Constraining cosmology with primordial black holes

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EXCELLENCE IN SCIENCE THE ROYAL SOCIETY

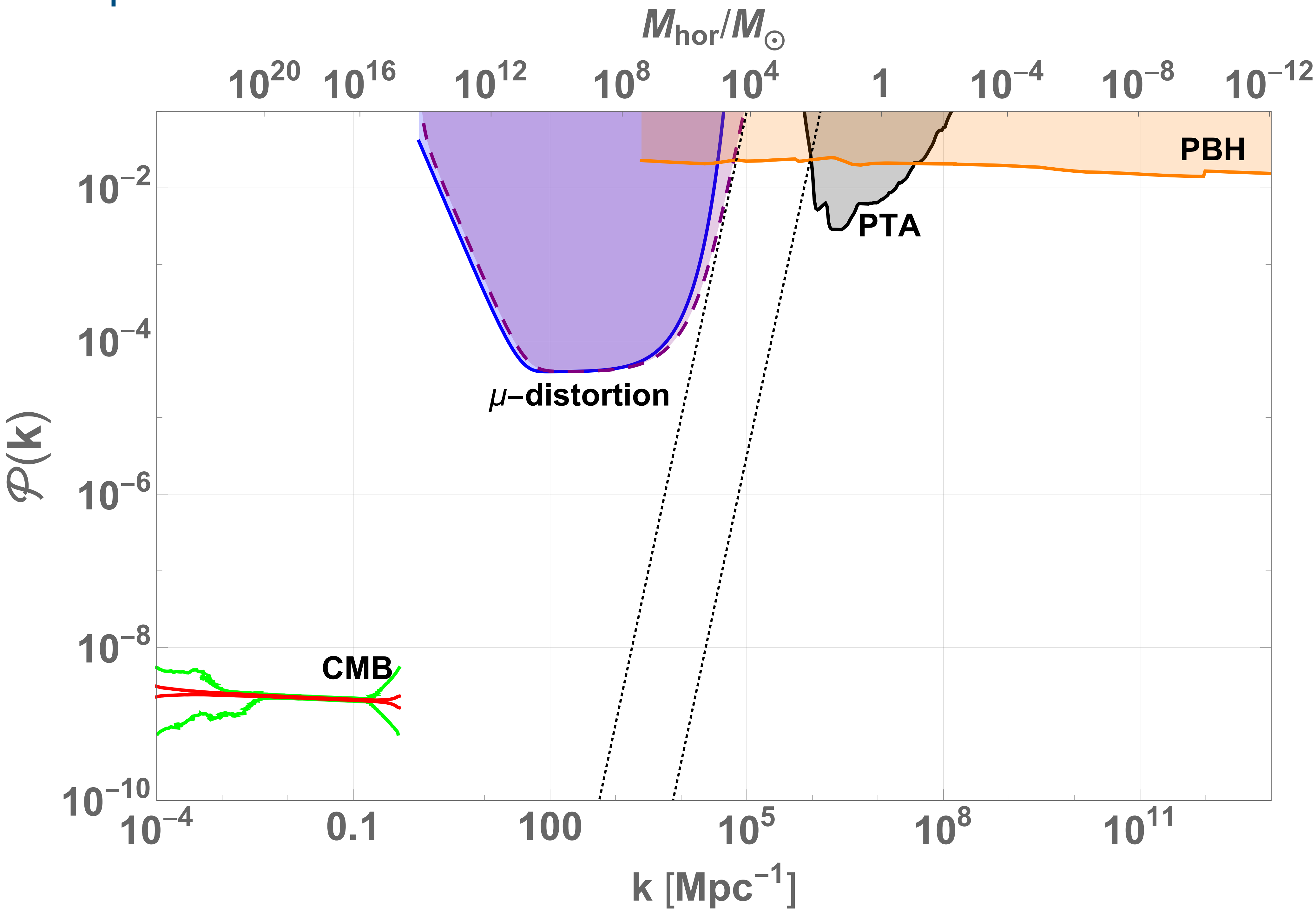
PBH collaborators include: J. Adamek, N. Bellomo, P. Cole, E. Copeland, A. Gow, M. Gosenca, A. Green, A. Hall, M. Hawkins, S. Hotchkiss, M. Hindmarsh, I. Musco, S. Patil, J. Peacock, D. Regan, M. Sasaki, S. Young

Solvay BH workshop, Brussels, 4 April 2019 (40’)

Intro

- **Like dark matter and dark energy, black holes (BHs) are hard to observe. Gravitational waves are a direct probe.**
- **Black holes are a fossil of dead stars**
  Perhaps also an early Universe fossil
- **Goal: Understand the initial conditions and constituents of the Universe**

Power spectrum constraints



CB, Cole & Patil `18
See also Kohri & Terada `18; Inomata and Nakama `18;
Clesse, Garcia-Bellido and Orani `18

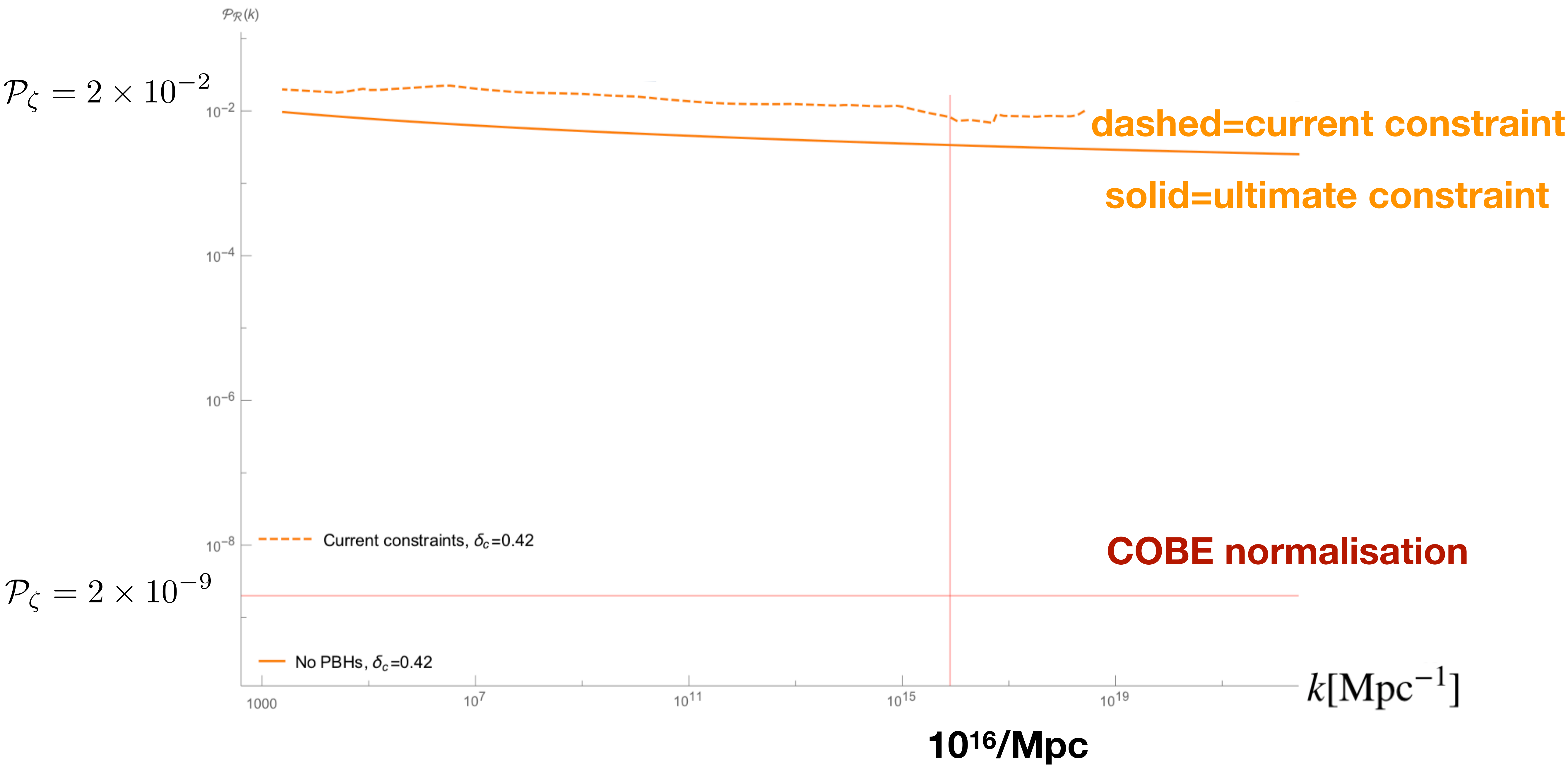
Constraints, constraints, constraints…

- If you care about PBHs being the DM, then f_{PBH} constraints are the key.

- If you care about the **initial conditions** then observational constraints are exponentially less important than properly understanding the formation mechanism and how to relate $P_{zeta}(k)$ to beta(M_{PBH}) and the equation of state. We don't know if the Universe was radiation dominated before BBN. Significant uncertainties remain despite a large, ongoing community effort: *Germani & Musco '18; Yoo et al '18; Kalaja, Bellomo, Musco, Verde… in preparation, etc*

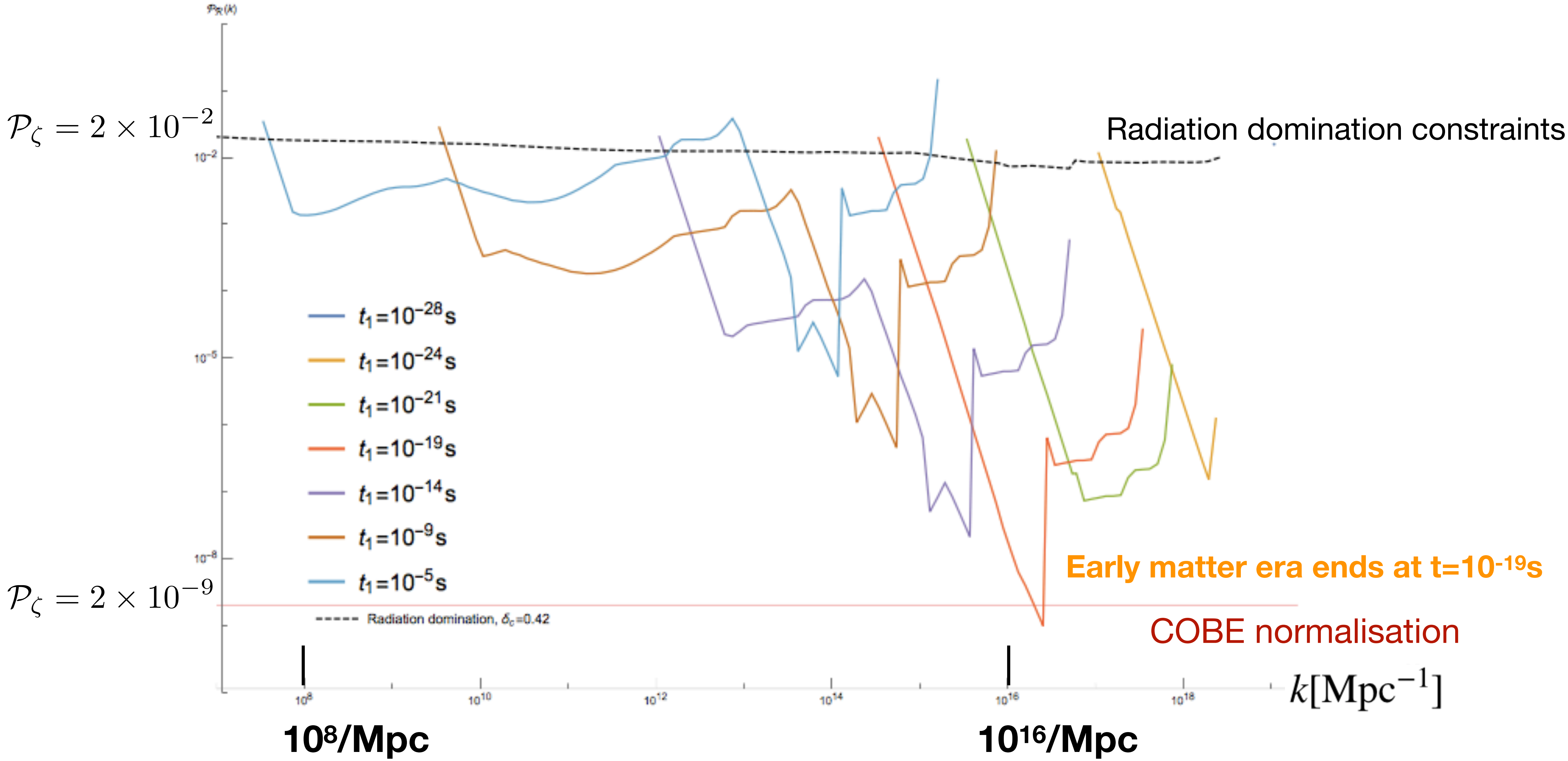
- Unlike *Bringmann, Scott & Akrami '12*, we do not include ultracompact minihalo constraints because they require DM be WIMPS. The steep density profile they assumed is unrealistic (simulated by *Gosenca, Adamek, CB, Hotchkiss `17*) but tight constraints still exist (*Delos et al `18*)

Ultimate constraints from PBHs



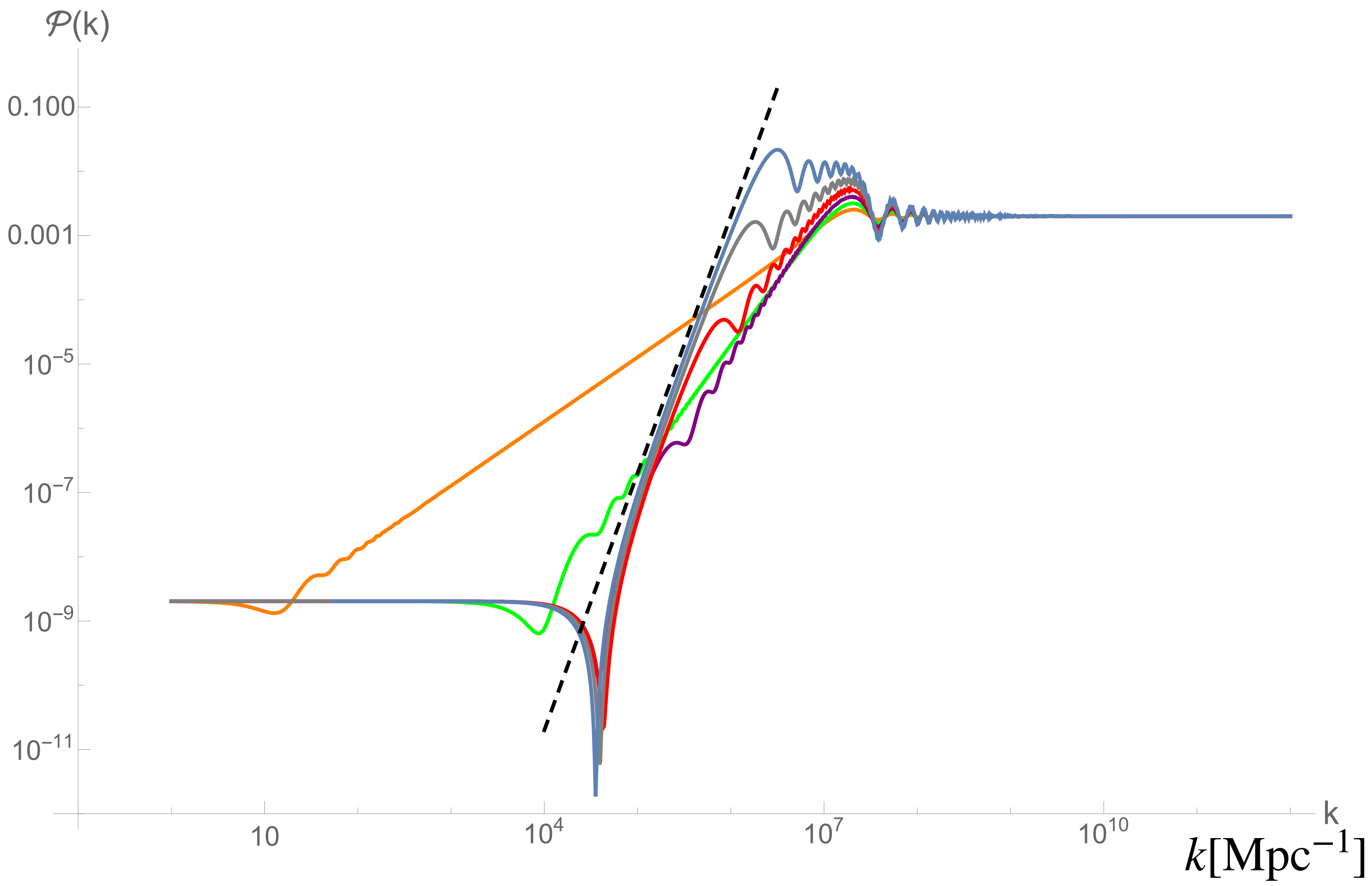
Pippa Cole & CB '17

Early matter domination dramatically tightens constraints



Pippa Cole & CB '17

Steepest possible power spectra

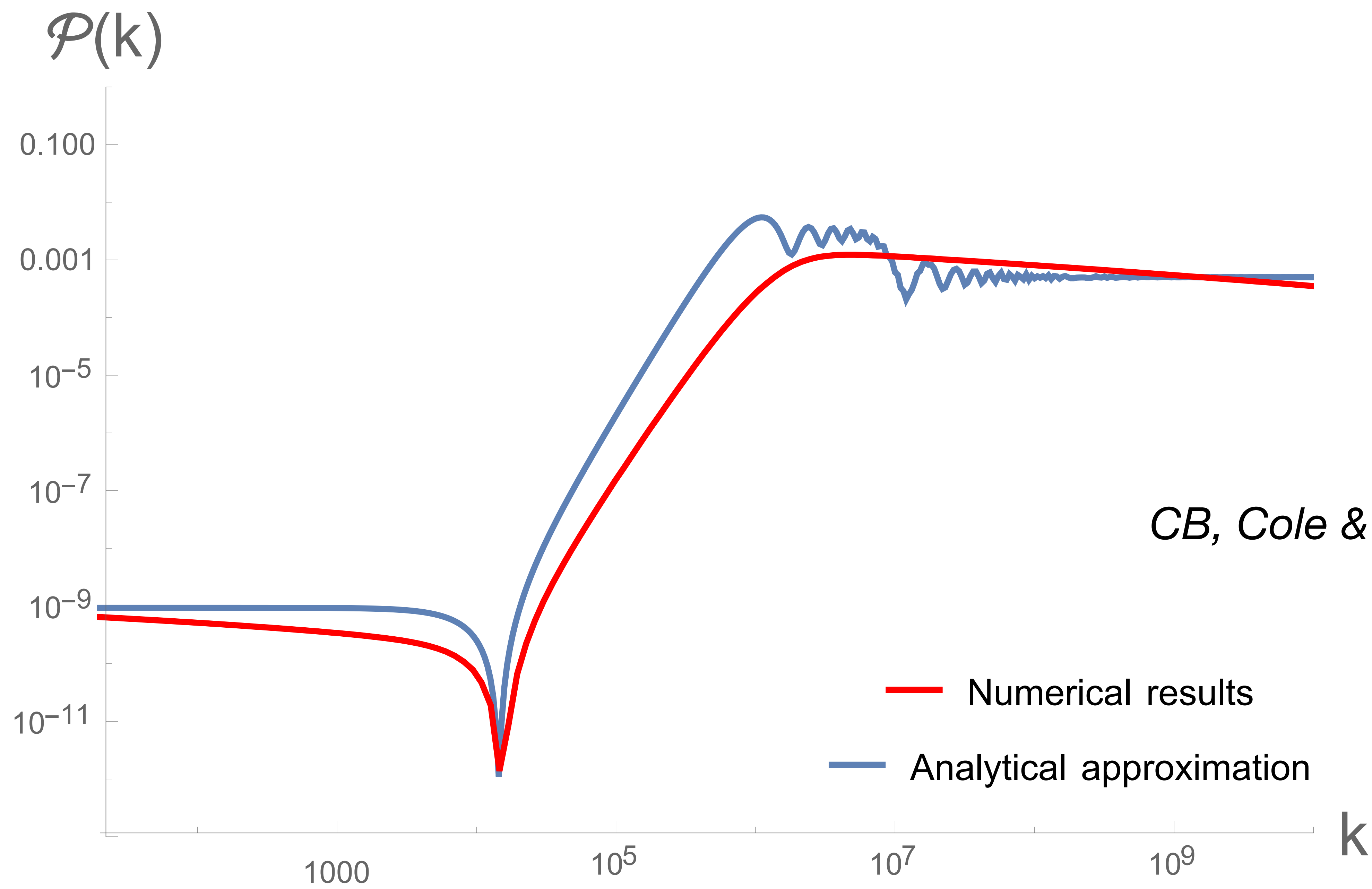


From single-field inflation with an inflection point (ultra slow-roll inflation).

