $\quad [\psi_\alpha, \bar\psi^\beta] = \delta_\alpha^\beta$

$\mathcal{H}_n$ = suitable irrep in Fock space

$$\mathcal{M}^{ab} = \bar\psi\Sigma^{ab}\psi, \qquad \gamma_0 = diag(1,1,-1,-1)$$
$$X^a = r\bar\psi\gamma^a\psi \qquad \text{cf. Hopf map}$$

$End(\mathcal{H}_n) \cong$ functions on $H_n^4 \;\cong\;$ harmonics on $S^2 \otimes$ functions on H^4

local stabilizer acts on $S^2 \;\Rightarrow\;$ harmonics = higher spin modes relation with $\mathfrak{hs}$:

constraints due to doubleton reps $\mathcal{H}_n$:

$$\begin{aligned} \eta_{cc'}\Theta^{ac}\Theta^{bc'} + (a \leftrightarrow b) &= r^2 \left(2R^2\eta_{ab} + \left(X^a X^b + X^b X^a\right)\right) \\ \epsilon_{abcde}\mathcal{M}^{ab}\mathcal{M}^{cd} &= 4nr^{-1}X_e \\ \epsilon_{abcde}\mathcal{M}^{ab}X^c &= nr\mathcal{M}_{de} \end{aligned}$$

(cf. Joseph-relations for $\mathfrak{so}(4,1)$)

here: part of simple matrix algebra $End(\mathcal{H}_n)$ for $\mathfrak{so}(4,2)$ (!) with

$$[\mathcal{M}_{ab}, X_c] = i(\eta_{ac}X_b - \eta_{bk}X_a)$$

$X^a = r\mathcal{M}^{a5}$ ... defines space H^4

$\mathcal{M}^{ab} \cong \{\theta^{\mu\nu}, t^\mu\}$ ... local $\mathfrak{so}(4,1)$ generators, define fiber over H^4

note:

$\exists$ UV scale $r \sim \frac{1}{n}R$ on S_N^4 (from NC)

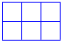
fuzzy "functions" on H_n^4:

$$End(\mathcal{H}_n) \rightsquigarrow \quad HS(\mathcal{H}_n) = \int\limits_{\mathbb{C}P^{1,2}} f(m)\,|m\rangle\langle m| \;\cong\; \bigoplus_{s=0}^{n} \mathcal{C}^s$$

$\mathcal{C}^0$ = scalar functions on H^4: $\quad \phi(X)$

$\mathcal{C}^1$ = selfdual 2-forms on H^4: $\quad \phi_{ab}(X)\theta^{ab} \;=$ 

$\vdots$

$End(\mathcal{H}_n) \cong$ fields on H^4 taking values in $\mathfrak{hs} = \oplus$  $\ni \theta^{a_1 b_1} \ldots \theta^{a_s b_s}$

cf. Vasiliev

higher spin modes = would-be KK modes on S^2

matrix model defines higher spin gauge theory, truncated at n

M. Sperling, HS 1806.05907

H_n^4 is starting point for cosmological quantum space-times $\mathcal{M}_n^{3,1}$:

- exactly homogeneous & isotropic, Big Bounce
- on-shell higher-spin fluctuations classified
- spin 2 metric fluctuations $\to$ gravitons
- nonlinear regime: torsion

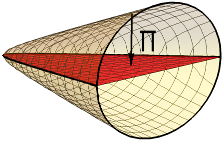
open FRW universe from H_n^4

HS 1710.11495

$\mathcal{M}_n^{3,1}$ = H_n^4 projected to $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \ \mathbb{C}P^{1,2} \to H^4 \subset \mathbb{R}^{1,4} \xrightarrow{\Pi} \mathbb{R}^{1,3} .$$

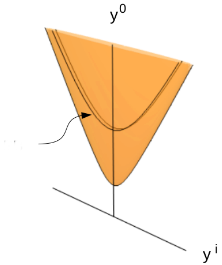
induced metric has Minkowski signature!



algebraically: $\mathcal{M}_n^{3,1}$ generated by

$$Y^\mu := X^\mu, \quad \text{for} \ \ \mu = 0,1,2,3 \qquad (\text{drop } X^4 \ )$$

geometric properties:



