Merger rate of primordial black hole binaries

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M.Sasaki, TS, T.Tanaka, S.Yokoyama, CQG 35, 063001(2018) [arXiv:1801.05235]

GWs from black hole binary!!

GWs show that BH-BH binaries exit and they merge in the age of the Universe.

(Until LIGO, we didn't know if they exist.)

1811.12907

Event	$m_1/\mathrm{M}_\odot$	$m_2/\mathrm{M}_\odot$	$\mathcal{M}/\mathrm{M}_\odot$	χ_{eff}	$M_{\mathrm{f}}/\mathrm{M}_\odot$	a_{f}	$E_{\mathrm{rad}}/(\mathrm{M}_\odot c^2)$	$\ell_{\mathrm{peak}}/(\mathrm{erg\ s^{-1}})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4}\times 10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	179
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7}\times 10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7}\times 10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9}\times 10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.0}_{-0.1}$	$3.5^{+0.4}_{-1.3}\times 10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5}\times 10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9}\times 10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5}\times 10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1\times 10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7}\times 10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9}\times 10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

So far, 10 merger events have been detected.

What is the origin of LIGO BHs?

The answer is not known yet.

- list possible scenarios
- propose ideas of how to test and distinguish them observationally.

Accumulation of data will tell us about the nature of the BH binaries.

Maybe, primordial black holes!

redshift

$z \simeq 10^{11}$ Formation of PBHs $(m \simeq 30 M_\odot)$

Initially mean separation is super-Hubble distance.

$z \simeq 10^{4}$ Formation of PBH binaries I

in the radiation dominated epoch

Nakamura et al. 1998, Ioka et al. 1999

Sasaki et al. 2016, Eroshenko 2016

Ali-Haimoud et al. 2017

Raidal et al. 2017, Raidal et al. 2018

Formation of PBH binaries II

inside DM halos at present epoch

$z \simeq 0$

Bird et al. 2016, Clesse and Garcia-Bellido 2016

Mergers of the PBH binaries

Two things need to be explained before including the PBH as a possible explanation of LIGO events.

- How PBHs formed binaries?
- Do their mergers explain the observed merger rate?

Binary formation in the RD era

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GRAVITATIONAL WAVES FROM COALESCING BLACK HOLE MACHO BINARIES

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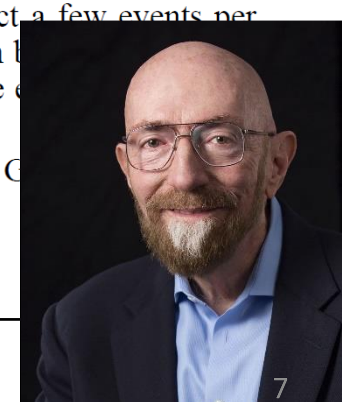
ABSTRACT

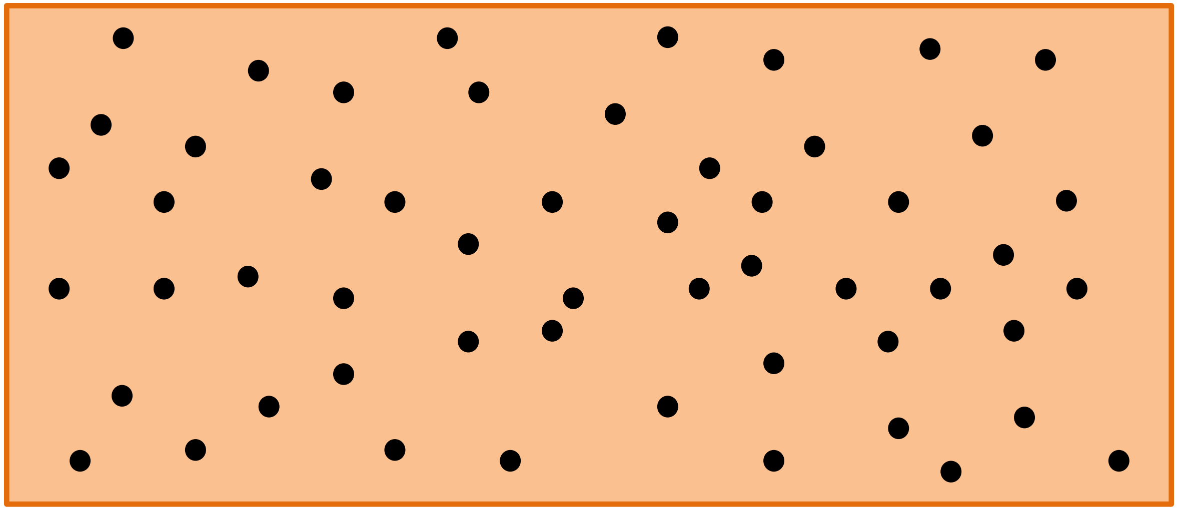
HOs are black holes of mass $\sim 0.5\ M_\odot$ they must have been formed in the early uni
e was ~1 GeV. We estimate that in this case in our Galaxy's halo out to ~ 50 kpc there exist ~5 ×
hole binaries the coalescence times of which are comparable to the age of the universe, so that the
e rate will be $\sim 5 \times 10^{-2}$ events yr^{-1} per galaxy. This suggests that we can expect a few events per
15 Mpc. The gravitational waves from such coalescing black hole MACHOs can b
tion of interferometers in the LIGO/VIRGO/TAMA/GEO network. Therefore, the
HOs can be tested within the next 5 yr by gravitational waves.

dings: black hole physics — dark matter — gravitation — gravitational lensing — G