The power spectrum cannot be steeper than k^4

CB, Cole & Patil 2018

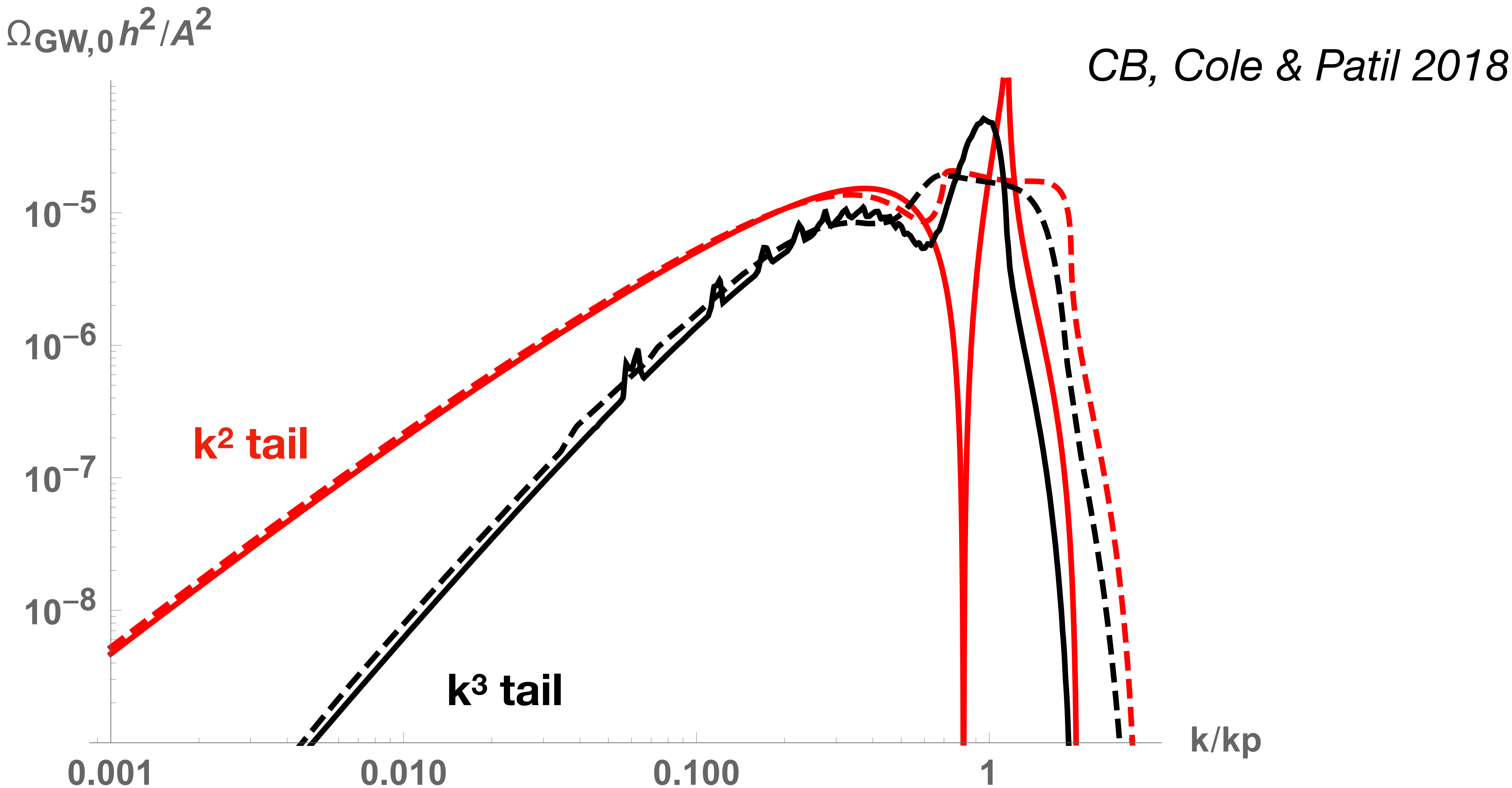
“Realistic” model with a smooth potential



CB, Cole & Patil 2018

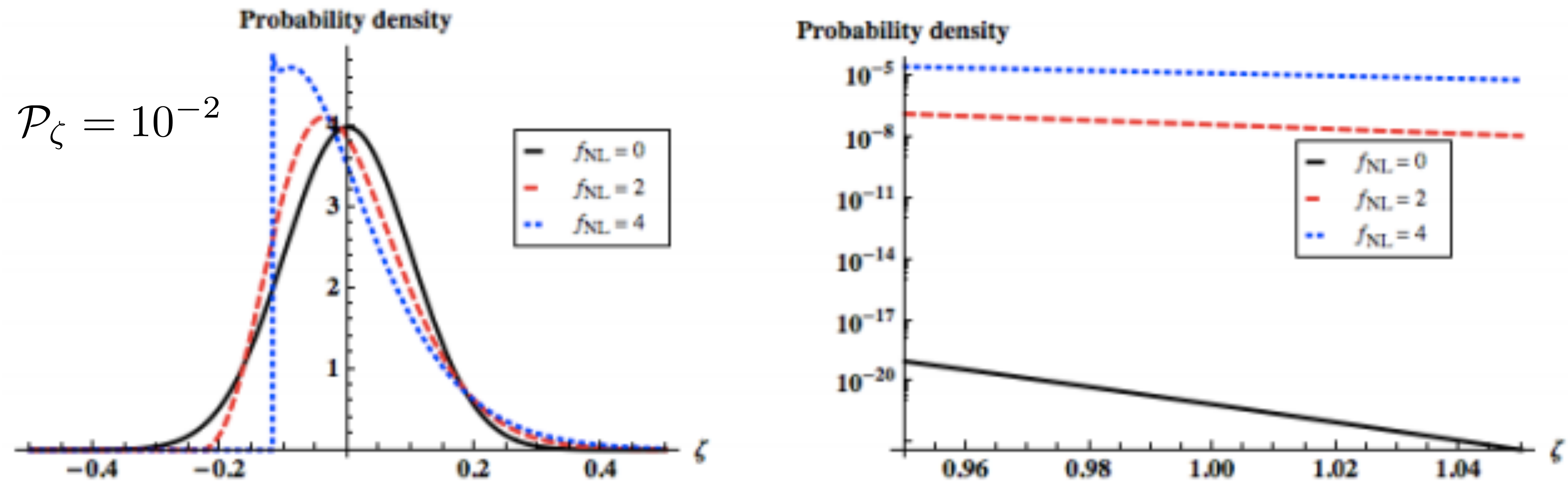
The red line is a full result from a smooth potential (*Germani & Prokopec* `17, modified from *Garcia-Bellido & Morales `17*), while the red line is based on a piecewise analytic calculation. Calculated using CPPTransport created by *David Seery 2016*

GW spectrum



Red line - delta function scalar power spectrum
Black line - k^4 scalar power spectrum with cut off
Surprisingly, the delta function scalar power spectrum has a shallower GW tail.
This is unphysical and a warning against using delta functions.

PBH abundance is exponentially sensitive to non-Gaussianity



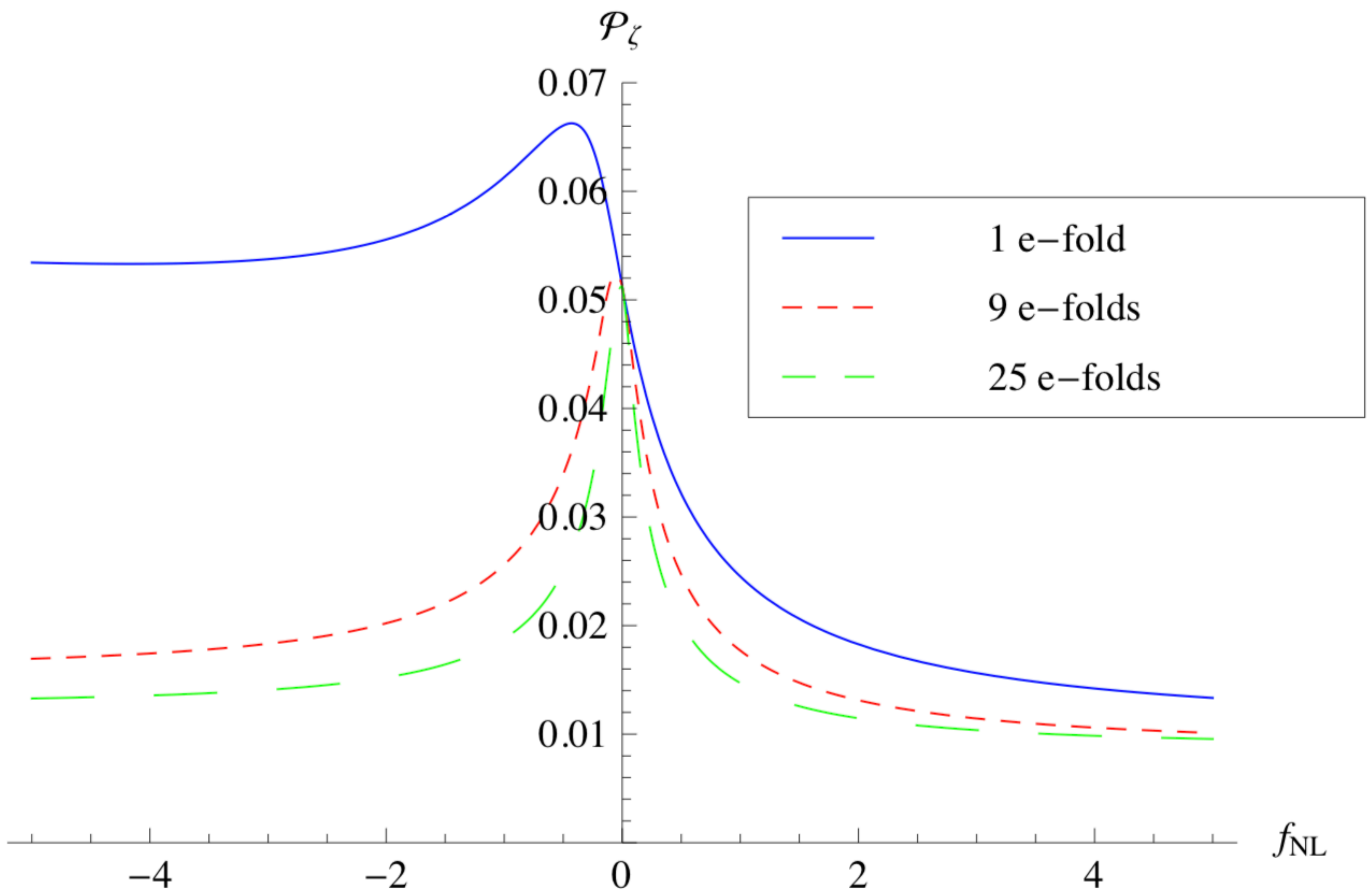
Young & CB 2013

Even a **tiny** amount of squeezed limit (local) non-Gaussianity correlating standard CMB scales with PBH formation scales will generate large scale DM isocurvature perturbations. This is strongly ruled out by Planck constraints

- *Tada & Yokoyama 2015, Young & CB 2015*

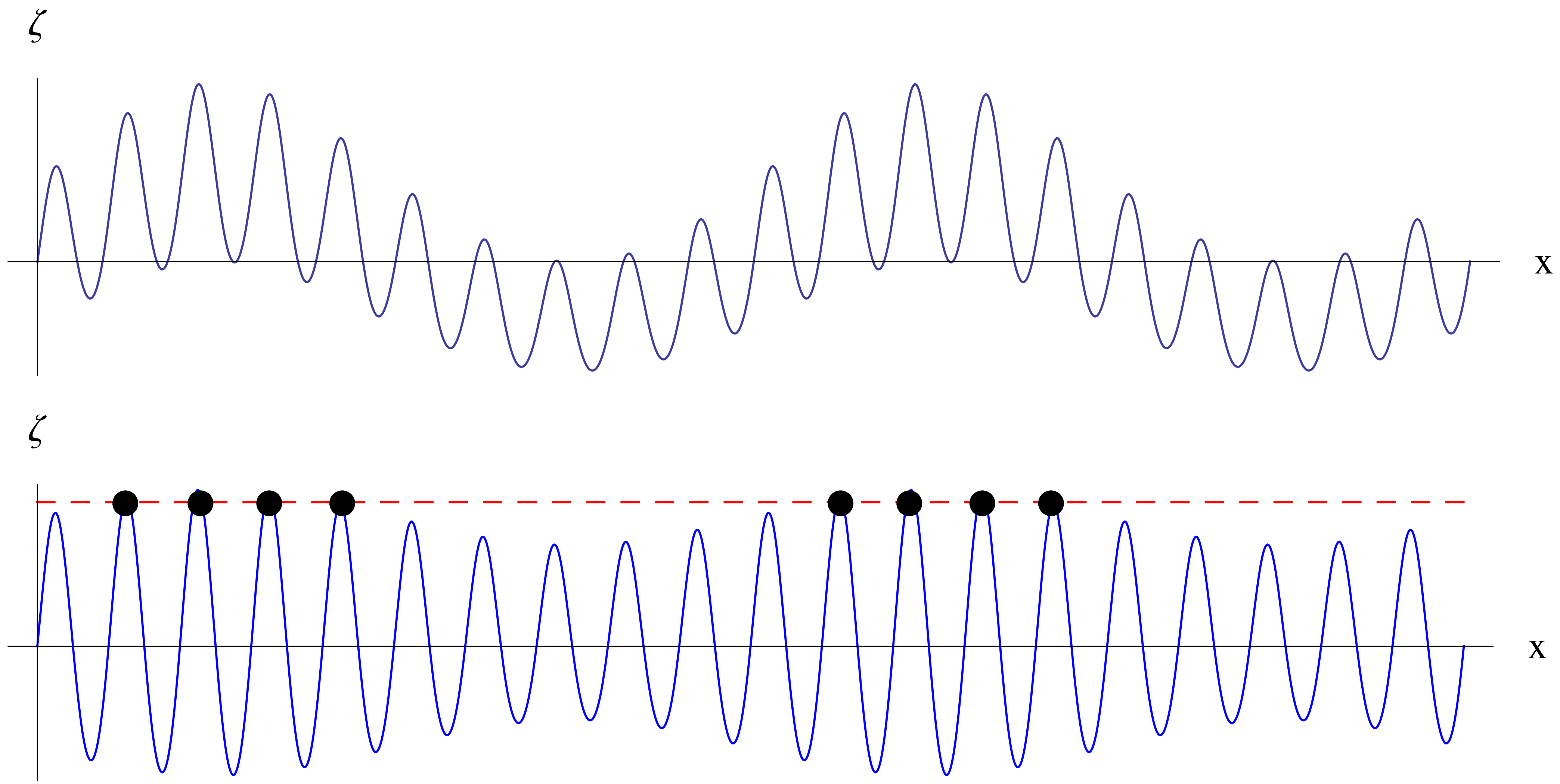
Non-Gaussianity strongly impacts the constraints

Considering local non-Gaussianity with a top-hat peak in the power spectrum



Young & CB `15

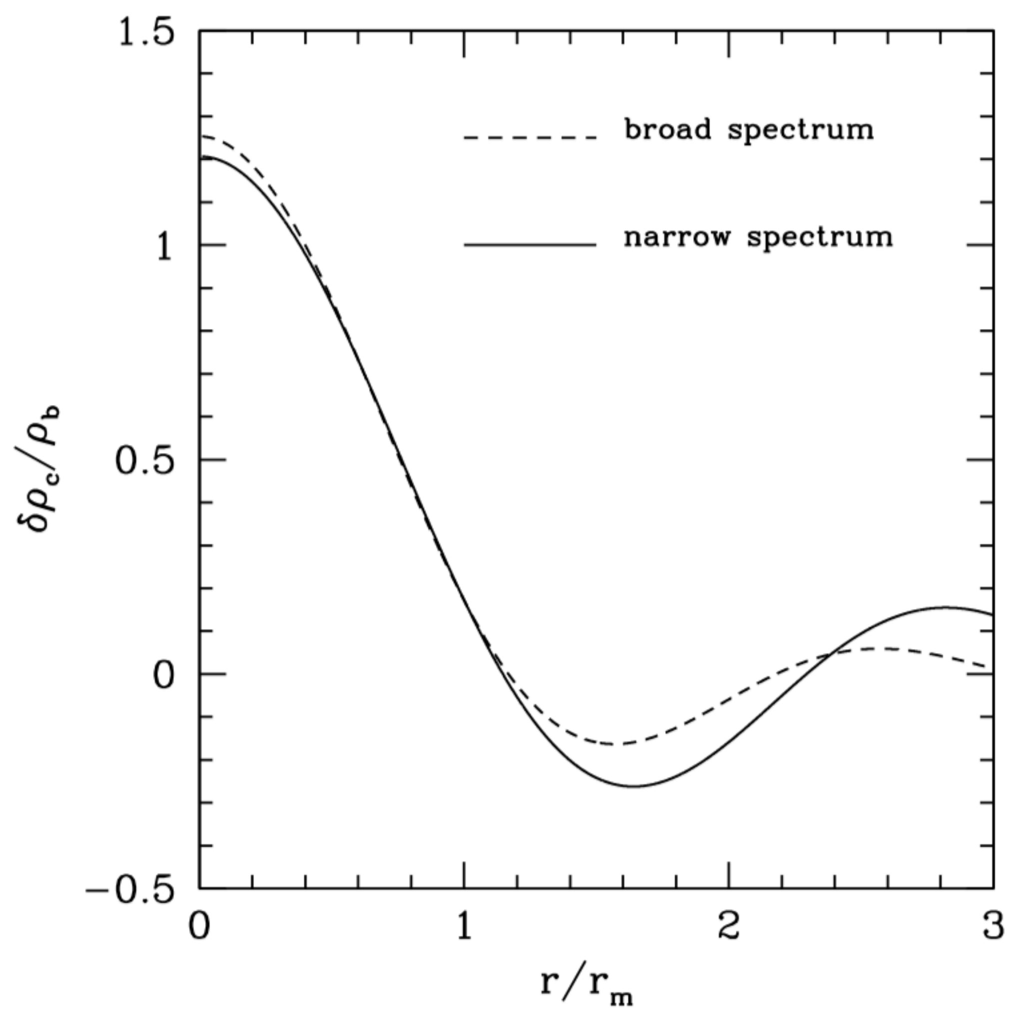
Isocurvature constraints



- Beware of primordial non-Gaussianity (of the curvature perturbation). If modes on observable scales are correlated to the PBH forming scales then a DM isocurvature perturbation will be generated and ruled out by Planck observations: - *Tada & Yokoyama 2015, Young & CB 2015*

The density profiles are related to the power spectrum shape

Due to the 2-point correlation function which tells you the density near peaks and shows that spherical symmetry is a good approximation
(*BBKS 1986* classic paper, non-spherical effects *Kühnel & Sandstad 2016*)
The density profile does not change strongly assuming a smooth peak in the primordial power spectrum, independently of the width