- manifest $SO(3,1)$ $\Rightarrow$ foliation into space-like 3-hyperboloids H^3_τ
- double-covered FRW space-time ($k=-1$)

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

$a(t) \propto t$ ... asympt. coasting

functions on $\mathcal{M}^{3,1}$:

generators:

$X^\mu = r\mathcal{M}^{\mu 5} \sim x^\mu$ ... base space

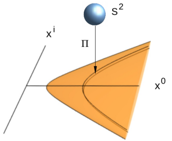
$T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$ ... fiber / momenta

commutation relations / Poisson brackets

$$\begin{aligned}\{t^\mu, x^\nu\} &= \sinh(\eta)\eta^{\mu\nu}\\ \{x^\mu, x^\nu\} &= \theta^{\mu\nu}\\ \{t^\mu, t^\nu\} &= -\frac{1}{r^2R^2}\theta^{\mu\nu}\end{aligned}$$

constraints:

$$\begin{aligned}x_\mu x^\mu &= -R^2\cosh^2(\eta), \qquad x^4 = R\sinh(\eta)\\ t_\mu t^\mu &= r^{-2}\cosh^2(\eta)\\ t_\mu x^\mu &= 0,\\ \theta^{\mu\nu} &= c(x^\mu t^\nu - x^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta}x_\alpha t_\beta\end{aligned}$$



t^μ ... generates space-like S^2 fiber functions as higher-spin modes:

$$\phi \in \mathrm{End}(\mathcal{H}_n) = \phi^{(0)} \oplus \phi^{(0)} \oplus \ldots \oplus \phi^{(n)}, \qquad \phi^{(s)} \in \mathcal{C}^s$$

(selected by spin Casimir $\mathcal{S}^2$)

2 points of view:

- functions on H_n^4: full $SO(4,1)$ covariance

  represent $\phi^{(s)}$ as

$$\begin{aligned} \phi^H_{a_1\ldots a_s} &\propto \{x^{a_1}, \ldots \{x^{a_s}, \phi^{(s)}\}\ldots\}_0 \\ \phi^{(s)} &= \{x^{a_1}, \ldots \{x^{a_s}, \phi^H_{a_1\ldots a_s}\}\ldots\} \end{aligned}$$

- functions on $\mathcal{M}_n^{3,1}$: reduced $SO(3,1)$ covariance

$$\phi^{(s)} = \phi_{\mu_1\ldots\mu_s}(x) t^{\mu_1} \ldots t^{\mu_s}$$

$t_\mu x^\mu = 0 \;\Rightarrow$ "space-like gauge" $\boxed{x^{\mu_i}\phi_{\mu_1\ldots\mu_s} = 0}$

($\rightarrow$ no ghosts!)

$SO(4,2)$ - invariant integral = trace, inner product

$$\langle\phi,\phi'\rangle := Tr(\phi^*\phi') = \int_{\mathbb{C}P^{1,2}} \omega^{\wedge 3}\, \phi\phi' = \int_{H^4} dV[\phi\phi']_0$$

$[\phi]_0$ ... average over S^2 fiber

$\mathcal{M}^{3,1}$ solution in IKKT model:

background solution:

$$T^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}$$

satisfies e.o.m.

$$\Box T^\mu = \frac{3}{R^2} T^\mu, \qquad \Box = [T^\mu, [T_\mu, .]]$$

- $[\Box, \mathcal{S}^2] = 0, \quad \mathcal{S}^2$ ... spin Casimir, selects spin sectors $\mathcal{C}^s$
- $\Box \sim \Box_G$ encodes eff. FRW metric $ds_G^2 = -dt^2 + a(t)^2 d\Sigma^2$

fluctuations & higher spin gauge theory

$$S[Y] = Tr([Y^\mu, Y^\nu][Y_\mu, Y_\nu] + \frac{6}{R^2} Y^\mu Y_\mu) = S[U^{-1} Y U]$$