Two assumptions **(Nakamura et al. 1997)**

1. After PBHs are formed, they distribute uniformly in space (Poisson).

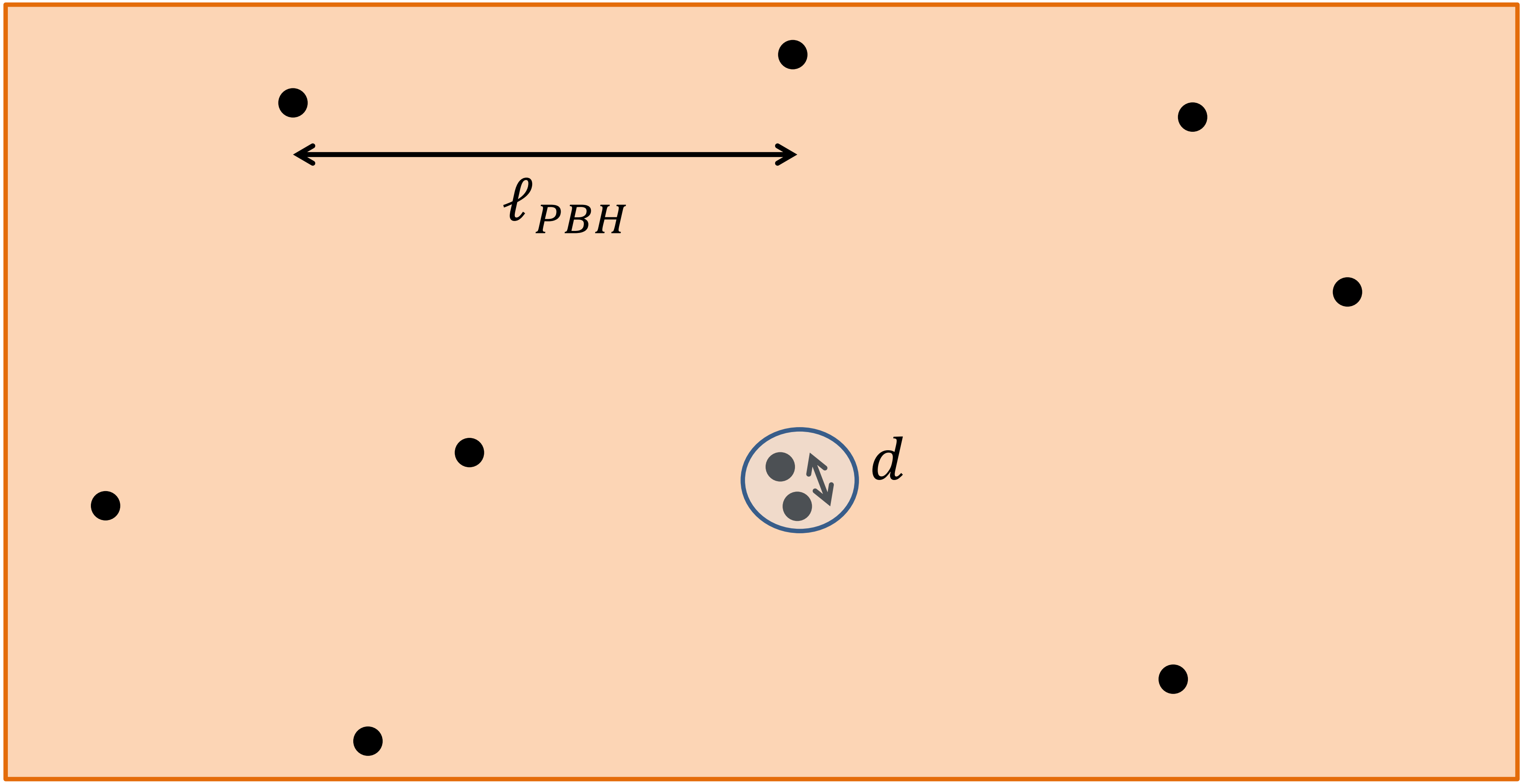


Initially, PBHs are separated by super-Hubble distance and on the flow of the cosmic expansion.

2. All PBHs have the same mass

Binary formation in RD era (Nakamura et al. 1997)

(The rest is not assumption but physical consequence.)

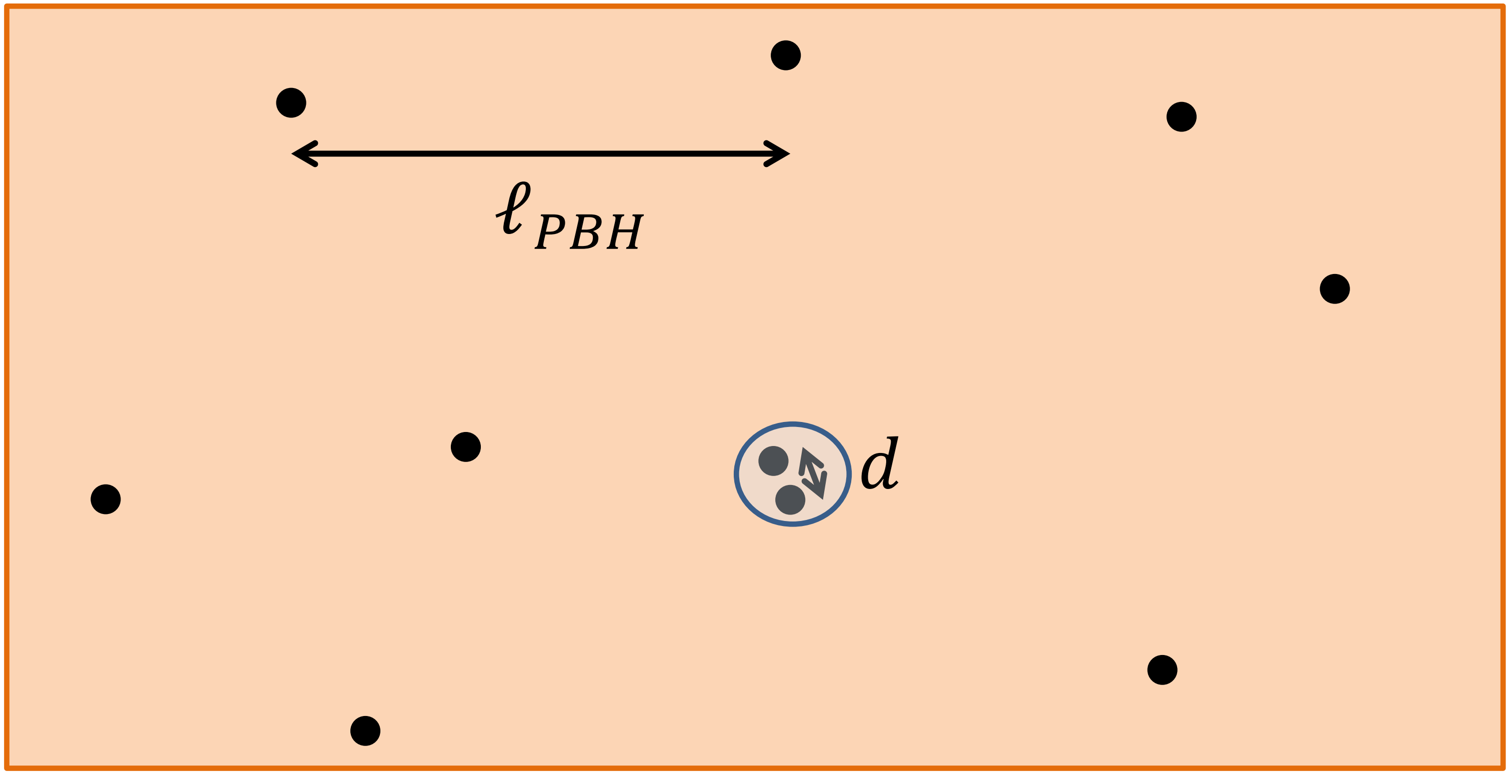


BH mass :2 M_{BH}

Radiation mass: $\rho_{rad} d^3 \propto 1/a$

Binary formation in RD era (Nakamura et al. 1997)

(The rest is not assumption but physical consequence.)