Germani & Musco 2018

The density profiles are related to the power spectrum shape

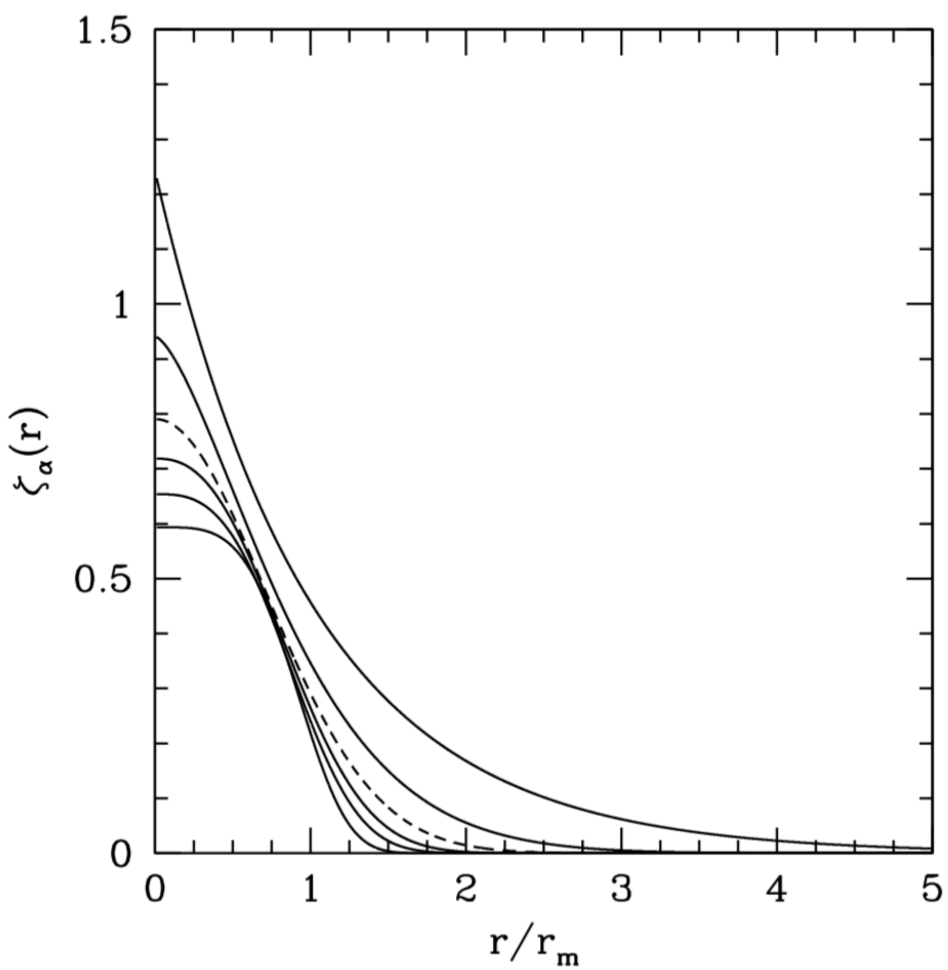
Curvature perturbation profiles:

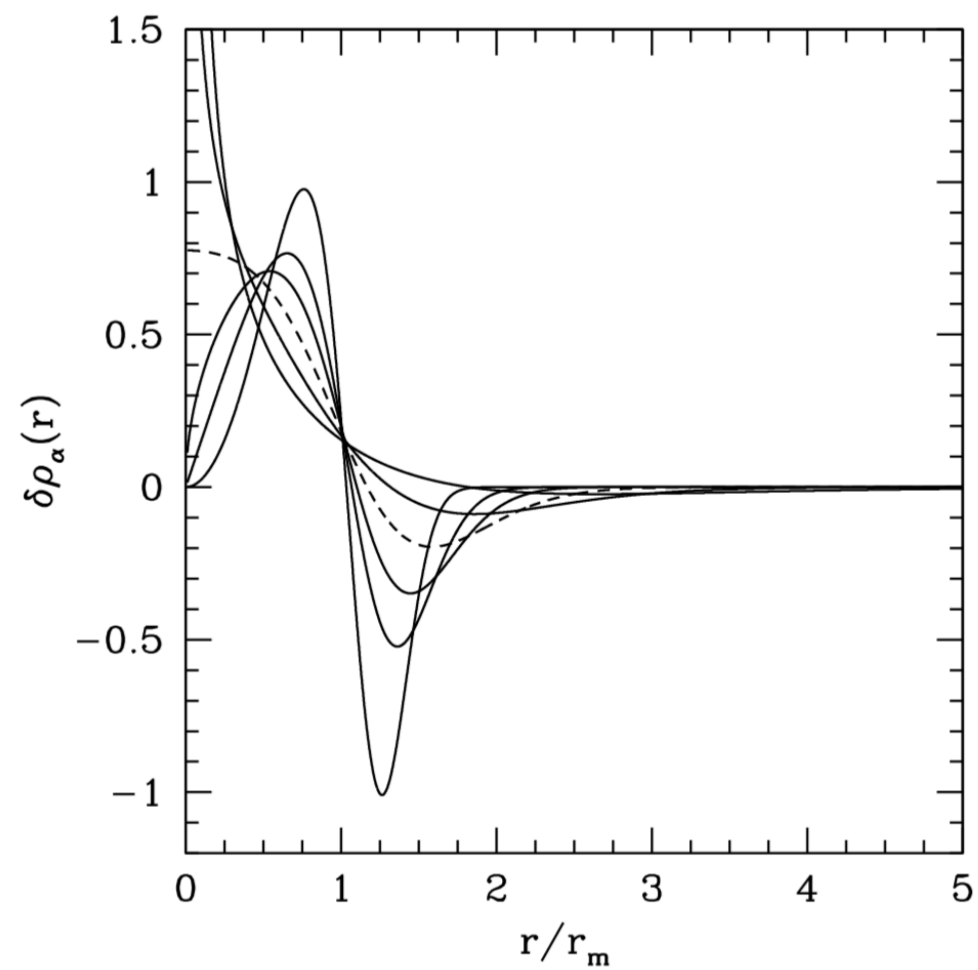
$$\zeta_\alpha(r) = \mathcal{A}\exp\left[-\left(\frac{r}{r_m}\right)^{2\alpha}\right]$$

r_m is the scale at which the compaction function is maximised

$$\mathcal{C}(r,t) \equiv 2\frac{M(r,t)-M_b(r,t)}{R(r,t)}$$

Profile dependence: *Musco 2018* (see also *Nakama, Harada, Polnarev & Yokoyama 2014)*





Young, Musco, CB 2019

The curvature and density perturbation

- The literature has almost exclusively assumed a linear mapping from the curvature perturbation, zeta, to the comoving density perturbation
- However, the large amplitude perturbations required for PBH formation mean that the linear relationship is unreliable
- This mapping is required because zeta is not a good variable for studying PBH formation (with some exceptions) and also to connect the inflationary and GR simulator communities

$$\frac{\delta\rho}{\rho_b} = -\frac{2(1+\omega)}{5+3\omega}\left(\frac{1}{aH}\right)^2 \nabla^2\zeta$$

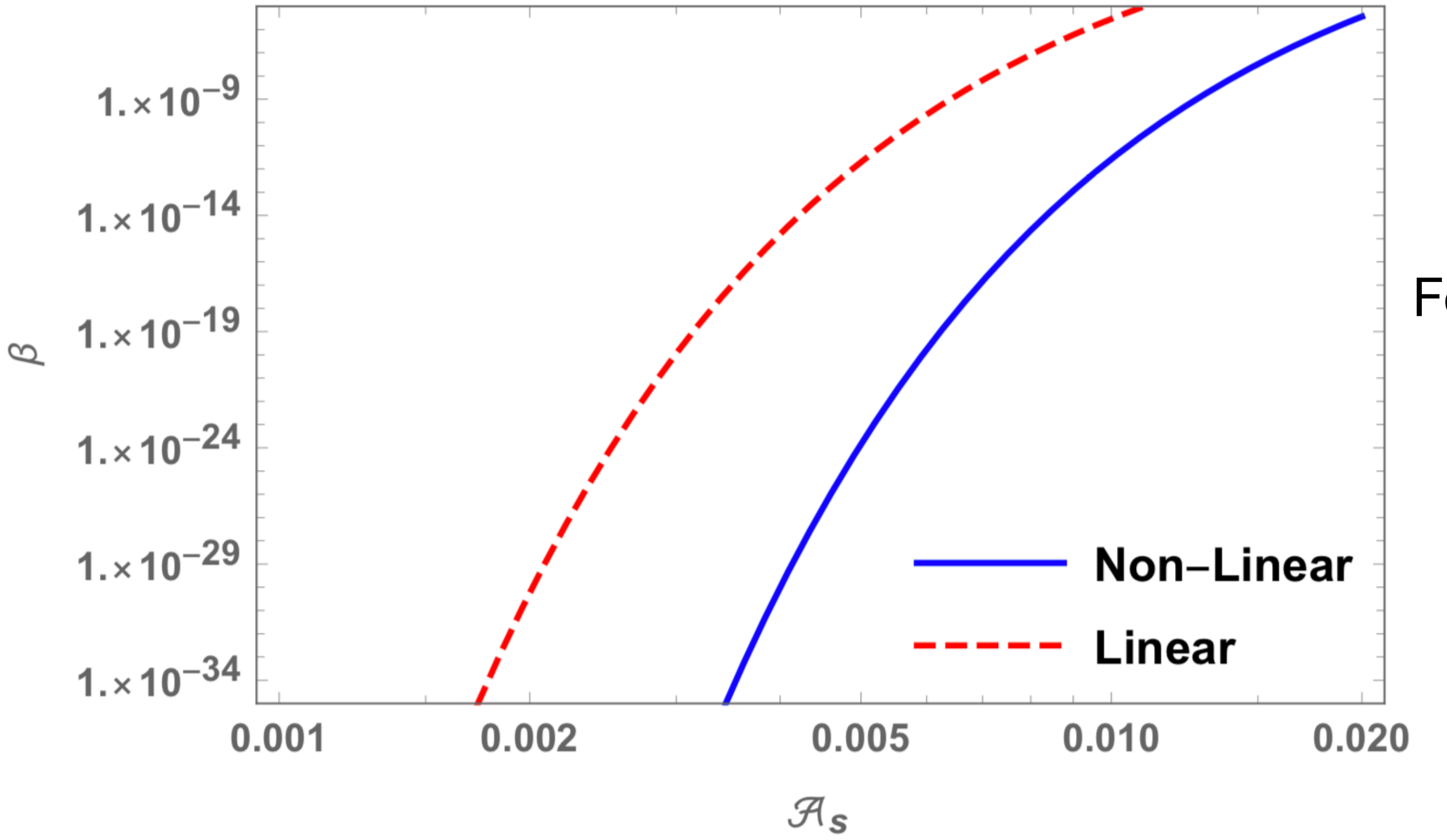
$$\frac{\delta\rho}{\rho_b}(r,t) = -\frac{4(1+\omega)}{5+3\omega}\left(\frac{1}{aH}\right)^2 e^{-5\zeta(r)/2}\nabla^2 e^{\zeta(r)/2}$$

The non-linear relation has been studied very recently
Kawasaki & Nakatsuka 2019 (last month)*;*
De Luca et al and *Young, Musco, CB* both on Tuesday this week

The non-linearity suppresses PBH formation

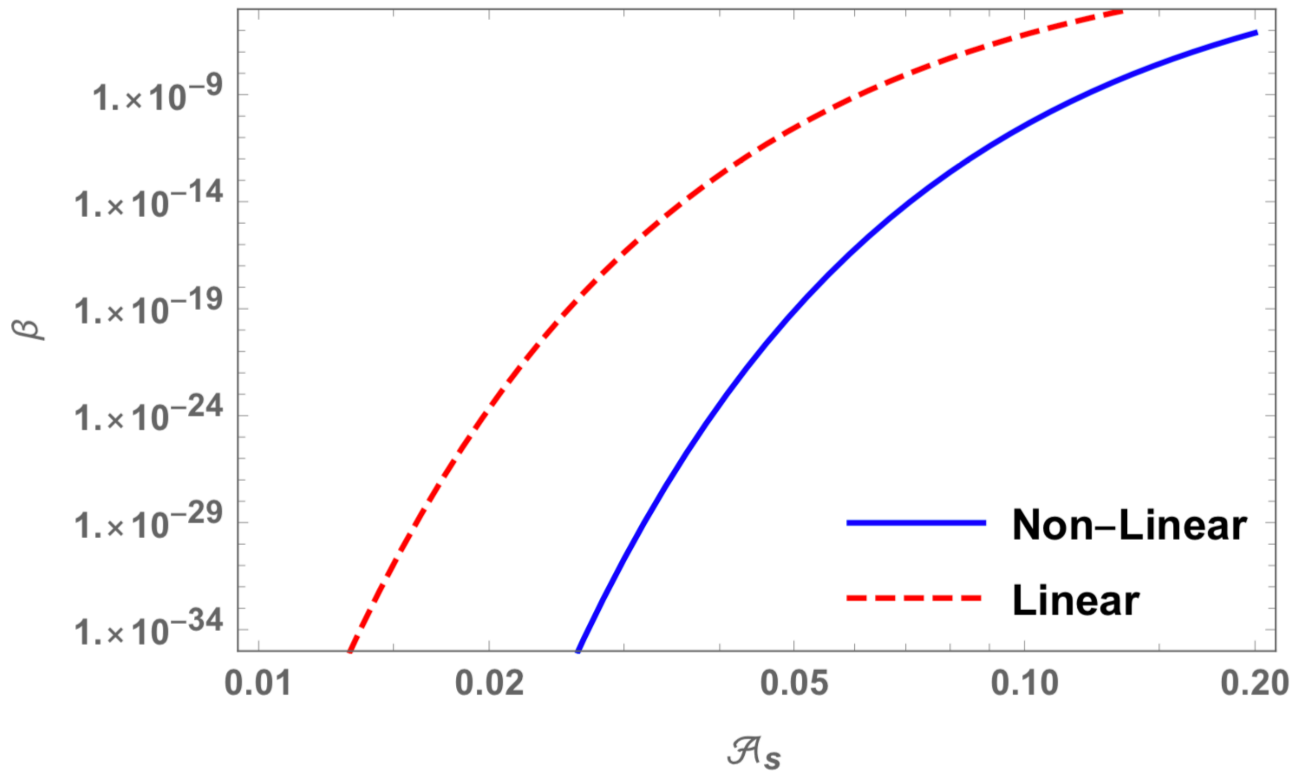
$$\delta_m = -\frac{2}{3} r_m \zeta'(r_m) \left[2 + r_m \zeta'(r_m)\right]$$

- This looks exactly like local non-Gaussianity, acts like a negative skewness on the 1-point distribution
- This non-Gaussianity is inevitable (ineludible)
- But the derivatives are uncorrelated on scales much larger than the PBH radii (r_m), so no large scale isocurvature perturbation generated (unless zeta is non-Gaussian)



For a (locally) scale invariant spectrum
Young, Musco, CB `19

Power spectrum constraints are weakened by a factor ~ 2



Delta function power spectrum
Young, Musco, CB `19

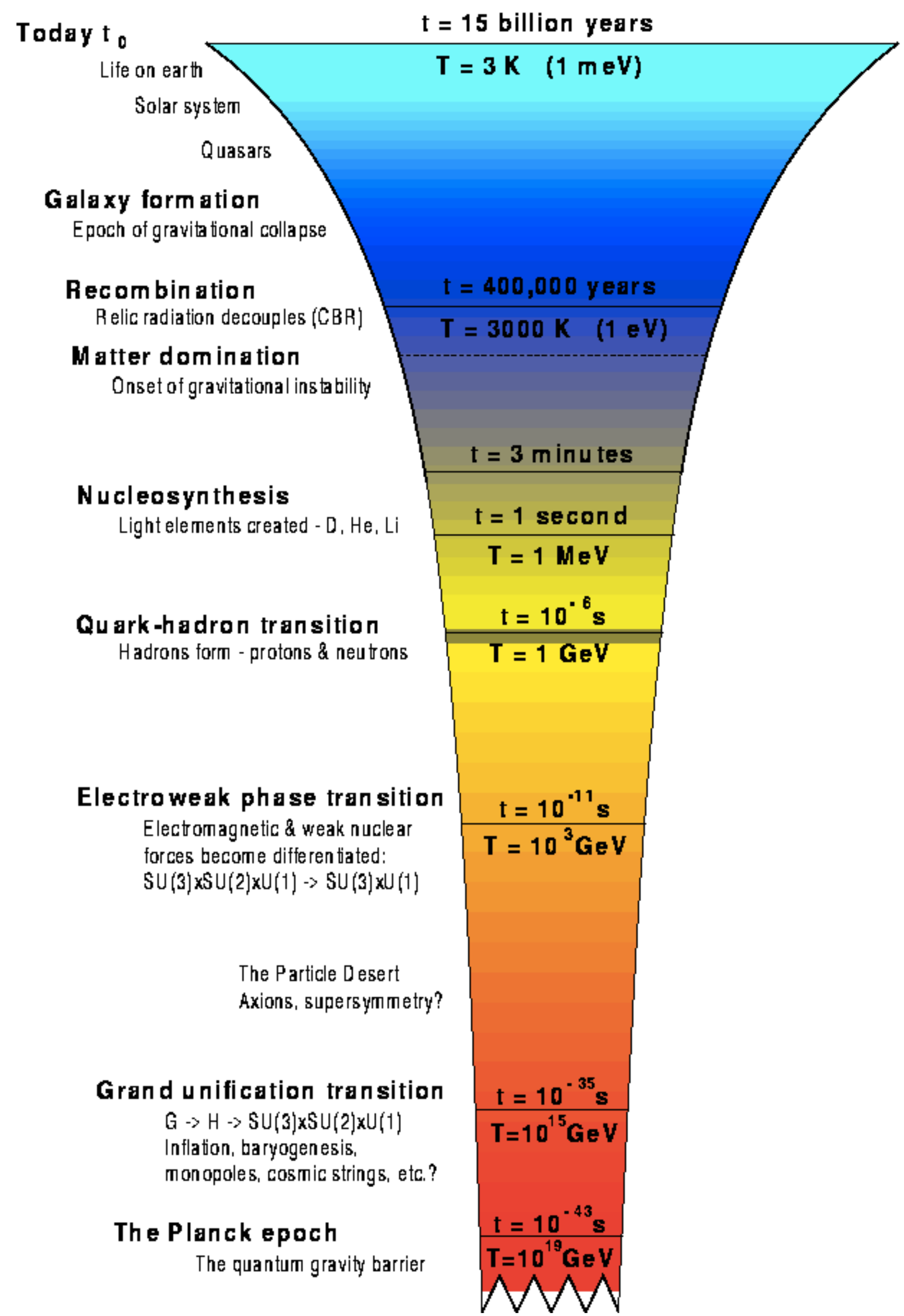
In order to generate the same number of PBHs when taking the non-linear (NL) relation into account, compared to the normal/wrong case that you use the linear relation, the power spectrum amplitude needs to increase by the ratio

$$1.5 \lesssim \frac{\mathcal{A}_{NL}}{\mathcal{A}_L} = \frac{16\left(1-\sqrt{\frac{2-3\delta_c}{2}}\right)^2}{9\delta_c^2} \lesssim 4$$

For the typical value of delta_c~0.55, power spectrum constraints are weakened by a factor of 2