background: $\overline{Y}^\mu = T^\mu$ ... space-time $\mathcal{M}_n^{3,1}$

add fluctuations $Y^\mu = \overline{Y}^\mu + \mathcal{A}^\mu$

$\mathcal{A}_\mu$ ... $\mathfrak{hs}$-valued 1-form on $\mathcal{M}^{3,1}$

expand action to second oder in $\mathcal{A}_\mu$

$$S[Y] = S[\overline{Y}] + \frac{2}{g^2} \mathrm{Tr} \mathcal{A}_\mu \Big(\underbrace{(\Box - \frac{3}{R^2})\delta^\mu_\nu + 2[[\overline{Y}^\mu, \overline{Y}^\nu], .]}_{\mathcal{D}^2} - \underbrace{[\overline{Y}^\mu, [\overline{Y}^\nu, .]]}_{g.f.} \Big) \mathcal{A}_\nu$$

generic eigenmodes: M. Sperling, HS: 1901.03522, HS 1910.00839

can show (using $\mathfrak{so}(4,2)$):

- 4 generic eigenmodes for each $\phi = \phi^{(s)}$ with $\Box\phi = \lambda\phi$:

$$\mathcal{D}^2 \mathcal{A}_\mu^{(i)}[\phi] = \lambda\, \mathcal{A}_\mu^{(i)}[\phi], \qquad i \in \{+,-,n,g\}$$

$$\mathcal{A}_\mu^{(i)}[\phi] = \begin{cases} \mathcal{A}_\mu^{(+)}[\phi] & := \{y_\mu, D^-\phi\}_+ \\ \mathcal{A}_\mu^{(-)}[\phi] & := \{y_\mu, D^+\phi\}_- \\ \mathcal{A}_\mu^{(n)}[\phi] & := D^+\{y_\mu, \phi\}_- \\ \mathcal{A}_\mu^{(g)}[\phi] & := \{t_\mu, \phi\} \end{cases}$$

(pure gauge)

- 4 towers of on-shell modes for each $s > 0$

$$\mathcal{D}^2 \mathcal{A}^{(i)}[\phi] = 0 \qquad \text{for} \qquad \Box\phi = 0, \qquad i \in \{+,-,n,g\}$$

universal propagation $\Box \sim \Box_G$

- 2 spin 0 modes

inner product matrix

$$\mathcal{G}^{(i,j)} = \left\langle \mathcal{A}^{(i)}[\phi'], \mathcal{A}^{(j)}[\phi] \right\rangle, \qquad i,j \in \{+,-,n,g\}$$

signature $(+++-)$

gauge-fixing $\{t^\mu, \mathcal{A}_\mu\} = 0$

physical Hilbert space

$$\mathcal{H}_{\text{phys}} = \{\mathcal{D}^2\mathcal{A} = 0, \ \ \mathcal{A} \ \text{ gauge fixed}\}/_{\{\text{pure gauge}\}}$$

results:

HS 1910.00839

- generically 2 physical modes for each $\Box\phi^{(s)} = 0, \ s \geq 1$
  would-be massive, $m^2 = 0$
- no ghosts, no tachyons
- same propagation for all modes

  (even though Lorentz invar only partially manifest)

nonlinear regime: frame and metric

any background $Y^{\dot\alpha}$ of the M.M. defines a $\mathfrak{hs}$ - valued frame

$$\boxed{E_{\dot\alpha}[\phi] := \{Y_{\dot\alpha}, \phi\},} \qquad \dot\alpha = 0, ..., 3$$

in coordinates: $\phi = \phi_{\underline{\alpha}}(x) t^{\underline{\alpha}}$

$$E_{\dot\alpha}[\phi] \sim E_{\dot\alpha}{}^{\mu}\partial_\mu\phi, \qquad E_{\dot\alpha}{}^{\mu} := \{Y_{\dot\alpha}, x^\mu\}$$

(in asymptotic regime: wavelength $\ll$ cosm.)

eff. metric from action:

$$-Tr[Y^{\dot\alpha}, \phi][Y_{\dot\alpha}, \phi] \sim \int E^{\dot\alpha}[\phi]E_{\dot\alpha}[\phi] = \int \sqrt{|G|}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad (\mathfrak{hs}-\text{valued})$$

where

$$G^{\mu\nu} = \rho^{-2}\eta^{\dot\alpha\dot\beta}E_{\dot\alpha}{}^{\mu}E_{\dot\beta}{}^{\nu}$$