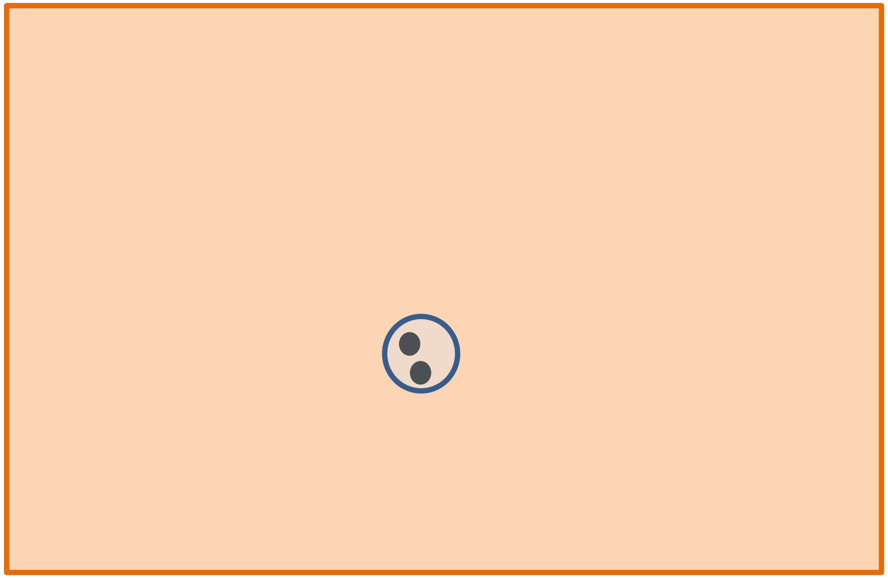


When 2 $M_{BH} > \rho_{rad} d^3$, the PBHs in pair becomes bound.

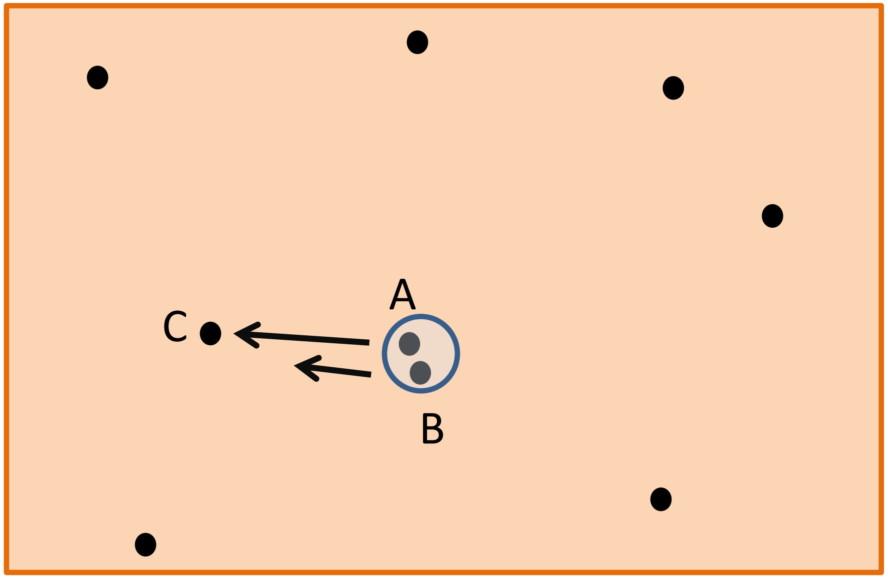
This happens for $d < f_{PBH}{}^{1/3} \ell_{PBH}$ and in the RD era. $(f_{PBH} = \frac{\Omega_{PBH}}{\Omega_{DM}})$

Only a fraction of PBHs (f_{PBH}) form a bound system.

Binary formation in RD era (Nakamura et al. 1997)



Ends up with direct collision
(no binary formation)

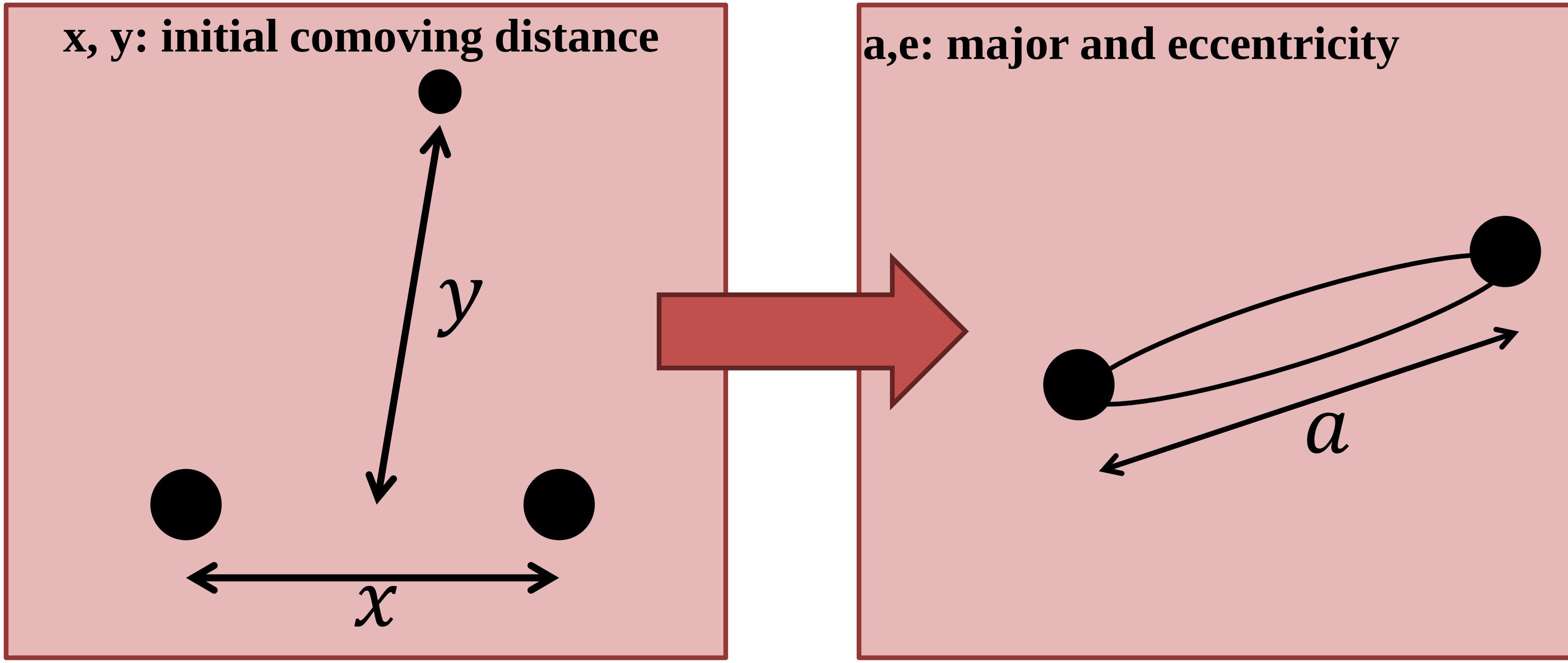


BH A is pulled more than BH B.