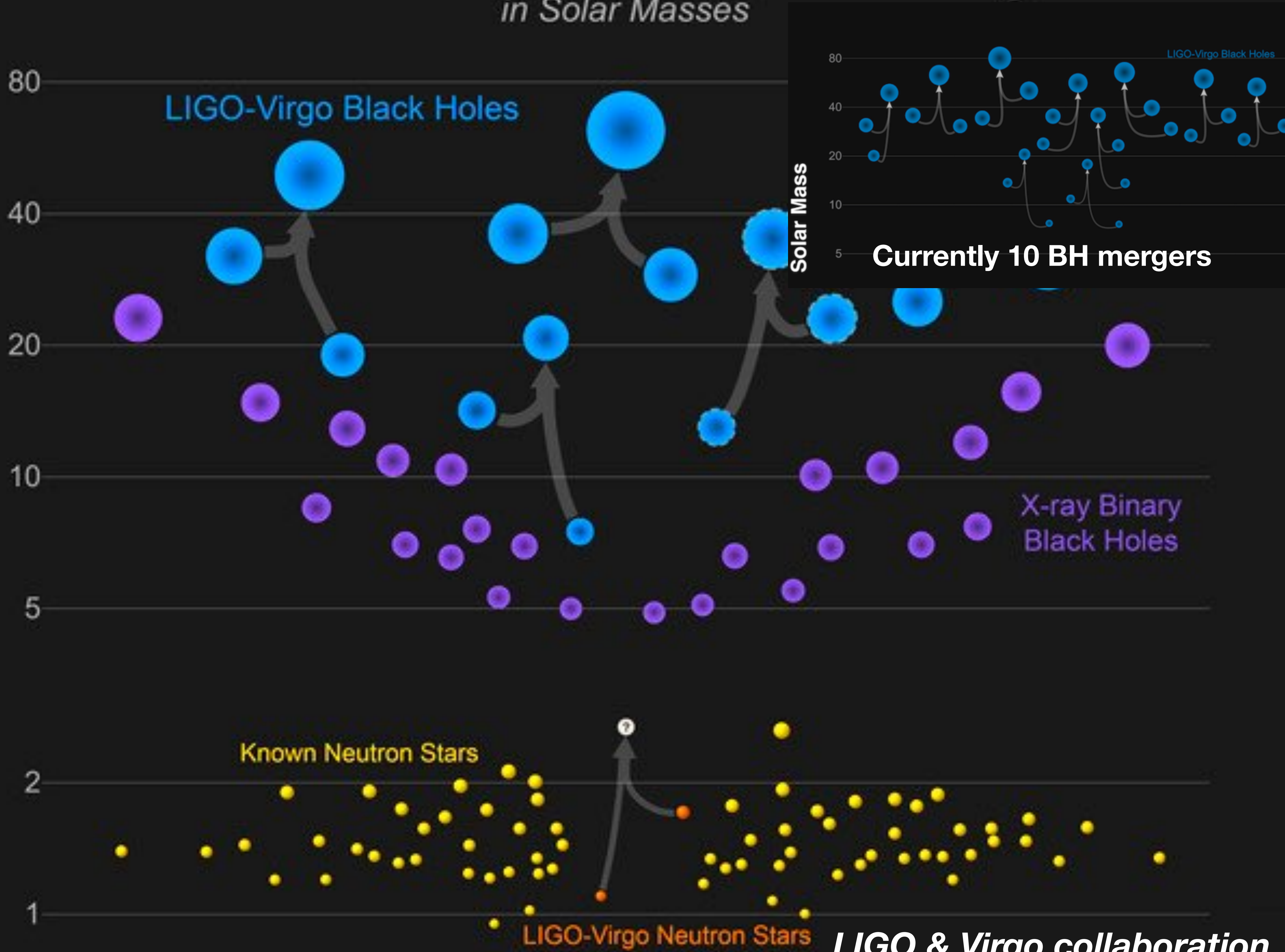
A cosmological coincidence

- The QCD transition occurs during the time when LIGO mass PBHs formed.
- Juan's talk: Another coincidence beta~eta~10^{-9}
- The horizon mass has grown by about 50 orders of magnitude since the end of inflation.
- QCD transition: t~10^{-6} s, T~200 MeV, M~1 $M_\odot$, k~10^7 Mpc^{-1}



Masses in the Stellar Graveyard

in Solar Masses

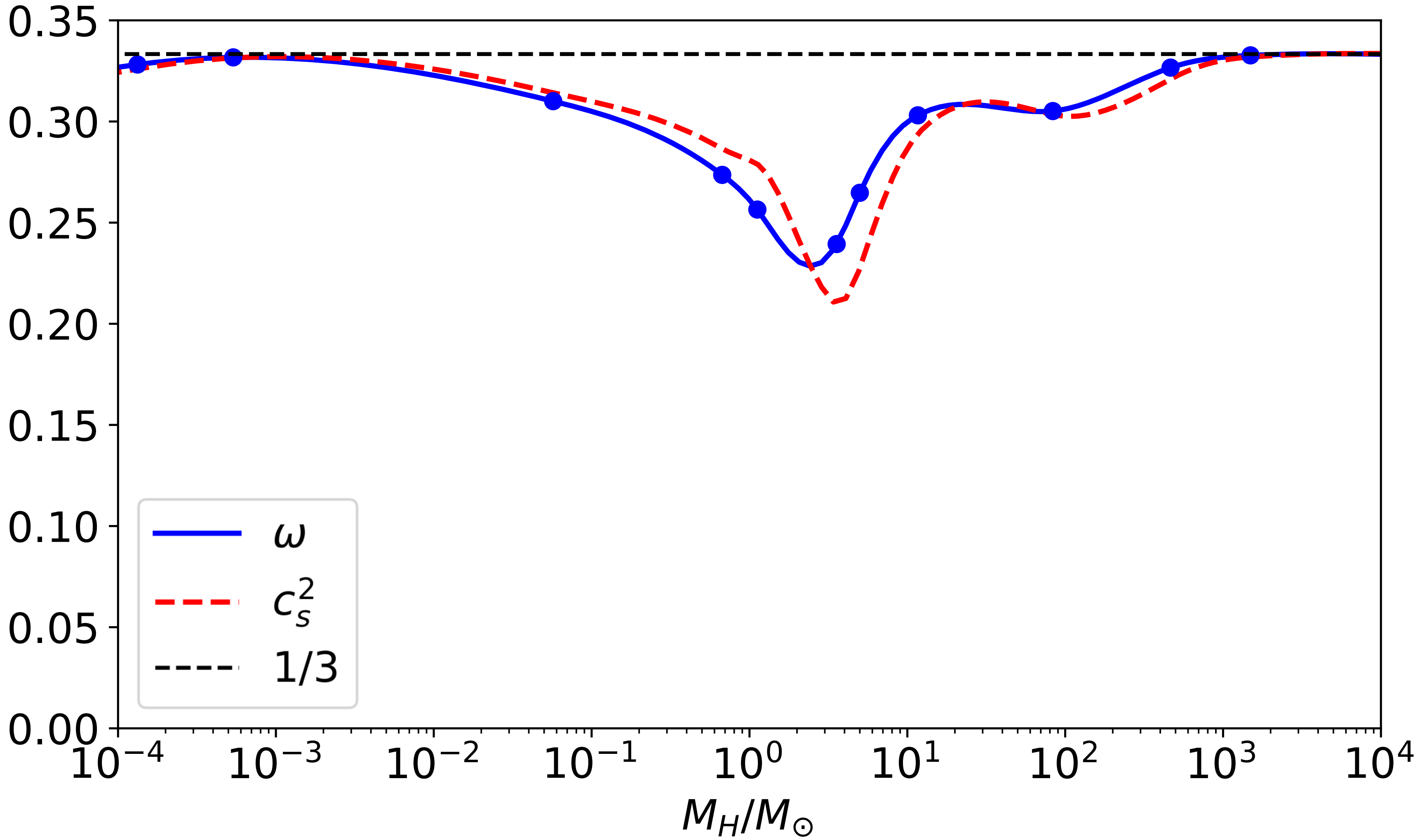


LIGO & Virgo collaboration

The QCD transition

- As the Universe cools below 1 GeV, strong interactions confine quarks into hadrons and the equation-of-state parameter w decreases. *Borsanyi et al (2016)* made the first definitive predictions of this period

- PBHs form with a mass comparable to the horizon mass - 1 solar mass (Crawford & Schramm `82,Jedamzik `98)

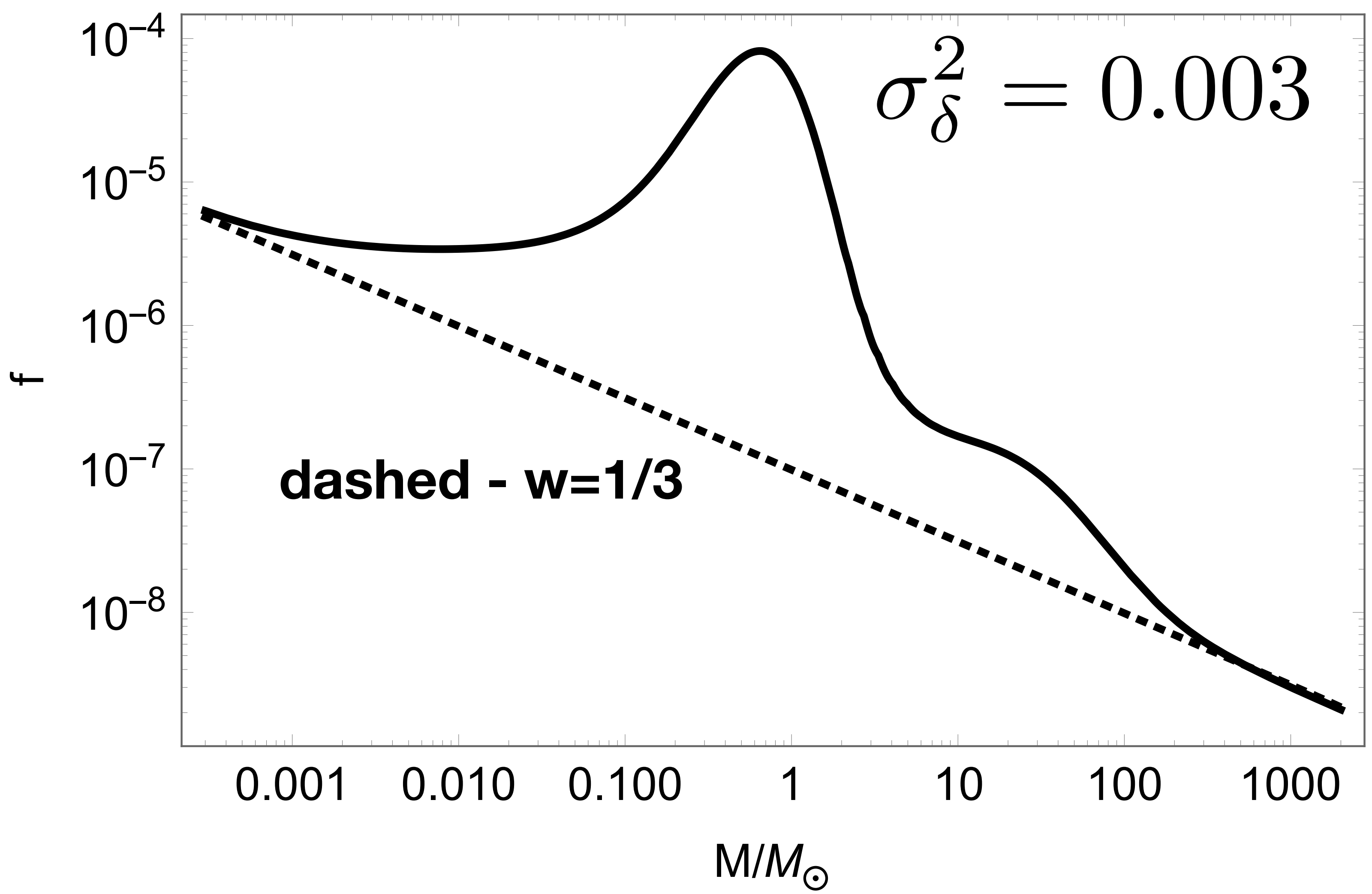


CB, Hindmarsh, Young & Hawkins 2018

The resultant PBH mass function

$$f(M) = \frac{1}{\Omega_{\rm CDM}} \frac{d\Omega_{\rm PBH}}{d \ln M_{\rm H}}$$

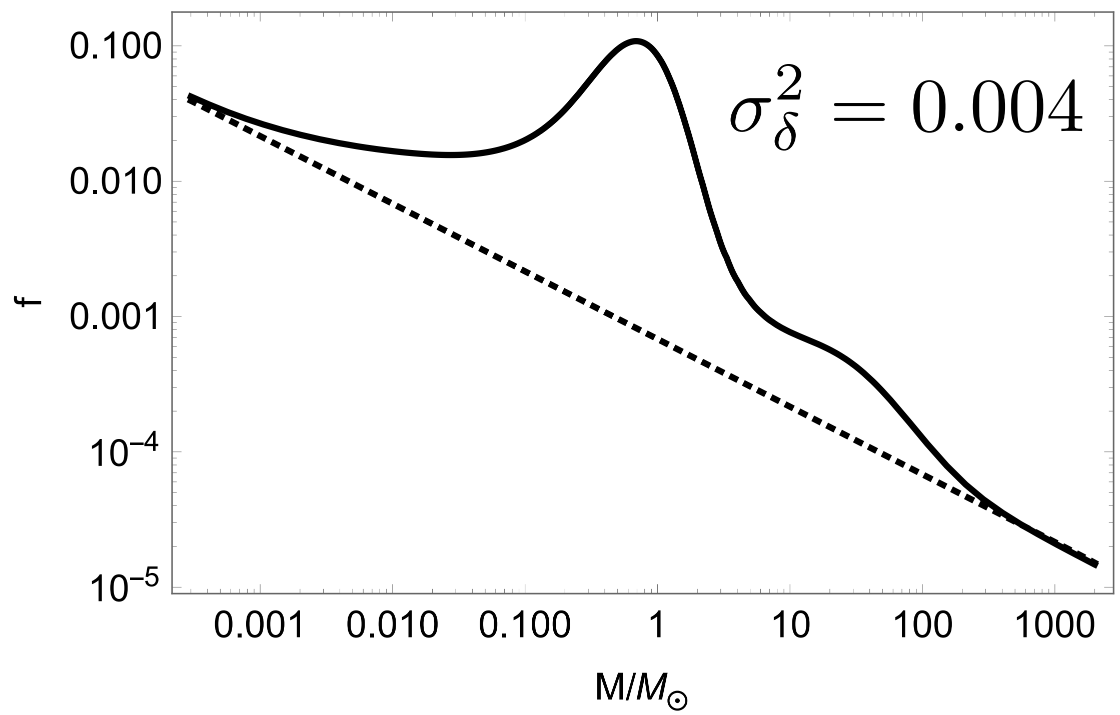
$$f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma_\delta^2}}$$

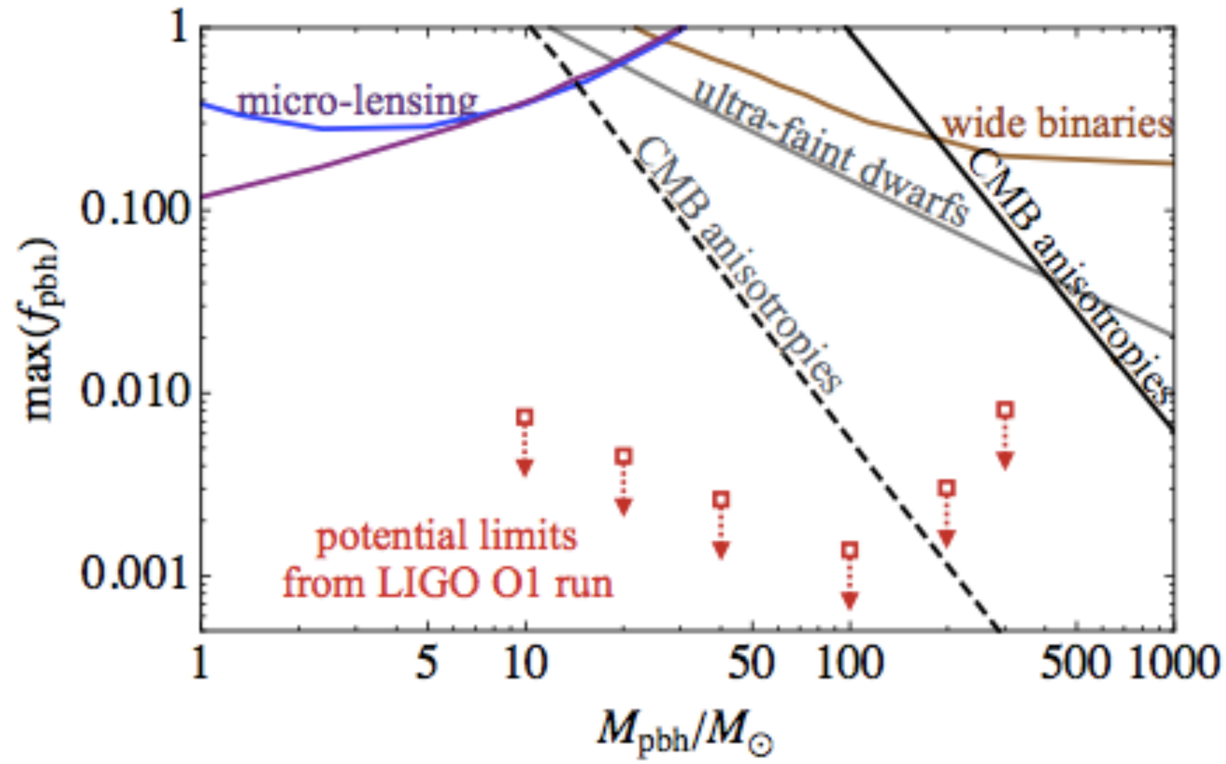


Despite the critical collapse threshold decreasing by only ~10%,
PBH formation is boosted by over two orders of magnitude

The resultant PBH mass function

$$f(M) = \frac{1}{\Omega_{\rm CDM}} \frac{{\rm d}\Omega_{\rm PBH}}{{\rm d}\ln M_{\rm H}}$$





CB, Hindmarsh, Young & Hawkins 2018

Haimoud et al 2017

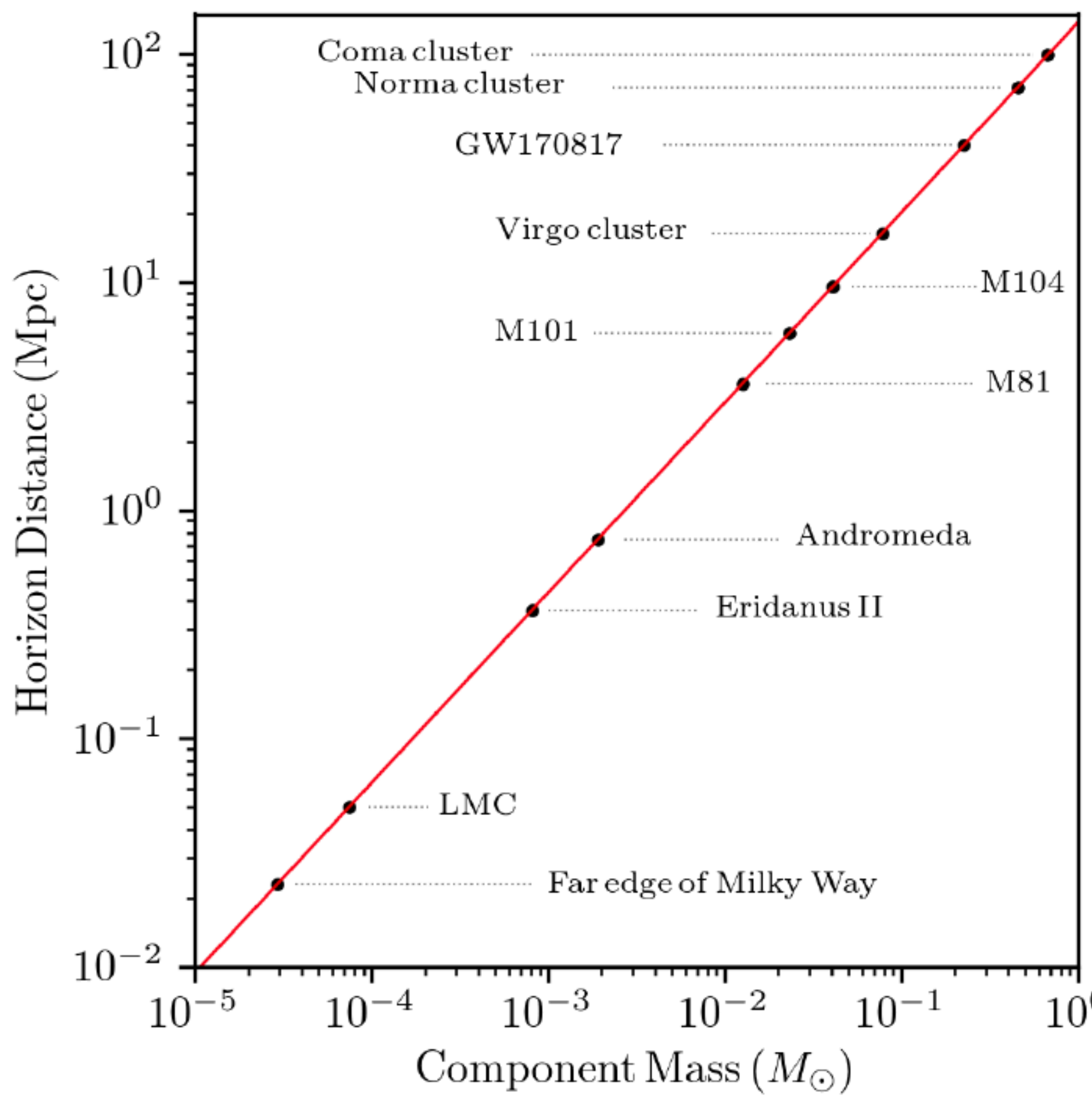
For the left plot, approx 10% of DM is made up of ~ solar mass PBHs and 0.1% lies in the LIGO mass range - enough to get the merger rate LIGO detects
Sasaki et al + Haimoud et al + Chen & Huang + Raidal et al + many more

- The BHs LIGO and Virgo detected might include PBHs (is the low spin a hint?) *Bird et al; Clesse & Garcia-Bellido; Sasaki, Suyama & Yokoyama; - all 2016*

- If the initial power spectrum was boosted on scales corresponding to LIGO mass BHs, the QCD transition naturally predicts a larger population of solar mass PBHs. These cannot form by stellar collapse.