<u>can show:</u>

$$\Box\phi = -\{Y^{\dot\alpha}, \{Y_{\dot\alpha}, \phi\}\} \sim \Box_G\phi$$

gauge transformations and $\mathfrak{hs}$-valued diffeos

scalar fields:

$$\delta_\Lambda \phi = \{\Lambda, \phi\} = \xi^\mu \partial_\mu \phi = \mathcal{L}_\xi \phi, \qquad \xi^\mu = \{\Lambda, x^\mu\}$$

$\{\Lambda, .\}$ ... Hamiltonian VF on $\mathbb{C}P^{1,2}$

push-forward by bundle projection

$$\Pi_* : \quad \Gamma\mathbb{C}P^{1,2} \to \Gamma\mathcal{M} \otimes \mathfrak{hs}$$

defines $\mathfrak{hs}$-valued volume-preserving diffeo $\{\Lambda, .\}$ on $\mathcal{M}$

vector fields (frame!):

$$\begin{aligned} \delta_\Lambda Y_{\dot\alpha} &= \{\Lambda, Y_{\dot\alpha}\} \\ (\delta_\Lambda E_{\dot\alpha})\phi &= \{\Lambda, \{Y_{\dot\alpha}, \phi\}\} - \{Y_{\dot\alpha}, \{\Lambda, \phi\}\} = (\mathcal{L}_\xi E_{\dot\alpha})\phi \end{aligned}$$

hence

$$\boxed{\delta_\Lambda E_{\dot\alpha}{}^\mu = \mathcal{L}_\xi E_{\dot\alpha}{}^\mu, \qquad \delta_\Lambda G^{\mu\nu} = \mathcal{L}_\xi G^{\mu\nu}}$$

$\mathfrak{hs}$-valued Weitzenböck connection & torsion

natural connection:

$$\nabla E_{\dot\alpha} = 0 \qquad \text{(Weitzenböck)} \qquad \Rightarrow \qquad \nabla G^{\mu\nu} = 0$$

flat (no curvature) but $\mathfrak{hs}$ - valued torsion:

$$T_{\dot\alpha\dot\beta} \equiv T[E_{\dot\alpha}, E_{\dot\beta}] = \nabla_{\dot\alpha} E_{\dot\beta} - \nabla_{\dot\beta} E_{\dot\alpha} - [E_{\dot\alpha}, E_{\dot\beta}]$$

can show:

$$\boxed{T_{\dot\alpha\dot\beta}{}^{\mu} = \{\hat\Theta_{\dot\alpha\dot\beta}, y^\mu\},} \qquad \hat\Theta_{\dot\alpha\dot\beta} := -\{Y_{\dot\alpha}, Y_{\dot\beta}\}$$

transforms covariantly under $\mathfrak{hs}$-valued diffeos,

$$\delta_\Lambda T_{\dot\alpha\dot\beta}{}^{\mu} = \mathcal{L}_\xi T_{\dot\alpha\dot\beta}{}^{\mu}$$

torsion encodes the field strength of the NC gauge theory

cf. Langmann Szabo hep-th/0105094

dynamical torsion and Ricci tensor

start with matrix eom in the form

$$\{Y^{\dot\alpha}, \hat\Theta_{\dot\alpha\dot\beta}\} = m^2 Y_{\dot\beta}$$

can recast as

$$\boxed{\nabla_\nu T^\nu{}_{\rho\mu} + T_\nu{}^\sigma{}_\mu T_{\sigma\rho}{}^\nu = m^2\rho^{-2} G_{\rho\mu}}$$

HS arXiv:2002.02742

trace gives

$$\rho\Box_G\rho + G^{\mu\nu}\partial_\mu\rho\partial_\nu\rho = 2m^2 - \frac{1}{2}T_\nu{}^\sigma{}_\mu T_{\sigma\rho}{}^\nu\gamma^{\mu\rho}$$