Ends up with eccentric binary

The surrounding PBHs (especially the nearest one) exert torque and the bound system acquires the angular momentum.

Binary formation in RD era (Nakamura et al. 1997)

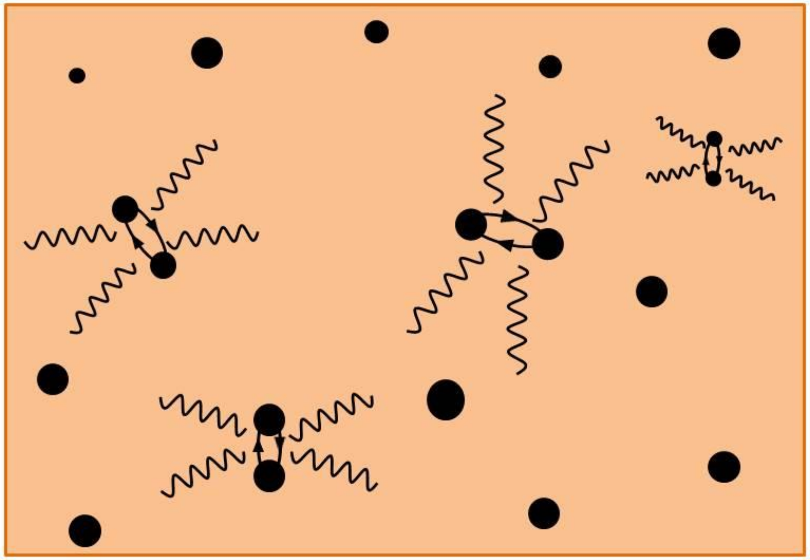


Once x and y are fixed, a and e are determined as

$$a = \frac{1}{f_{PBH}} \frac{x^4}{\bar{x}^3} \qquad e = \sqrt{1 - \frac{x^6}{y^6}}$$

Binary formation in RD era (Nakamura et al. 1997)

We can compute probability distribution of (a,e).



Uniform distribution

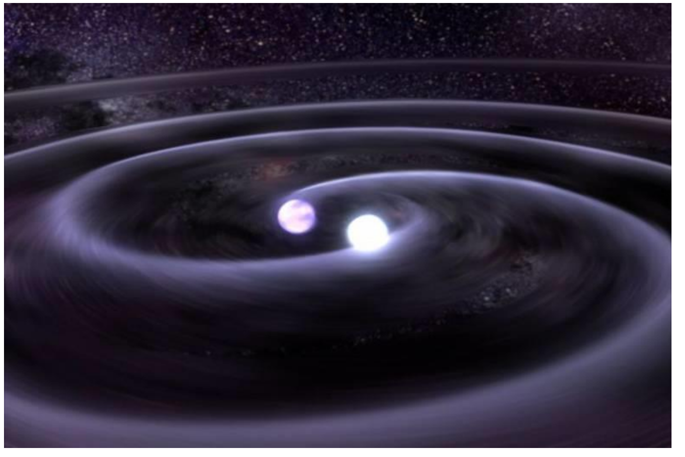
$$dP = \frac{9}{\bar{x}^6} x^2 y^2 dx dy \qquad 0 < x < y < \bar{x}$$

Probability in $(a, a + da)$ and $(e, e + de)$

$$\boxed{dP = \frac{3}{4} f^{3/2} \bar{x}^{-3/2} a^{1/2} e (1 - e^2)^{-3/2} da\, de.}$$

Life time of the binary

The next thing to do is to convert the probability in (a, e) to the merger probability in $(t, t + dt)$.



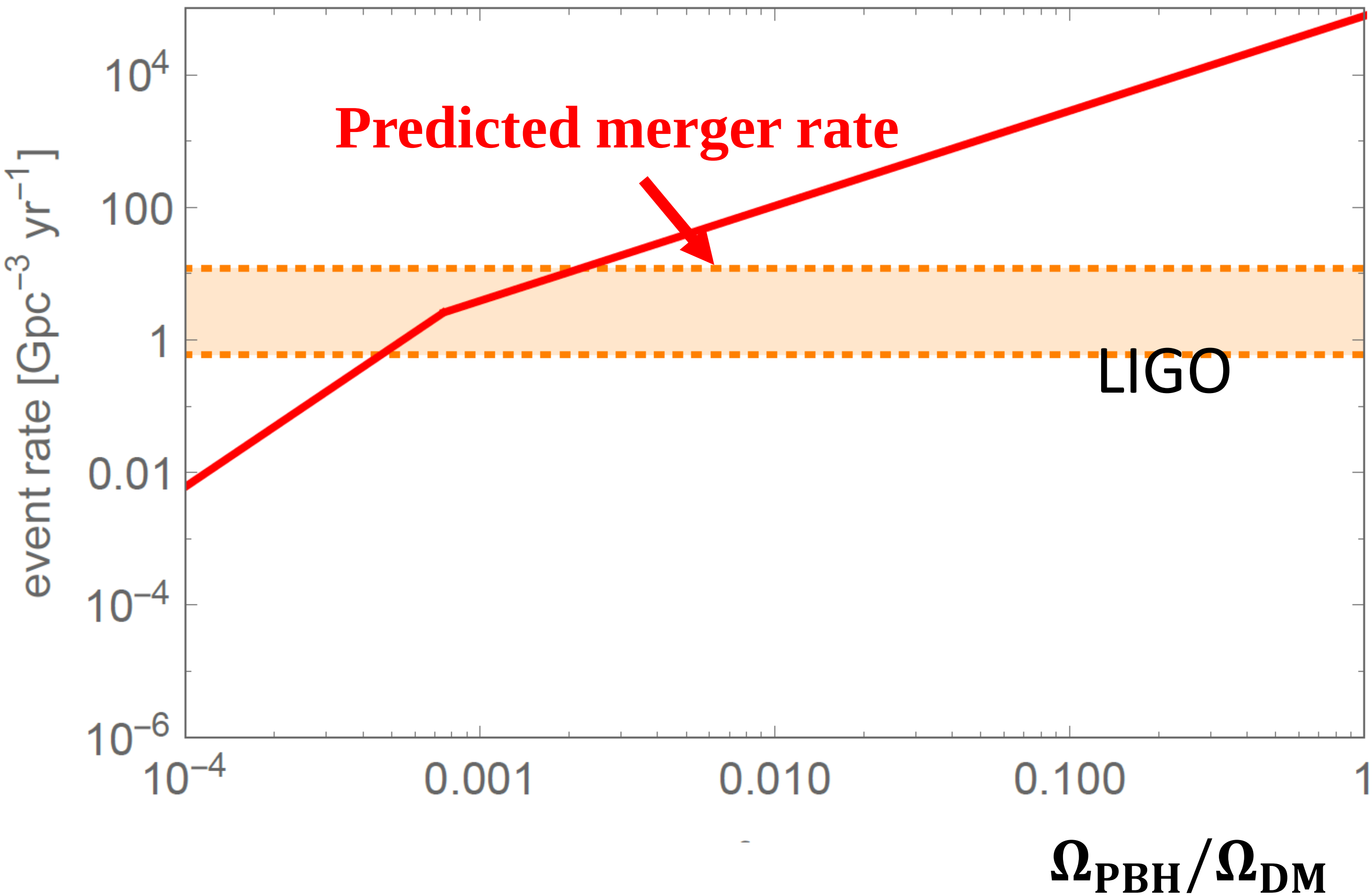
Life time of the binary is a function of major axis a and eccentricity e.

$$t = Qa^4(1 - e^2)^{7/2}, \qquad Q = \frac{3}{170}(GM_{\mathrm{BH}})^{-3}$$

In the paper by Nakamura et al. 1997, $M_{BH} = 0.5M_{\odot}$ and $\Omega_{PBH} = \Omega_{DM}$ was considered.