LIGO sensitivity for equal mass mergers

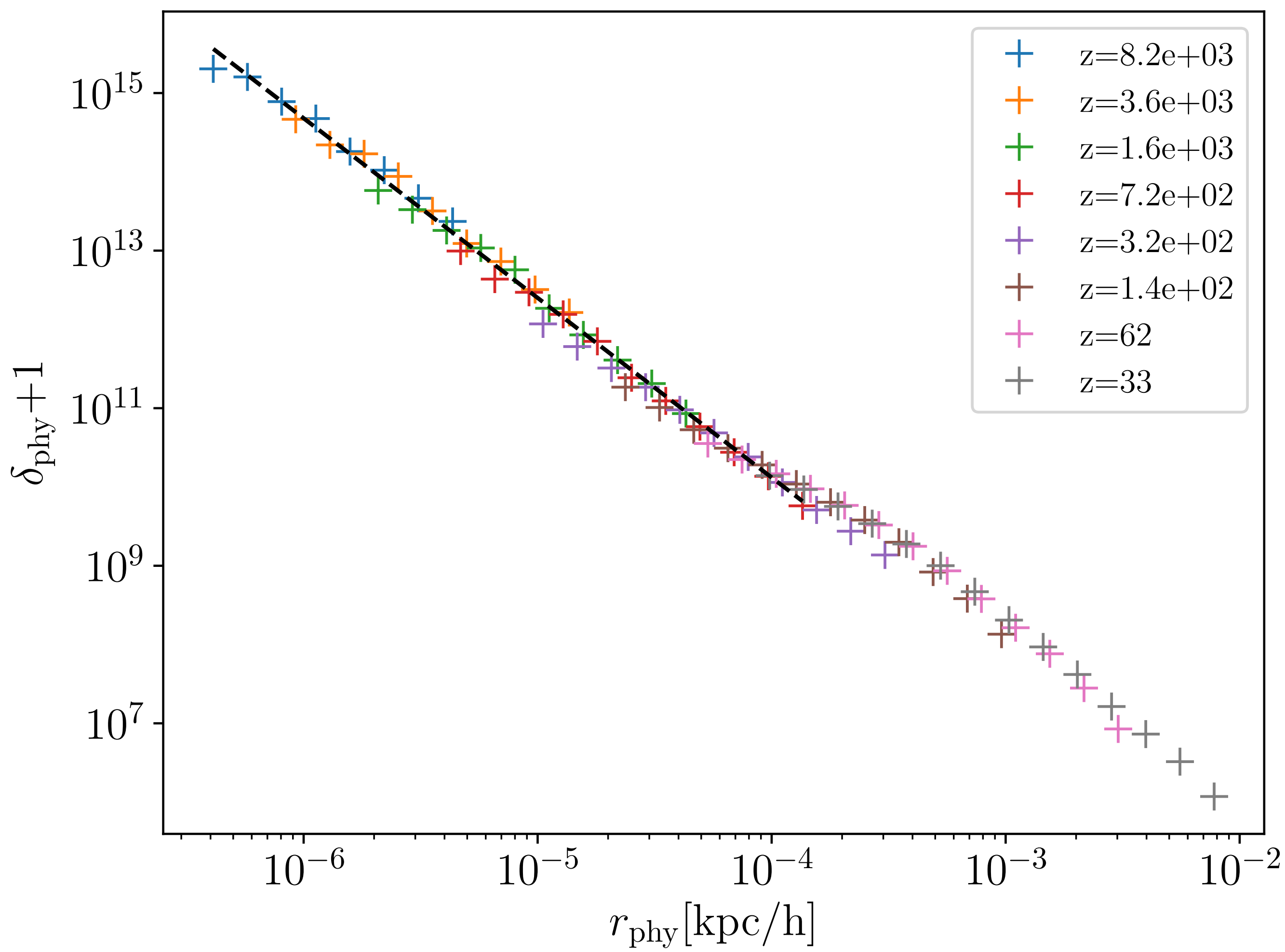
Magee et al 2018



If PBHs are more than one billionth of the dark matter, what is the rest made of?

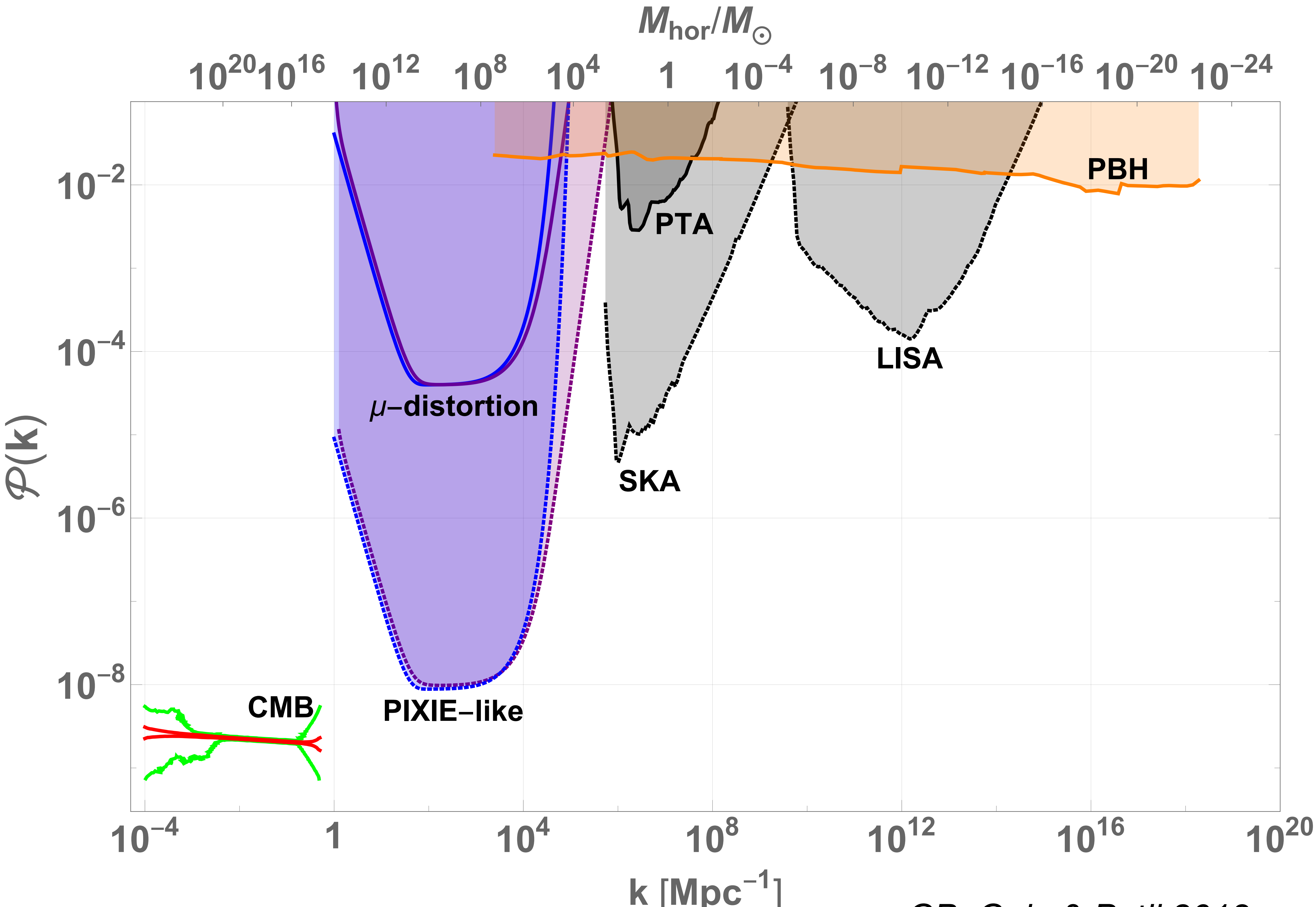
- Not WIMPs!
- If $f_{PBH}<1$, then another DM component is inevitable. It matters what it is
- Steep and high density profiles form around PBHs
- In contrast to UCMHs without a PBH seed
- We showed analytically and by the first N-body simulations of this process that the DM density profile scales as $r^{-9/4}$
- WIMPs in the innermost parts of the halo would annihilate and generate a strong gamma-ray signal

- A detection of WIMPs or PBHs would rule out the existence of the other (unless $M_{PBH}<10^{-6}\ M_{sun}$)

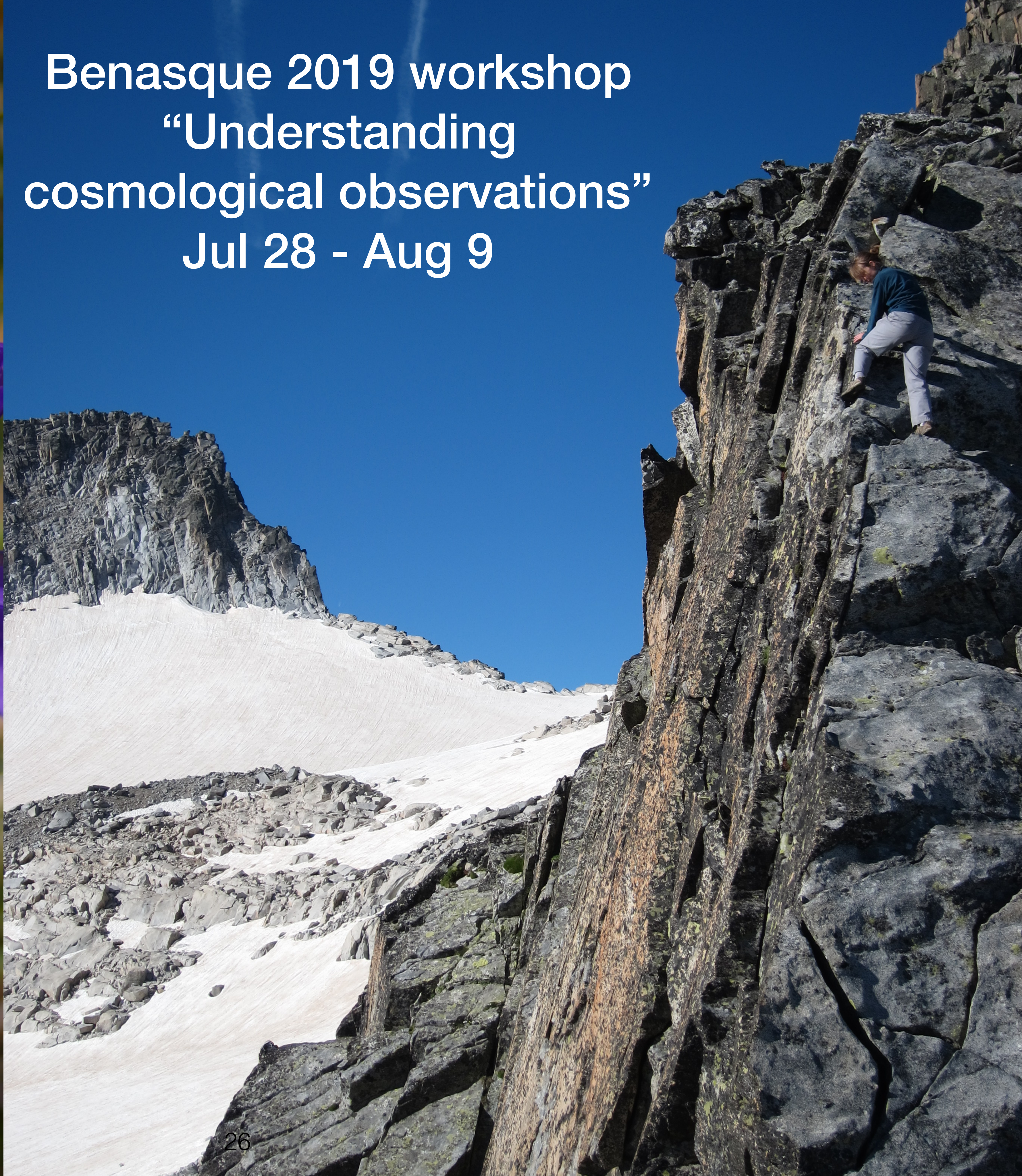


Adamek, CB, Gosenca & Hotchkiss 2019; see also Lacki & Beacom 2010; Eroshenko 2016; Boucenna, Kühnel, Ohlsson & Visinelli 2017

Future constraints



CB, Cole & Patil 2018



Benasque 2019 workshop "Understanding cosmological observations"

Jul 28 - Aug 9

Organizers:
D. Blas
C. Byrnes
M. Crocce
R. Flauger
A. Lewis
S. Nadathur
H. Peiris

Plenary Speakers
- Daniel Baumann
- Kfir Blum
- Camille Bonvin
- Cora Dvorkin
- Stephen Feeney
- Shirley Ho
- Julien Lesgourgues
- Samaya Nissanke
- Blake Sherwin
- Tracy Slatyer
- Anze Slosar
- Sherry Suyu

Summary

- Even if PBHs do not exist, Bernard has not wasted 45 years of his life!
- The power spectrum cannot grow faster than k^4. The commonly assumed delta function power spectrum is unrealistic in important ways (e.g. stochastic GW constraints).
  "Steepest growth of the power spectrum and primordial black holes",
  CB, P. Cole, S. Patil; ArXiv:1811.11158
- Brand new!
  The non-linear delta to zeta relationship weakens the PBH constraints by a factor of 2-3
  Primordial black hole formation and abundance: contribution from the non-linear relation between the density and curvature perturbation; S. Young, I. Musco, CB; ArXiv:1904.00984
- If LIGO detected *any* primordial black hole, then there should exist a larger population of solar mass black holes
  "Primordial black holes with an accurate QCD equation of state";
  CB, M. Hindmarsh, S. Young, M. Hawkins; JCAP (2018); ArXiv:1801.06138
- Mixed dark matter models are required if f<<1. PBHS and WIMPs cannot coexist
  "WIMPs and stellar-mass primordial black holes are incompatible";
  J. Adamek, CB, M. Gosenca, S. Hotchkiss; arXiv:1901.08528