Bianci identity:

$$\{Z_{\dot\gamma}, \hat\Theta_{\dot\alpha\dot\beta}\} + (\mathrm{cycl}) = 0$$

implies

$$0 = \nabla_\sigma T_{\lambda\rho}{}^\mu + T_{\rho\sigma}{}^\nu T_{\nu\lambda}{}^\mu + (cycl)$$

Ricci tensor: can show:

$$\mathcal{G}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}G_{\mu\nu}\mathcal{R} = 8\pi \mathbf{T}_{\mu\nu}$$

in vacuum, where (using eom for torsion)

$$\begin{aligned} 8\pi\mathbf{T}_{\mu\nu} &= -\tfrac{1}{2}(T_{\rho}{}^{\delta}{}_{\nu}\,T_{\mu\delta}{}^{\rho} + T_{\rho}{}^{\delta}{}_{\mu}\,T_{\nu\delta}{}^{\rho}) - K_{\delta}{}^{\rho}{}_{\mu}\,K_{\rho}{}^{\delta}{}_{\nu} + 2\rho^{-2}\partial_\mu\rho\partial_\nu\rho \\ &\quad + G_{\mu\nu}\Big(- \tfrac{1}{4}T^{\sigma\delta\rho}T_{\delta\rho\sigma} + \tfrac{1}{8}T^{\delta\sigma\rho}T_{\delta\sigma\rho} - \rho^{-2}\partial\rho\cdot\partial\rho - 3R^{-2}\rho^{-2}\Big) \\ &= O(TT) \end{aligned}$$

... effective e-m tensor due to torsion HS arXiv:2002.02742

note:

- Riemann tensor $\mathcal{R}_{\mu\nu\rho\sigma} \sim \partial T + TT$, but $\mathcal{R}_{\mu\nu} \sim TT$

  hence $\boxed{\mathcal{R}_{\mu\nu} = 0}$ in "weak gravity regime" in vacuum (on-shell)
- expect significant contribution from torsion only for very large, massive objects (galaxies?)

cosmic background torsion

$$\bar{T}_{\rho\sigma}{}^{\mu} = \frac{1}{a(t)^2}\left(\delta^{\mu}_{\sigma}\tau_{\rho} - \delta^{\mu}_{\rho}\tau_{\sigma}\right)$$

where τ ... time-like VF on FLRW

back-of-the-envelope estimate for e-m tensor due to torsion:

$$\mathbf{T}_{\mu\nu} \sim \frac{1}{(r+c)^2}$$

leads to $\approx$ const. rotation velocities, *as long as torsion above cosm. background*

... not too bad as "dark matter" ?!?

taking into account matter:

not yet understood, but conjecture:

$$\mathcal{G}_{\mu\nu} \sim 8\pi \mathbf{T}_{\mu\nu}^{\text{matter}} + \{.,.\}$$

(higher deriv. of matter)

remarks:

- action $S \sim \int \Theta^{\dot\alpha\dot\beta}\Theta^{\dot\alpha\dot\beta}$ rather than $S_{EH} = \int T_{\nu\lambda}{}^{\mu} T_{\nu\lambda}{}^{\mu} + ...$
  $\Rightarrow$ different from teleparallel gravity, GR !
  seems reasonably close in intermediate scales
  quite different on cosmic scales
- hints that $G_N \sim \frac{1}{a(t)}$ (?)

linearized metric fluctuation modes

$$h_{\mu\nu}[\mathcal{A}] \propto \{\mathcal{A}_\mu, y_\nu\} + (\mu \leftrightarrow \nu)$$

off-shell: most general metric fluct.

- $\mathcal{A}^{(-)}[\phi^{(2)}]$ ... 5, $\approx$ massive spin 2
- $\mathcal{A}^{(+)}[\phi^{(0)}], \mathcal{A}^{(n)}[D\phi^{(0)}]$ ... 2 scalars
- $\nabla_\mu \mathcal{A}_\nu + \nabla_\mu \mathcal{A}_\nu$ ... 3 (!) pure gauge

physical (vacuum):

$\mathcal{A}^{-}[\phi^{(2,0)}]$ ...2 graviton modes (massless !)