In the paper by Sasaki et al. 2016, $M_{BH} = 30M_{\odot}$ and the formula was extended to the case $\Omega_{PBH} < \Omega_{DM}$.

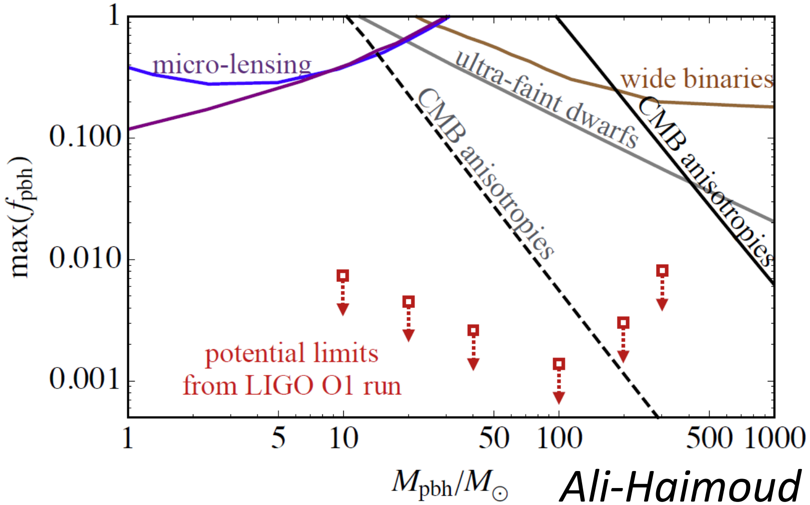
Merger event rate *Sasaki et al. 2016*



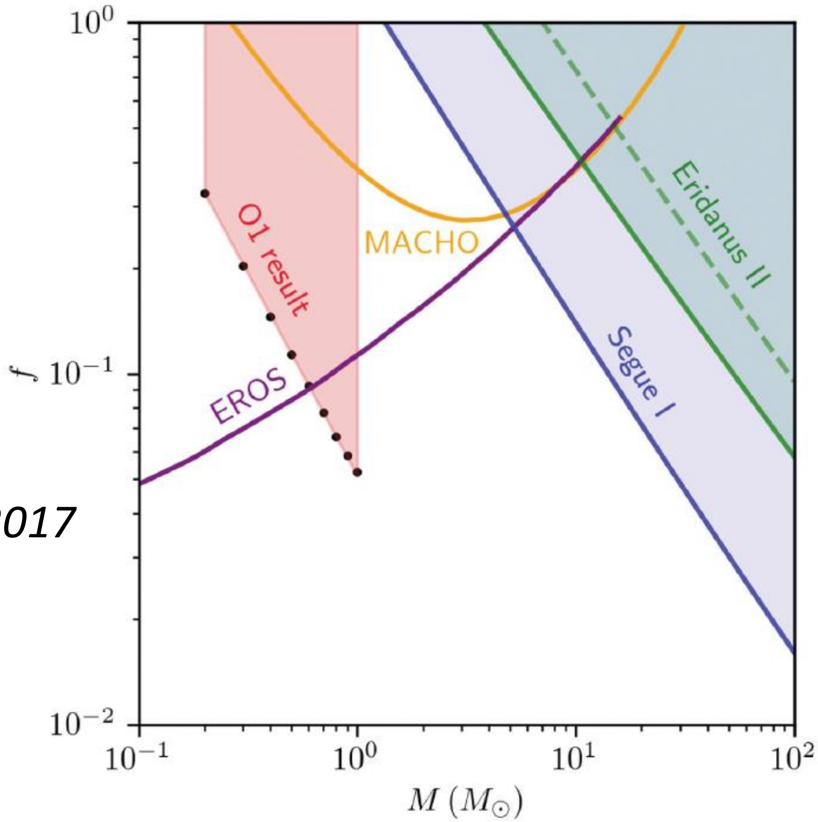
Consistent with LIGO if PBHs constitute about 0.1% of DM.

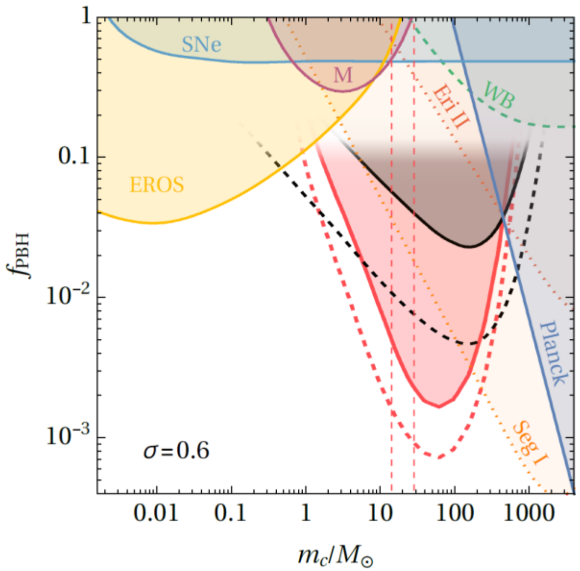
Monochromatic mass function is assumed.
Additional consideration is necessary for the extended mass function. **(Carr et al. 2017)**

Recently, the same mechanism has been used to place upper limit on Ω_{PBH} from the LIGO observations.



Ali-Haimoud et al. 2017





Raidal et al. 2018

LIGO-Virgo Collaboration 2018

Various effects that are ignored have been evaluated in other papers.