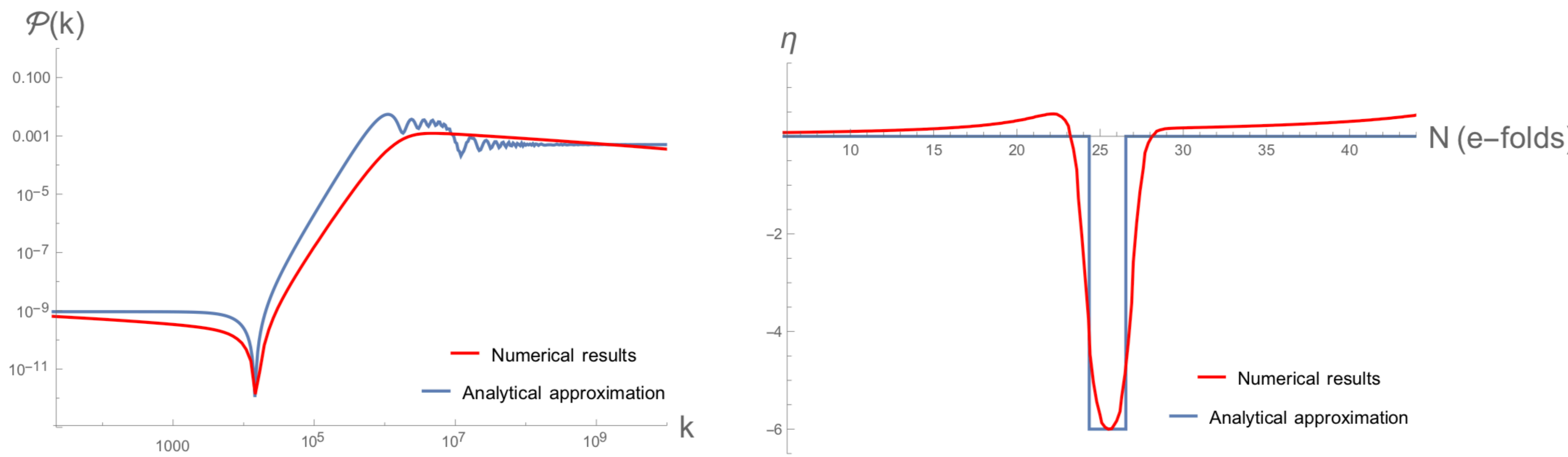
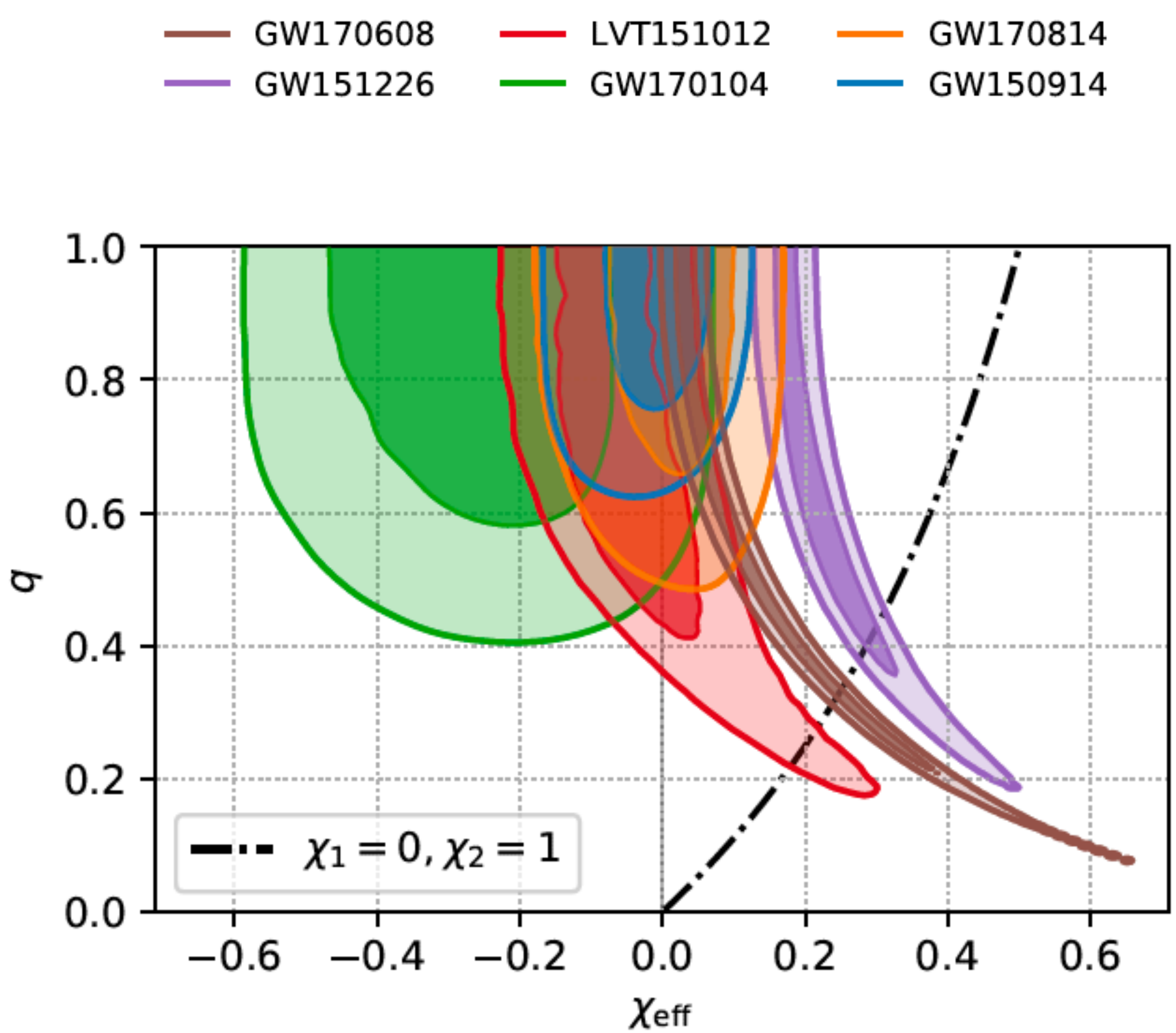
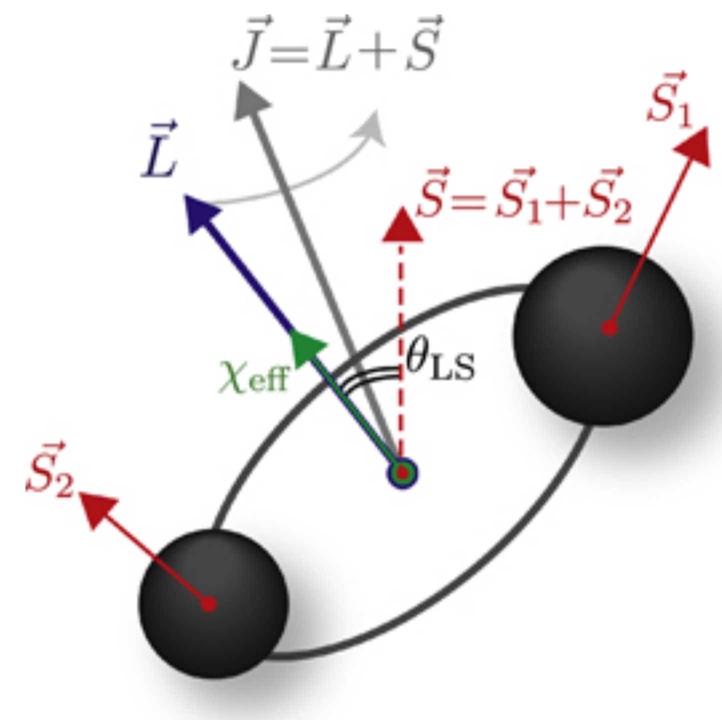


FIG. 4: Left-hand plot: Numerical results for the potential in [31] are plotted in red and our analytical approximation is plotted in blue. The analytical approximation involves 3 constant phases of η from 0 to -6 and back to 0. The right-hand plot shows the piecewise form for η used for the analytical approximation in blue, with 2.2 e-folds of $\eta = -6$. The full numerical evolution of η for the potential in [31] is shown in red. Note that the units in e-folds have been defined arbitrarily, and we have chosen to centre the phase of $\eta = -6$ in our analytical approximation at the time N when the numerical η reaches -6 instantaneously.

CB, Cole & Patil 2018

Black hole spin





$$\chi_{\rm eff} = \frac{c}{G(m_1+m_2)}\left(\frac{\vec{S}_1}{m_1}+\frac{\vec{S}_2}{m_2}\right)\cdot\vec{L}$$

$$a_* = \frac{c\,|\,\vec{S}\,|}{Gm^2} \leq 1$$

All except one event is consistent with zero effective spin, arguably a surprise if the BHs are astrophysical but expected for PBHs: *Chiba & Yokoyama 2017; Mirbabyi et al 2019; De Luca et al 2019; Belczynski et al. 2017*

Primordial Black Hole (PBH) formation

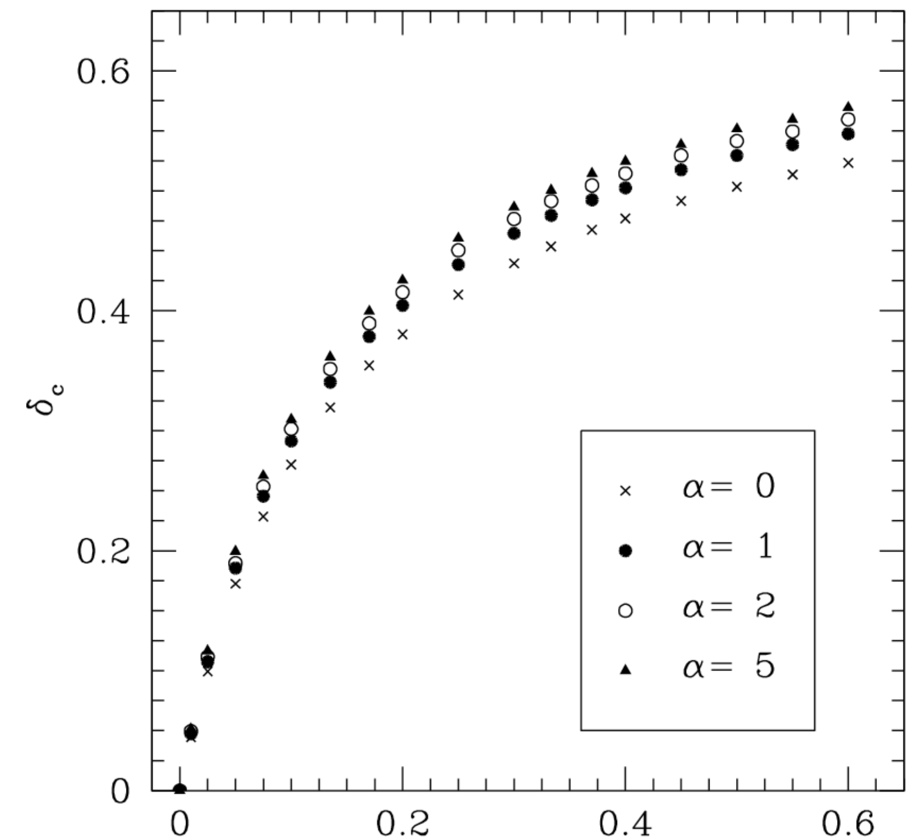
1. They form from large amplitude density perturbations

2. Causality prevents collapse while the perturbations are super-horizon

3. Shortly after horizon entry they collapse and form with a mass comparable to the horizon mass

4. 1-to-1 relation between horizon entry time, horizon length and PBH mass

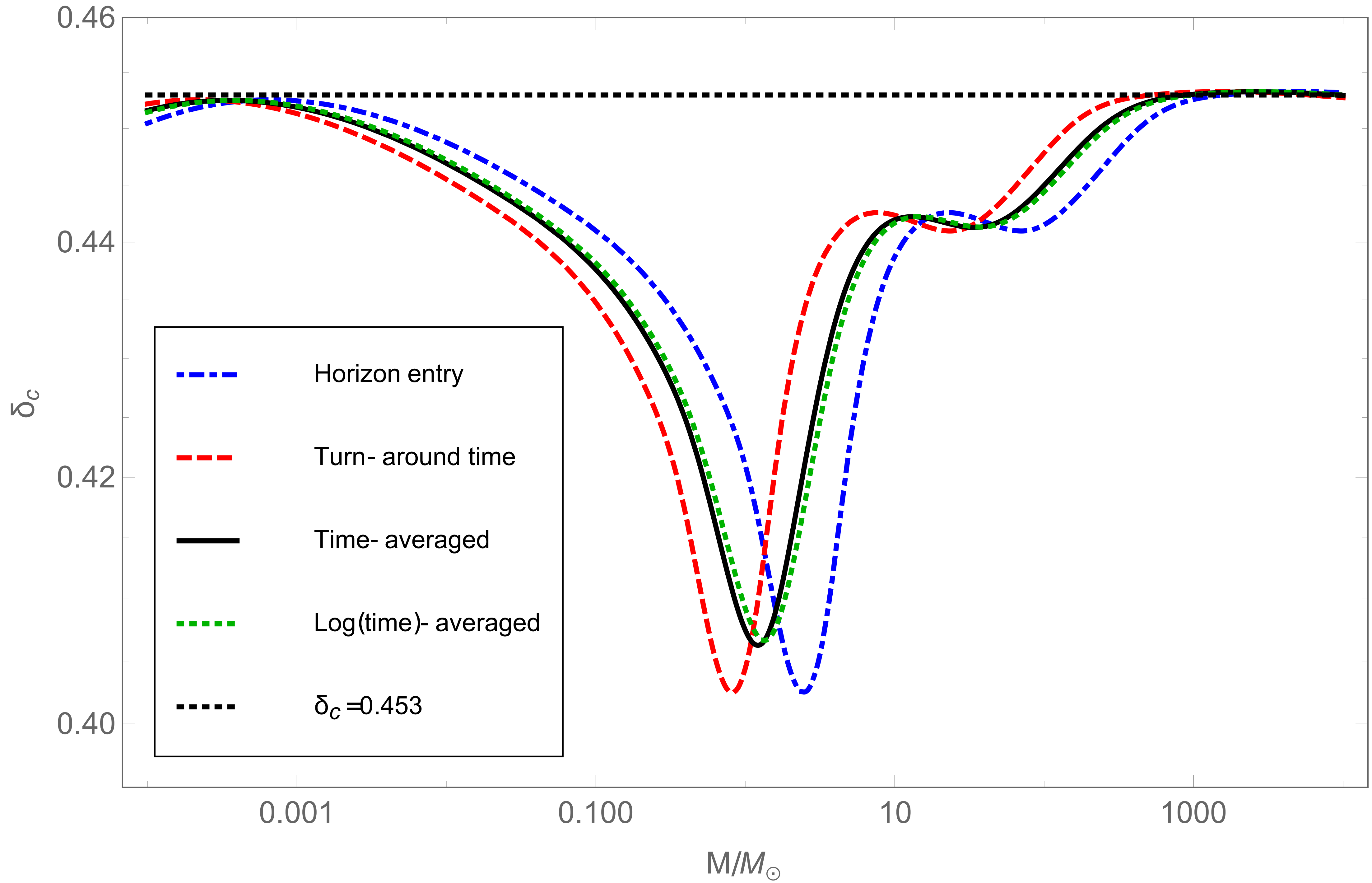
Collapse threshold



equation of state

Musco and Miller 2013

Collapse threshold vs horizon mass

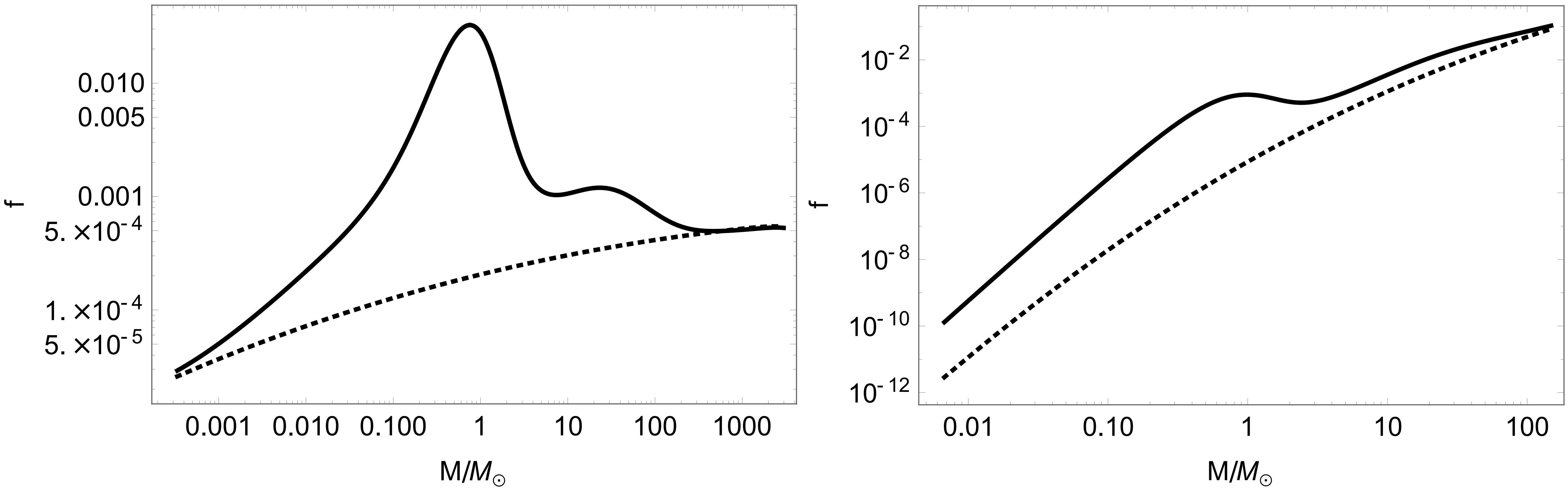


CB, Hindmarsh, Young & Hawkins 2018

Varying the primordial perturbations

If the primordial power spectrum is not scale invariant on the relevant scales then the mass function changes, but a peak remains

$n_s - 1 = -0.05$ $\qquad$ $n_s - 1 = -0.2$



The PBH merger rate

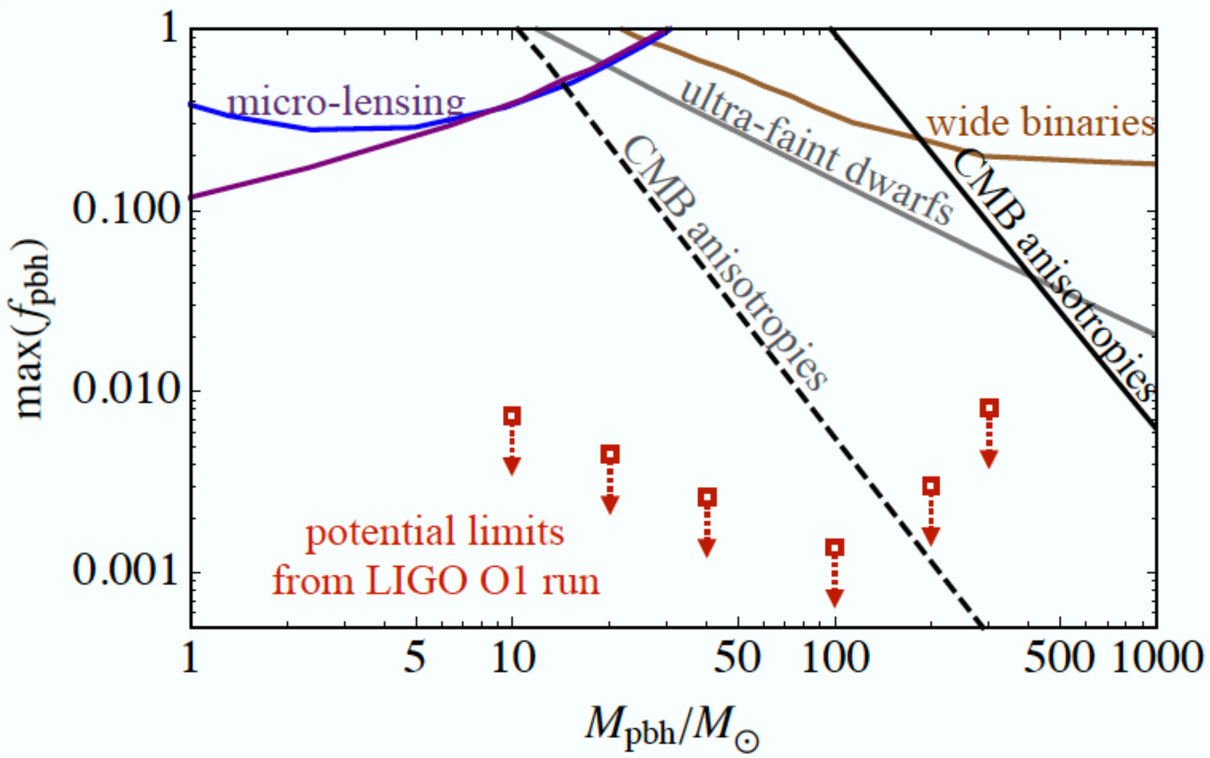


FIG. 7. Potential upper bounds on the fraction of dark matter in PBHs as a function of their mass, derived in this paper (red arrows), and assuming a narrow PBH mass function. These bounds need to be confirmed by numerical simulations. For comparison we also show the microlensing limits from the EROS [21] (*purple*) and MACHO [20] (*blue*) collaborations (see Ref. [74] for caveats and Ref. [32] for a discussion of uncertainties), limits from wide Galactic binaries [22], ultra-faint dwarf galaxies [25], and CMB anisotropies [24].