$\mathcal{A}^{-}[\phi^{(2,1)}]$ ...2 "vector" modes, $\approx$ pure gauge

$\mathcal{A}^{-}[\phi^{(2,2)}]$ ..."scalar" mode $\ni$ (lin. Schwarzschild !)

$$\boxed{\mathcal{R}^{\mu\nu}_{(\mathrm{lin})}[h[\mathcal{A}^{(-)}]] \approx 0}$$ on-shell (up to cosm. scales)

linearized Schwarzschild solution

HS 1905.07255

Ricci-flat "scalar" metric perturbation from $\mathcal{A}^{(-)}[D^+ D^+ \phi]$

$$ds^2 = (G_{\mu\nu} - h_{\mu\nu})dy^\mu dy^\nu = -dt^2 + a(t)^2 d\Sigma^2 + \phi'(dt^2 + a(t)^2 d\Sigma^2)$$

$$\phi' \sim \frac{e^{-\chi}}{\sinh(\chi)} \frac{1}{a(t)^2} \sim \frac{1}{\rho} \frac{1}{a(t)^2}$$

$\approx$ lin. Schwarzschild (Vittie) solution on FRW, eff. mass $m(t) \sim \frac{1}{a(t)}$

linearized approx. valid only in quasi-static case $\tau = -2$,
otherwise large pure gauge contribution (cf. massive graviton)
(similar for vector modes)

aspects of resulting gravity:

- standard gravitons recovered, lin. Schwarzschild
- expect $\approx$ GR at intermediate scales (to be elaborated)
- torsion:
  - is a physical, dynamical tensor field
  - governed by nonlin. PDE
  - encodes NC field strength
  - source for gravity (Einstein tensor)
  - similar to dark matter ?
- lin. metric modes contain extra (scalar & vector) modes,

  not Ricci-flat (due to torsion?)
- significant differences at cosmic scales,

  reasonable (coasting) cosmology without any fine-tuning

summary

- matrix models:
  natural framework for quantum theory of space-time & matter
- 3+1D covariant quantum cosmological FRW space-time solution

  $\rightarrow$ higher spin gauge theory,
  similarities with Vasiliev theory

  reg. BB, finite density of microstates
- fluctuations fully consistent (no ghosts or tachyons)
  all ingredients for gravity
- Yang-Mills-like theory, good UV behavior (SUSY)
- $\rightarrow$ emergent gravity rather than GR
  new physics (torsion), possibly dark matter/energy...

... looks intriguing, needs to be elaborated

introductory review: HS arXiv:1911.03162 relation with string theory:

- solutions = branes (here: novel types of branes ...)
- quantum effects $\rightarrow$ $\frac{1}{r^8}$ interactions $\sim$ IIB sugra in target space

coupling to matter & eom:

for physical transverse traceless spin 2 modes $h_{\mu\nu}[\phi^{(2,0)}]$:

$$S_2[\mathcal{A}^{(-)}[\phi^{(2,0)}]] \propto -\int h^{\mu\nu}[\phi^{(2,0)}](\Box - R^{-2})(\Box_H - 2r^2)^{-1} h_{\mu\nu}[\phi^{(2,0)}]$$

leads to eom

$$(\Box - 2R^{-2})h_{\mu\nu} \sim -(\Box_H - 2r^2)T_{\mu\nu}$$

breaking $SO(4,1) \to SO(3,1)$ and sub-structure

consider

$$D\phi := -i[X^4, \phi], \qquad \text{respects } SO(3,1)$$

acts on spin s modes as follows

$$D = \underbrace{div^{(3)}\phi}_{D^-\phi} + \underbrace{t^\mu \nabla^{(3)}_\mu \phi}_{D^+\phi} : \quad \mathcal{C}^s \to \mathcal{C}^{s+1} \oplus \mathcal{C}^{s-1}$$

decomposition into $SO(3,1)$ irreps on $H^3 \subset H^4$

$$\mathcal{C}^{(s)} = \mathcal{C}^{(s,0)} \oplus \mathcal{C}^{(s,1)} \oplus \ldots \oplus \mathcal{C}^{(s,s)}$$

D^- resp. D^+ act as

$$D^- : \ \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s-1,k-1)}, \qquad D^+ : \ \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s+1,k+1)} .$$