(Ioka et al. 1998, Hayasaki et al. 2009, Sasaki et al. 2016, Eroshenko 2016, Ali-Haimoud et al. 2017, Raidal et al.2018)

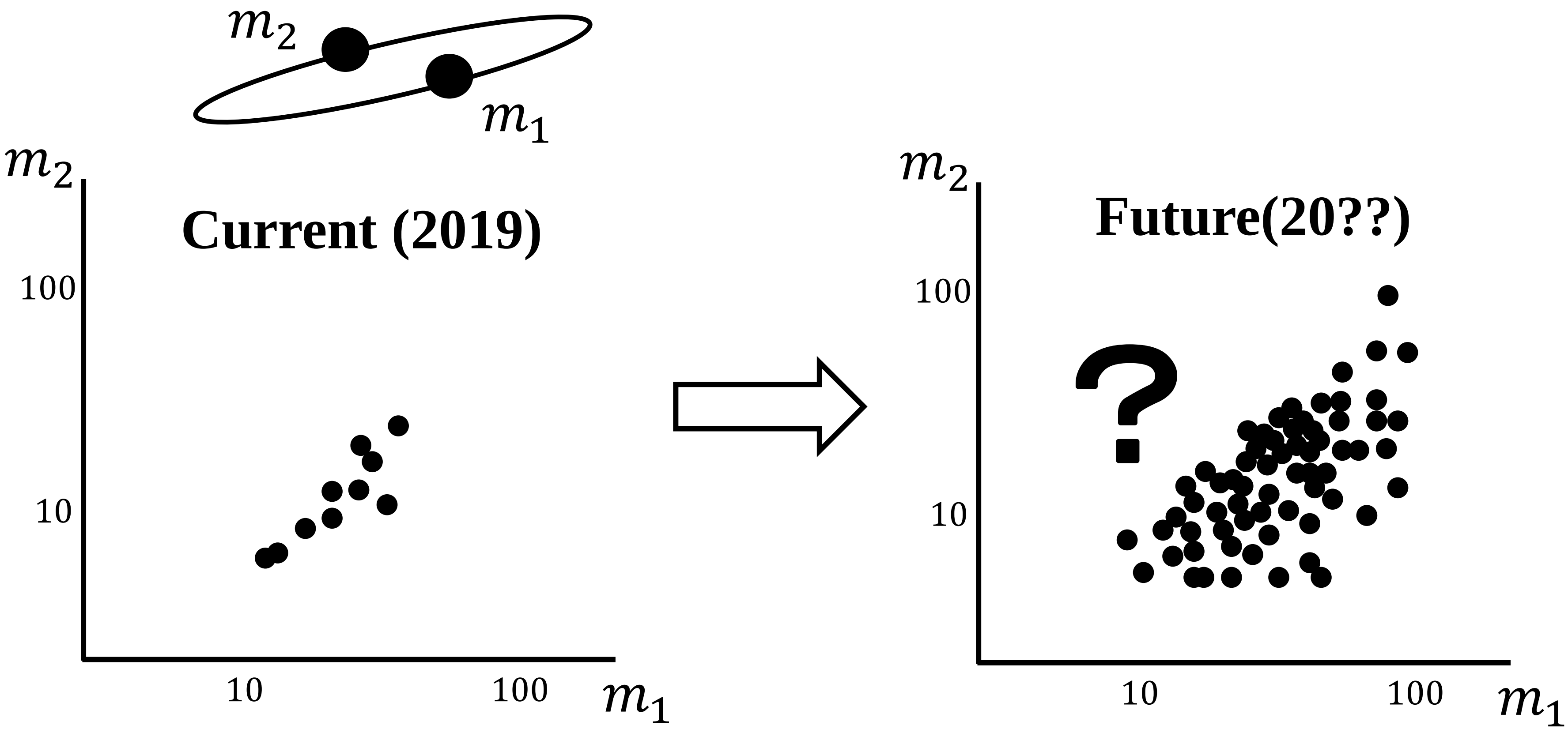
- Tidal force from outer BHs
- Initial peculiar velocity of PBHs
- Three body collisions
- Additional tidal force from dark matter perturbations
- Encounters of other PBHs (later time effect)
- Tidal force from halos (later time effect)
- Dynamical friction from DM and baryon (later time effect)

Simple analytical estimation suggest that those effects do not lead to the significant change of the result.

We have to keep in mind that these studies adopt the two assumptions.

How do we test the PBH scenario?

- Cosmic evolution of merger rate *T.Nakamura et al. 2016*
- Spin distribution *T.Chiba and S.Yokoyama 2016*
- Stochastic GWs *K.Ioka et al 1999, S.Wang et al. 2016, M.Raidal et al. 2017*
- **Merger distribution in BH mass plane** *B.Kocsis, TS, T.Tanaka, S.Yokoyama 2017*



In the future, we will observe many merger events and will be able to discuss about the distribution in the PBH mass plane (m_1,m_2).

In order to derive $\mathcal{R}(m_1, m_2)$, we first generalized the formula to the case of the extended PBH mass function $f(m_{BH})$.

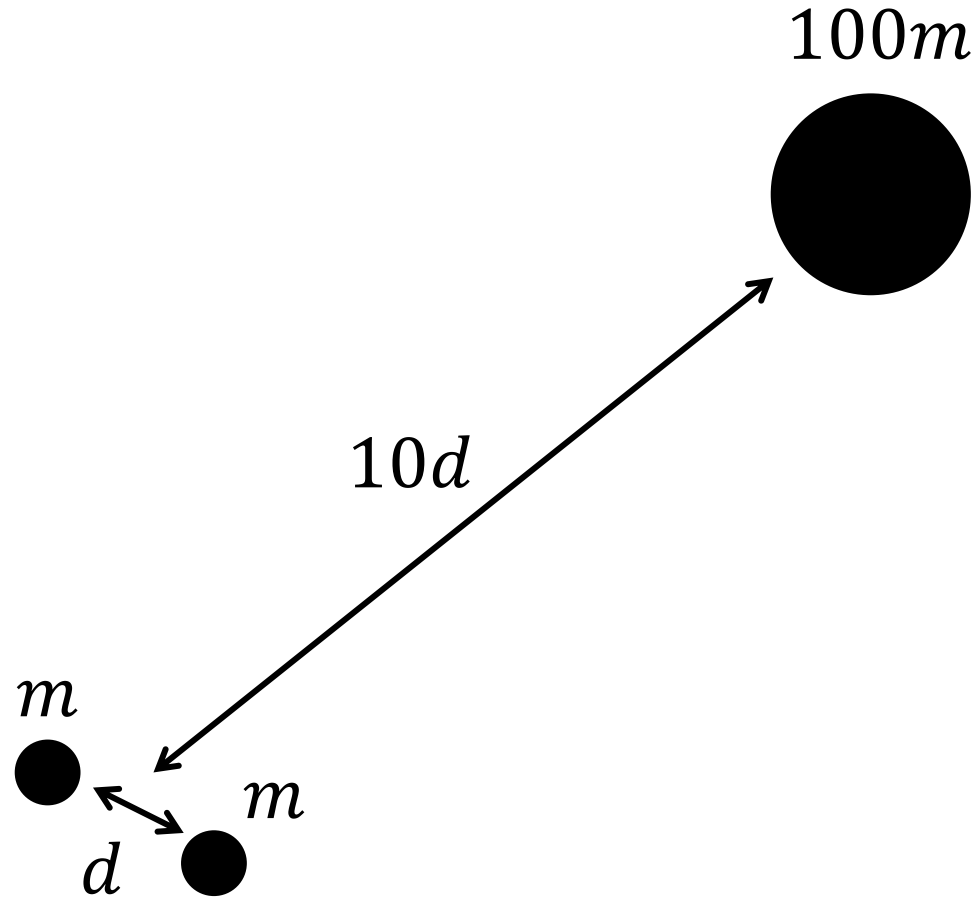
※*There is no unique prediction on the shape of the PBH mass function.*

Two assumptions

・$f(m_{BH})$ is not so broad ($\frac{m_{max}}{m_{min}} \lesssim O(10)$). It is not clear at all if the same mechanism of the binary formation can still work dominantly for very broad mass function.