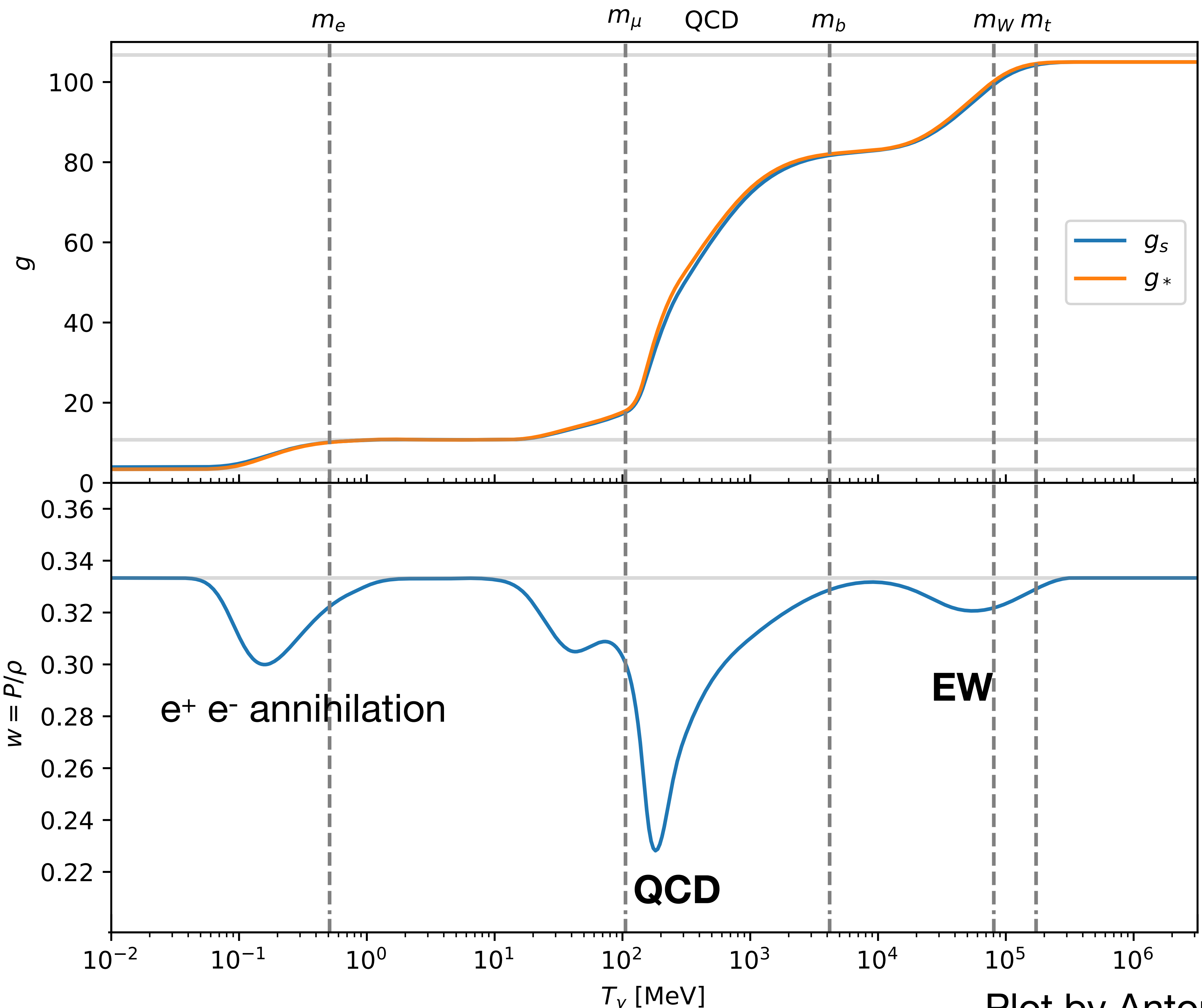
Haimoud et al 2017

- Caveats:

1. Assumes a monochromatic mass spectrum
2. Assumes PBHs are randomly placed initially
3. Assumes BH binaries are not disrupted
4. Neglects accretion around the BHs

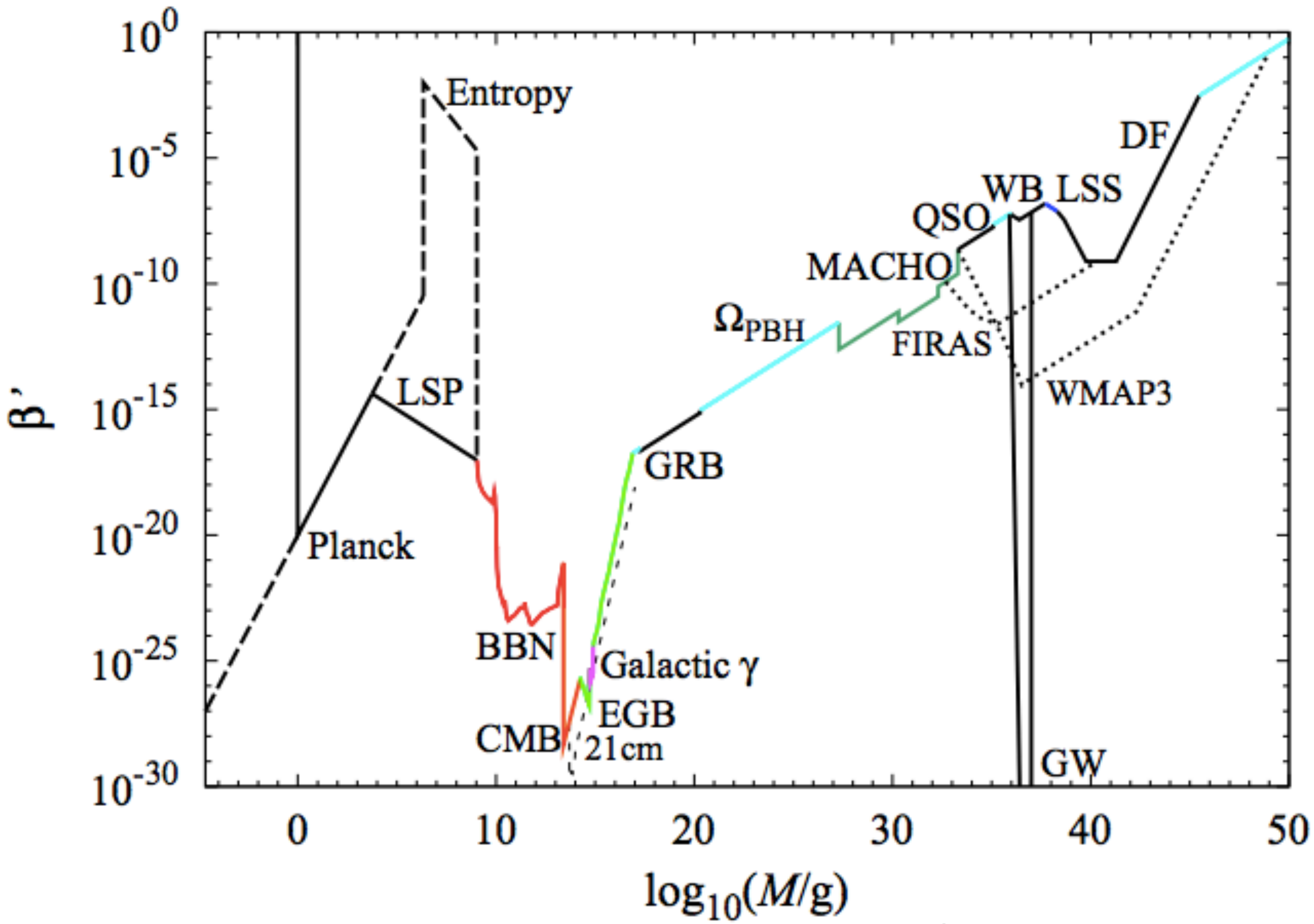
First cosmological evolution simulations
Raidal, Spethmann, Vaskonen & Veermäe `18

Thermal history



Plot by Antony Lewis

PBH constraints at formation



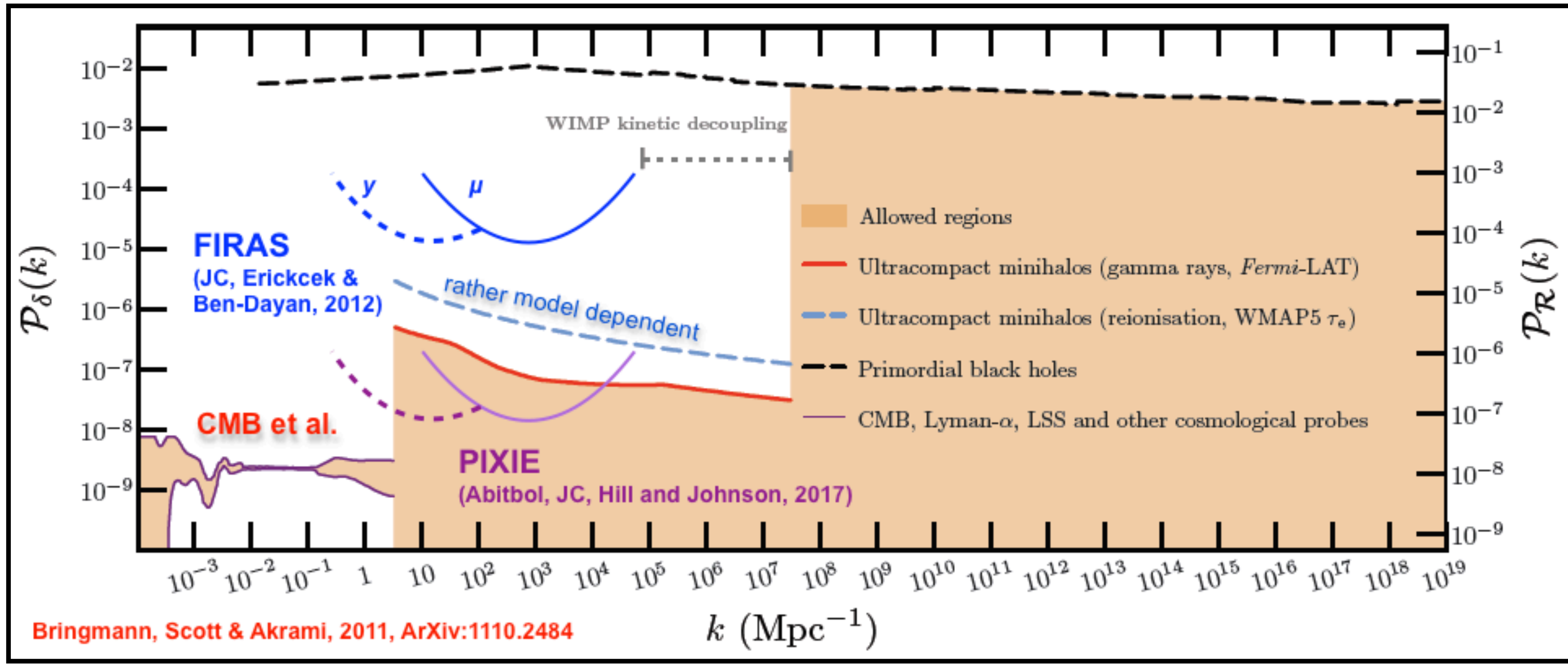
Carr et al 2009

$$\rho_{PBH} \propto a^{-3}$$

$$\rho_{rad} \propto a^{-4}$$

Ω_{PBH} grows like the scale factor from formation until radiation-matter equality

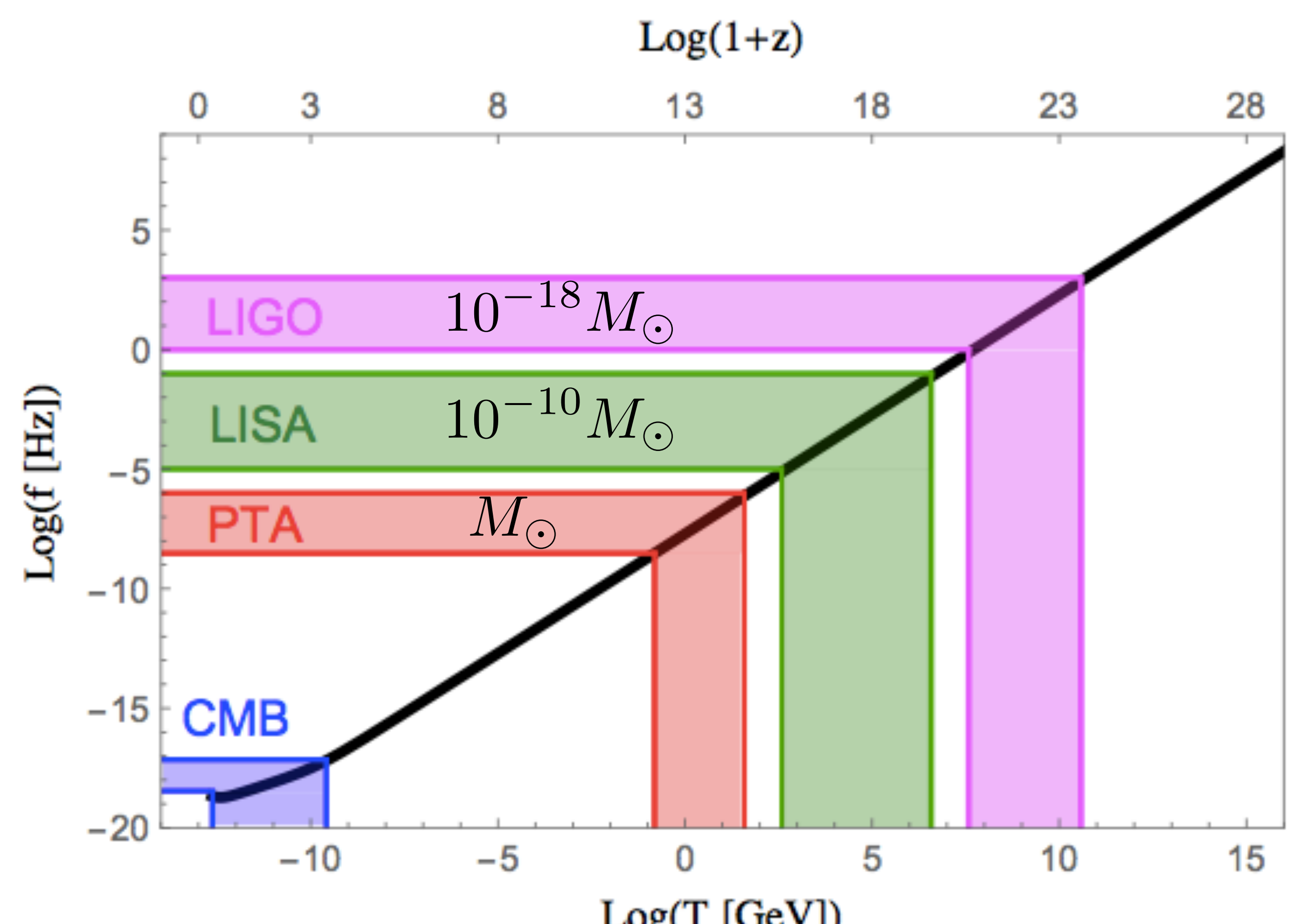
GWs



- At linear order in perturbation theory; scalar, vector and tensor modes decouple
- Second-order tensor perturbations are sourced by scalar perturbations squared

$$f = \frac{c_s k}{2\pi} \qquad \Omega_{GW}(k) \simeq 30\Omega_{\rm rad}\mathcal{P}_\zeta(k)^2$$

$$M_{\rm PBH} \simeq 0.1 \left(\frac{\rm GeV}{T}\right)^2 M_\odot$$



Review: Caprini and Figueroa 2018

(Disputed) observational constraints

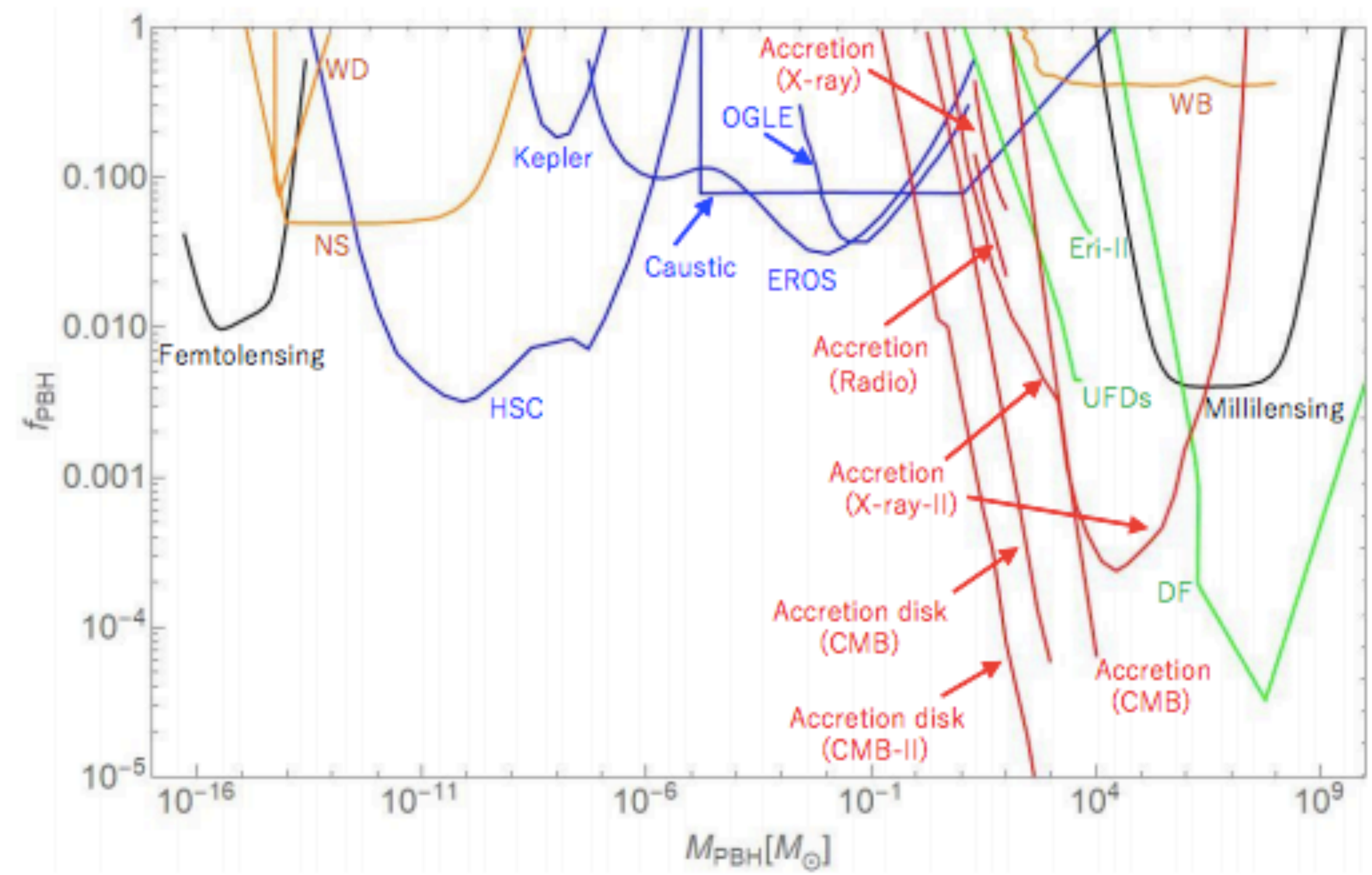


Figure 11: Upper limit on $f_{PBH} = \Omega_{PBH}/\Omega_{DM}$ for various PBH mass (assuming monochromatic mass function). Blue curves represent lensing constraints by EROS [116], OGLE [119], Kepler [122], HSC [123] and Caustic [125] (see 3.1.1). Black curves represent constraints by the millilensing [132] (3.1.2) and the femtolensing [138] (3.1.3). Orange curves represent dynamical constraints obtained by requiring that existent compact objects such as white dwarfs (WDs) [141] (3.2.1) and neutron stars (NSs) [142] (3.2.2) as well as the wide binaries (WBs) [151] (3.2.3) are not disrupted by PBHs. Green curves represent constraints by the dynamical friction (DF) on PBHs [152] (3.2.6), the ultra-faint dwarfs (UFDs) [153], and Eridanus II [153] (3.2.5). Red curves represent constraints by the accretion onto the PBHs such as CMB for the case of the spherical accretion [166] and the case of the accretion disk [171] with two opposite situations where the sound speed of the baryonic matter is greater (labeled by CMB) or smaller (labeld by CMB-II) than the relative baryon-dark matter velocity (3.3.1), radio, and X-rays [173, 180] (3.3.2).