・No correlation between different PBH masses.

Apart from this, we do not assume a specific form of $f(m)$.



If $\frac{m_{max}}{m_{min}} \gtrsim 100$, force from the third BH could become dominant.

Merger event rate distribution in (m_1, m_2) plane

PBH mass function

$$\mathcal{R}(m_1, m_2, t) = \frac{n_{\rm BH}}{2} f(m_1) f(m_2) P_{\rm intr}(m_1, m_2, t)$$

Observable in the future

Probability that given BH pair (m_1, m_2) form a binary and merge at time t.

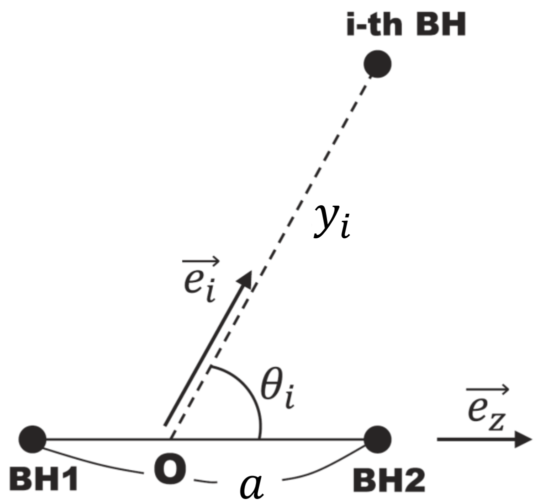
Non-trivial task is to evaluate P_{intr}.

To derive $P_{intr}(m_1, m_2)$, we need to know the probability distribution of (a, e).

Distribution of (a, e) is determined by statistical variables: $\{x, y_i, M_i, \boldsymbol{e}_i\}$

$$a = Ax^4, A = \frac{1}{1+z_{eq}} \frac{\rho_c \Omega_m}{m_1 + m_2}$$

$$1 - e^2 = \frac{9}{4}\zeta^2, \quad \zeta = \sum_{i=1}^{N} \frac{x^3}{y_i^3} \frac{M_i}{m_t} \sin(2\theta_i) \frac{(\boldsymbol{e}_z \times \boldsymbol{e}_i)}{|\boldsymbol{e}_z \times \boldsymbol{e}_i|},$$



Probability density of (x,ζ): $F(x,\zeta)dxd\zeta$

$$P_{\rm intr}(m_1,m_2,t)=\int_0^{e_{\rm m}} de\, F(x(a),\zeta(e))\underline{\frac{dx}{da}\frac{d\zeta}{de}\frac{\partial a}{\partial t}}$$

Known functions

$$F(x(a),\zeta(e))=\Theta(a_{\max}-a)\frac{4\pi x^2(a)}{n_{\rm BH}^{-1}}$$

$$\times\int\lim_{N\to\infty}\prod_{i=1}^{N}\frac{dV_i}{n_{\rm BH}^{-1}}\frac{f(M_i)dM_i}{n_{\rm BH}}\frac{\sin\theta_i d\theta_i d\phi_i}{4\pi}$$

$$\times\Theta(y_i-y_{i-1})e^{-\frac{4\pi}{3}n_{\rm BH}y_N^3}\delta(\zeta-g(x,y_i,M_i,\theta_i,\phi_i))$$

Evaluation of F is a non-trivial task.

We evaluated the merger rate under two different approximations.

- Nearest BH only ($N = 1$), analytically
- Flat mass function ($N \gg 1$) (numerically)

- <u>Nearest BH only ($N = 1$), analytically</u>

$$\boxed{P_{\rm intr}(m_1, m_2, t) = \frac{2}{37t} \int dM_1 \, \frac{1}{\beta} \frac{m_{\rm t}}{M_1} \frac{f(M_1)}{n_{\rm BH}} K^{\frac{16}{37}} \times \left[G(K) - G\left(\frac{M_1}{m_c}\right)\right],}$$

$$K = \left(\frac{170}{3}\right)^{\frac{3}{16}} \left(\frac{3}{\pi}\right)^{\frac{1}{4}} (1+z_{\rm eq})^{\frac{3}{4}} \pi \Omega_m^{\frac{1}{4}} f_{\rm PBH} (G m_{\rm BH} H_0)^{\frac{5}{16}} \beta$$

$$\sim 3 \times 10^{-4} \, f_{\rm PBH} \beta \left(\frac{m_{\rm BH}}{10 {\rm M}_\odot}\right)^{\frac{5}{16}},$$

$$G(x) = \Gamma\left(\frac{58}{37}, x\right)$$

$$m_c \equiv \frac{m_{\rm t}}{2\pi n_{\rm BH}} \frac{1}{\beta} \left(\frac{t}{Q}\right)^{1/7} (1+z_{\rm eq})^{4/7} \left(\frac{\rho_{c,0}\Omega_m}{m_{\rm t}}\right)^{\frac{25}{21}}$$

- <u>Nearest BH only ($N = 1$), analytically</u>

$$\frac{m_{\rm t}}{M_1}\frac{f(M_1)}{n_{\rm BH}}K^{\frac{16}{37}}\left[G(K)-G\left(\frac{M_1}{m_c}\right)\right]\propto\begin{cases}\underline{m_{\rm t}^{\frac{22}{21}}}(m_1m_2)^{-\frac{1}{7}}, & \frac{M_1}{m_c}<1\\ \underline{m_{\rm t}^{\frac{36}{37}}}(m_1m_2)^{\frac{3}{37}}, & \frac{M_1}{m_c}>1.\end{cases}$$

$$m_t \equiv m_1 + m_2$$

$$\mathcal{R}(m_1,m_2,t)=\begin{cases}C_A m_{\rm t}^{\frac{22}{21}}h_A(m_1)h_A(m_2), & (M_1<m_c\\ C_B m_{\rm t}^{\frac{36}{37}}h_B(m_1)h_B(m_2), & (M_1>m_c\end{cases}$$