Sasaki et al 2017 Review

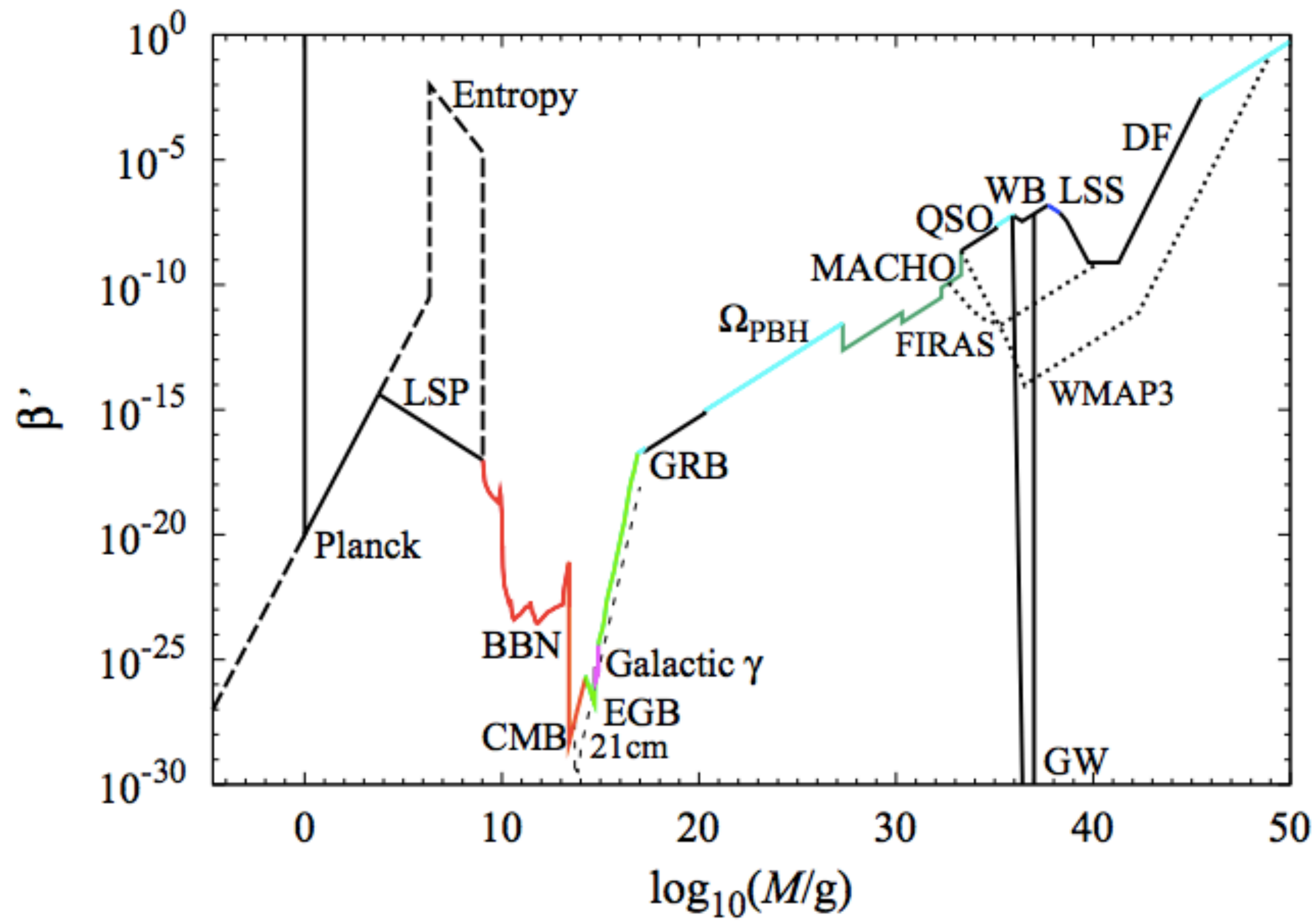
The Gaussian calculation

$$P(\zeta) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{\zeta^2}{2\sigma^2}\right)$$

$$\beta \simeq \int_{\zeta_c}^{\infty} P(\zeta)d\zeta \simeq exp\left(-\frac{\zeta_c^2}{2\sigma^2}\right)$$

$$\frac{\sigma}{\zeta_c} = \sqrt{\frac{1}{2\ln(1/\beta)}} \qquad \zeta_c \simeq 1$$

$$\mathcal{P}_\zeta \lesssim 10^{-2}$$



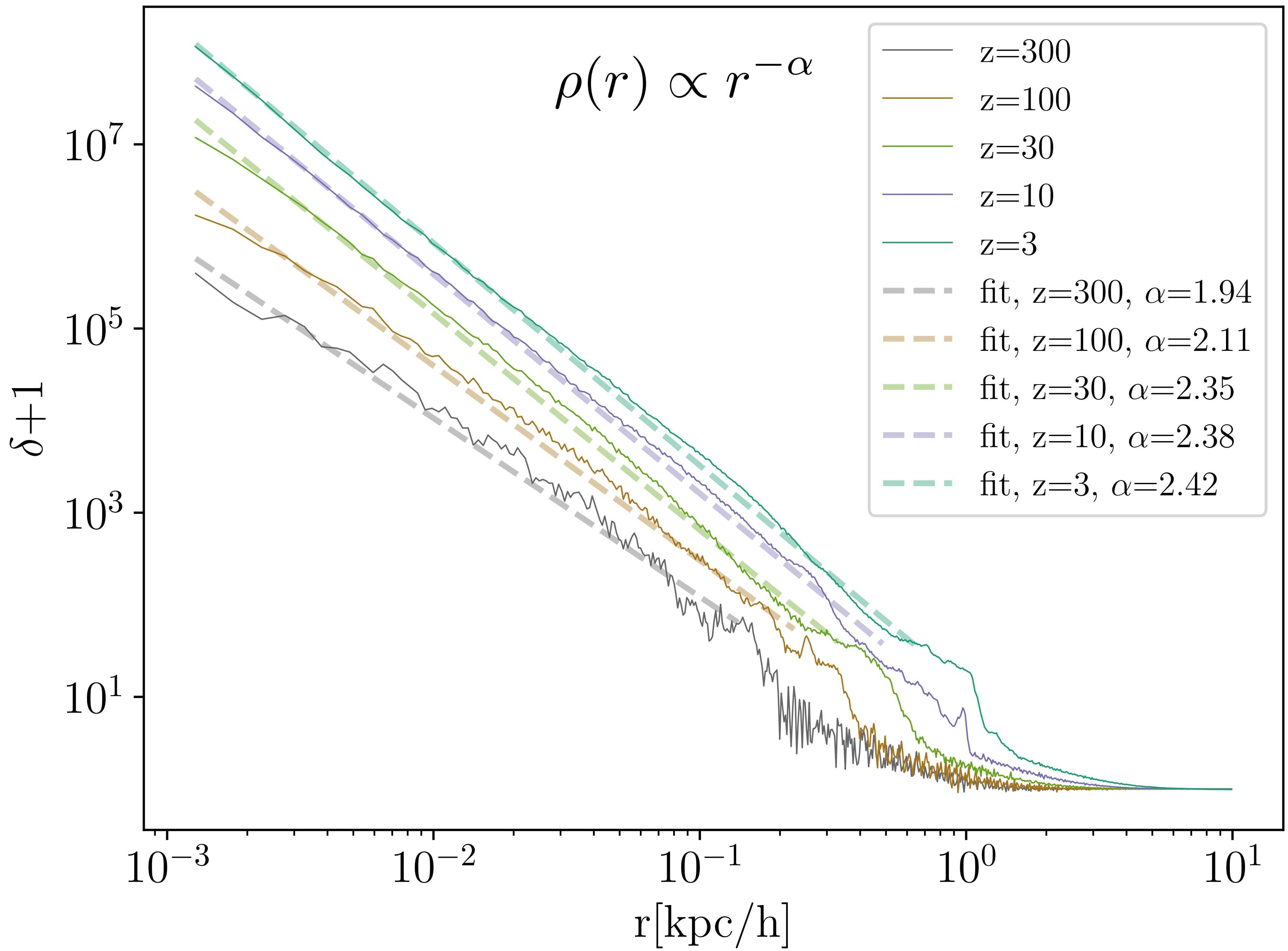
This shows why we probe ~10 sigma fluctuations, and why constraints only mildly depend on beta

The tail is very sensitive to non-Gaussianity (skewness, kurtosis, etc)

Ultracompact minihalos

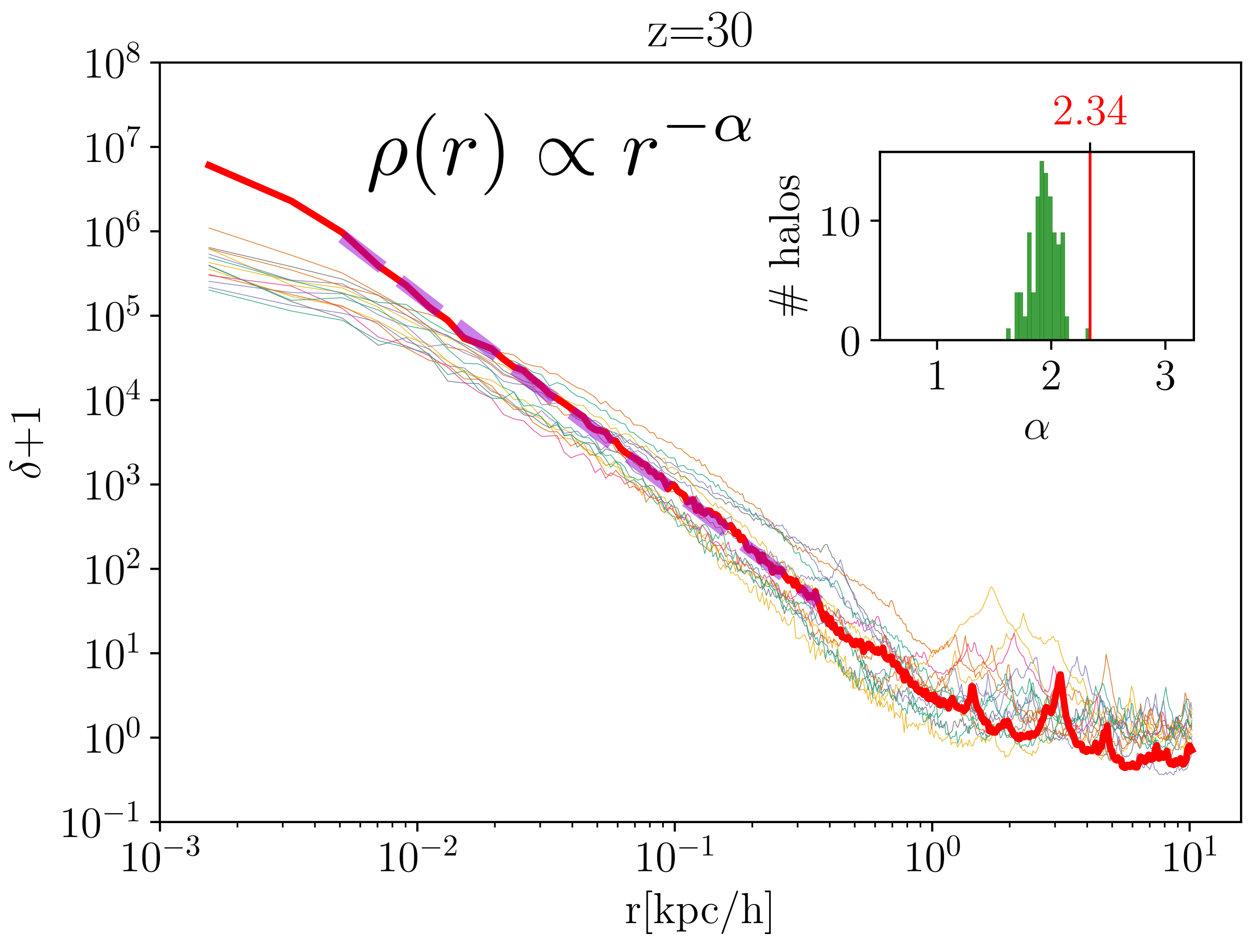
- relic of the early Universe imprinted in dark matter: primordial information is preserved
- form around matter-radiation equality, if $\delta > 10^{-3}$
- large central density *Ricotti and Gould, arXiv:0908.0735*
- steep power-law profile: $\rho(r) \propto r^{-9/4}$ *Bertschinger, 1985*
- $10^{-3} M_{\odot} \lesssim M \lesssim 10^{7} M_{\odot}$ today
- analytical approximations: spherically symmetric, isolated halo with homogeneous background

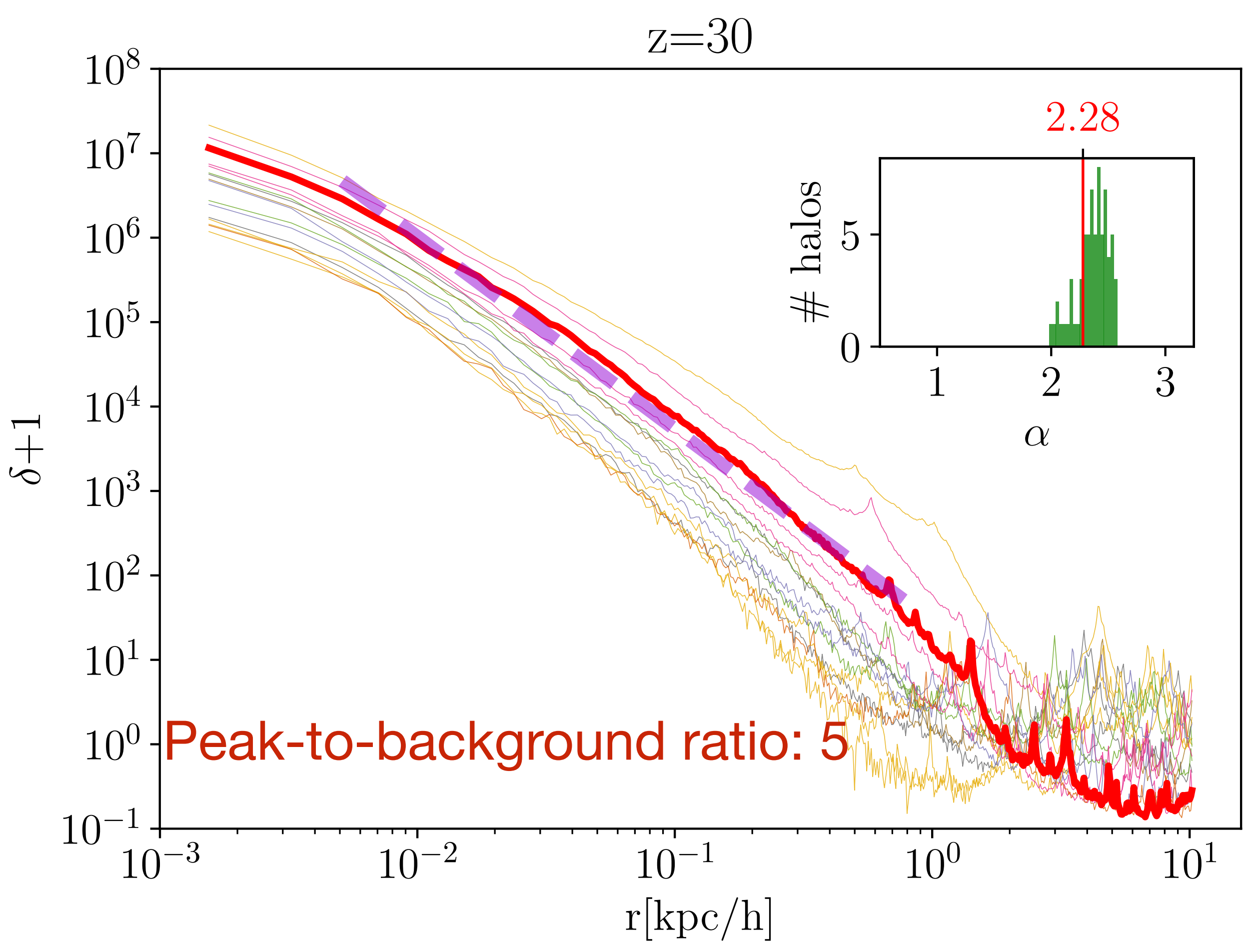
Spherically-symmetric halo in a homogenous background



The first 3D N-body UCMH simulations: Gosenca, Adamek, CB, Hotchkiss, 2017

Peak-to-background ratio: 15 (power-law fit)





- The red line corresponds to the same "proto-halo" as on the previous slide but with 15 or 5 times smaller background perturbations added
- UCMH formation is heavily disrupted
- No collapse threshold (δ_c) exists
- NFW profile results in realistic cases
- WIMP annihilation signal is reduced
- *Lacki & Beacom 2010, Eroshenko 2017, Boucenna et al 2017 all have different constraints. Florian's talk*
- Observational constraints from microlensing and pulsar timing are important

Gosenca, Adamek, CB, Hotchkiss, 2017

Allowed power spectra assuming the "standard" parametrisation

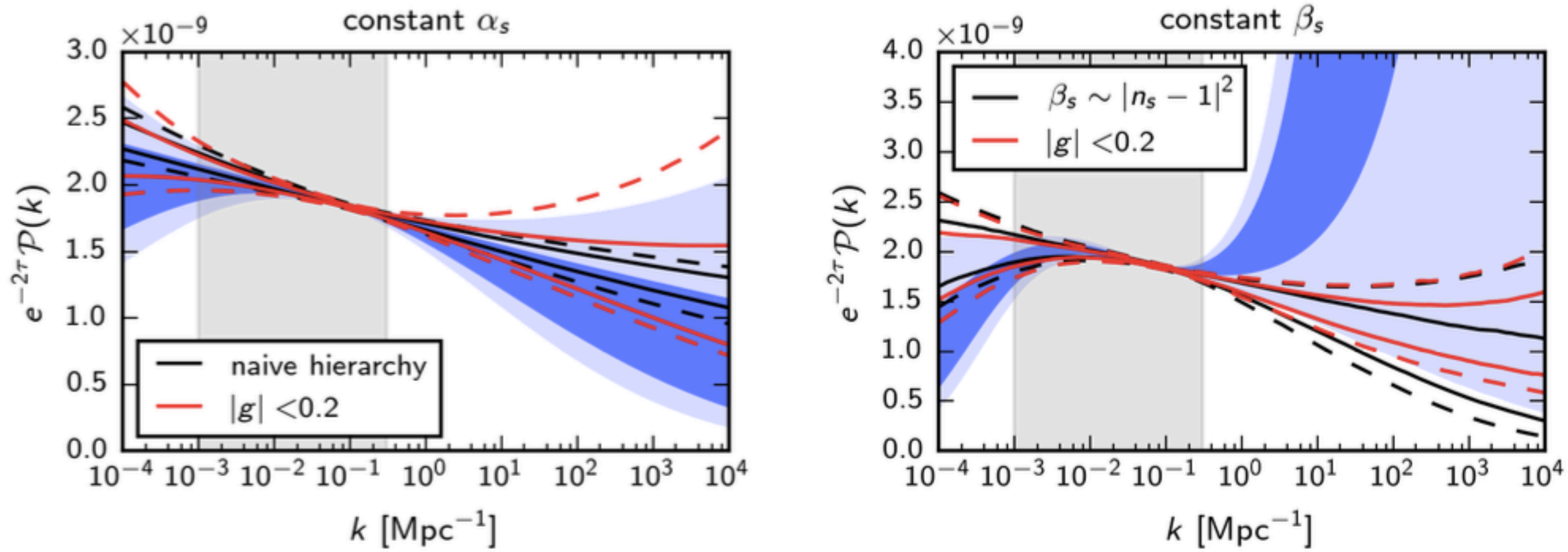


Figure 6. Consequences of the imposition of slow roll (defined by the smallness of g) for the power spectrum scaled by $e^{-2\tau}$, where τ is the optical depth (whose value affects the amplitude of the spectrum, but not its shape). The blue contours represent the 68% (dark blue) and 95% (light blue) limits on the allowed values of the power spectrum (rescaled by a factor of $e^{-2\tau}$) extrapolated from *Planck* 2015 TT+lowTEB constraints (over gray shaded scales) assuming a constant α_s (left) and a constant β_s (right), for different values of k. The solid and dashed red contours represent the 68% and 95% limits on the fraction of these spectra for which $|g| < 0.2$ for the range of scales corresponding to $10^{-3}\text{Mpc}^{-1} < \text{k} < 10^4\text{Mpc}^{-1}$. The solid and dashed black contours represent the 68% and 95% limits on the fraction of these spectra corresponding to the unshaded regions in figure 1 (note that for the plot on the right the limits of this region already violate the naive expectation for the magnitude of β_s).

Vieira, CB, Lewis 2017