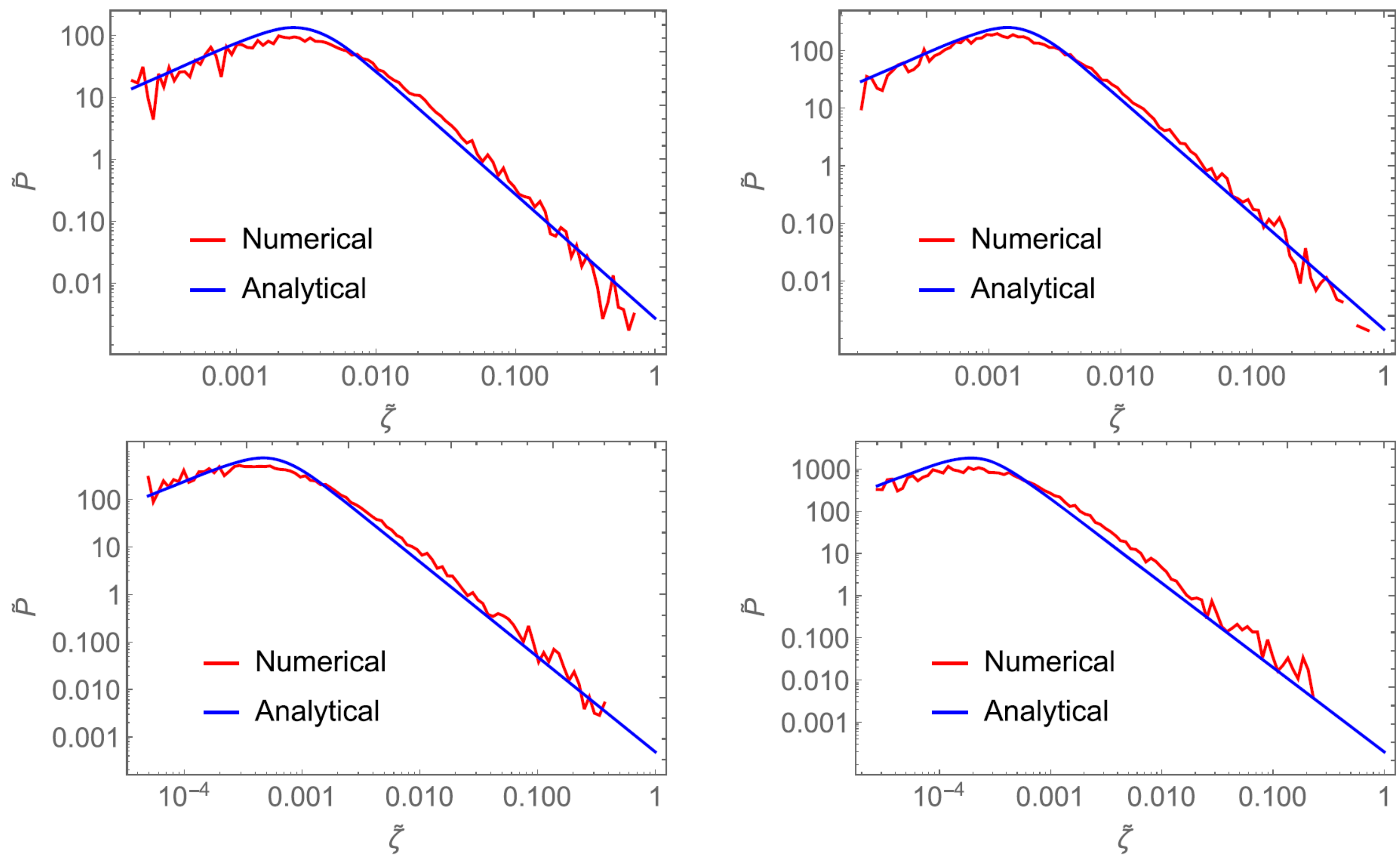
$$h_A(m)\equiv m^{-\frac{1}{7}}f(m),\;\; h_B(m)\equiv m^{\frac{3}{37}}f(m)$$

C_A, C_B:independent of m_1 and m_2

- <u>Flat mass function ($N \gg 1$) (numerically)</u>

We found an approximate fitting formula for the probability distribution of the eccentricity.

$$F(x,\zeta) = 6\sqrt{3}\xi^{1/3} n_{\mathrm{BH}} \tilde{\sigma}^2 \zeta x^2 \left(\frac{m_{\mathrm{t}}}{m_{\max}}\right)^2 \left[\left(\frac{m_{\mathrm{t}}}{m_{\max}}\right)^3 \zeta^3 + \xi\tilde{\sigma}^6\right]^{-1}$$



In both cases, we found that the merger rate distribution is given by

$$\mathcal{R}(m_1, m_2, t) = C\tilde{f}(m_1)\tilde{f}(m_2)(m_1 + m_2)^{\alpha}$$

$C, \tilde{f}(m)$: sensitive to the PBH mass function

Dependence on the total mass is not sensitive to the mass function!!

$$\frac{\mathbf{36}}{\mathbf{37}} < \boldsymbol{\alpha} < \frac{\mathbf{22}}{\mathbf{21}} \qquad (0.97 < \alpha < 1.05)$$

$$\ln\mathcal{R} = \ln C + \ln\tilde{f}(m_1) + \ln\tilde{f}(m_2) + \alpha\ln(m_1 + m_2)$$

$$\Rightarrow \quad \frac{\partial^2}{\partial m_1 \partial m_2}\ln\mathcal{R} = -\frac{\alpha}{(m_1 + m_2)^2}$$

Hidden Universality of $R(m_1, m_2, t)$

Statement

Construct a quantity α out of the distribution $R(m_1, m_2, t)$ as

$$\alpha(m_1, m_2, t) \equiv -(m_1 + m_2)^2 \frac{\partial^2}{\partial m_1 \partial m_2} \ln \mathcal{R}(m_1, m_2, t)$$

Then, the PBH mergers predict

$$0.97 \lesssim \alpha \lesssim 1.05$$

for any PBH mass function (as long as it is not broad).

Different formation mechanisms predict different value

$\alpha \approx 1.43$ **PBH binary formation at low redshift. (Bird et al. 2016, Clesse, Garcia-Bellido 2016)**

$\alpha \sim 4$ **Dynamical formation scenario (astrophysics BHs)**

1812.05376

Effects of the surrounding primordial black holes on the merger rate of primordial black hole binaries

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(Dated: December 20, 2018)

We develop an analytic formalism for computing the merger rate of primordial black hole binaries with a general mass function by taking into account the torques by the surrounding primordial black holes and linear density perturbations. We find that $\alpha = -(m_i + m_j)^2 \partial^2 \ln \mathcal{R}(m_i, m_j)/\partial m_i \partial m_j = 36/37$ is independent of the mass function. Moreover, the ratio of the merger rate density of primordial black hole binaries by taking into account the torques by the surrounding primordial black holes to by the nearest primordial black hole is independent of the masses of binaries.

Recent study also confirmed out result!!

Summary

GW astronomy has just begun.

LIGO might have detected PBHs for the first time.

The PBH scenario can be tested in the future by GW data.