Scattering Theory for Quantum Gravity

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The S Operator as a Map Between Current Algebras

- QFT with a Mass Gap
- Currents on the Conformal Boundary
- The Awada Gibbons Shaw Algebra
- Local Physics as a Cutoff
- Holographic Space Time
- Black Holes

QFT With a Mass Gap

- Infinite Set of Asymptotically Conserved Currents $f^* \partial_\mu \phi_{in/out} - \phi^*_{in/out} \partial_\mu f$. Energy $\pm$ for *out*/*in*. .

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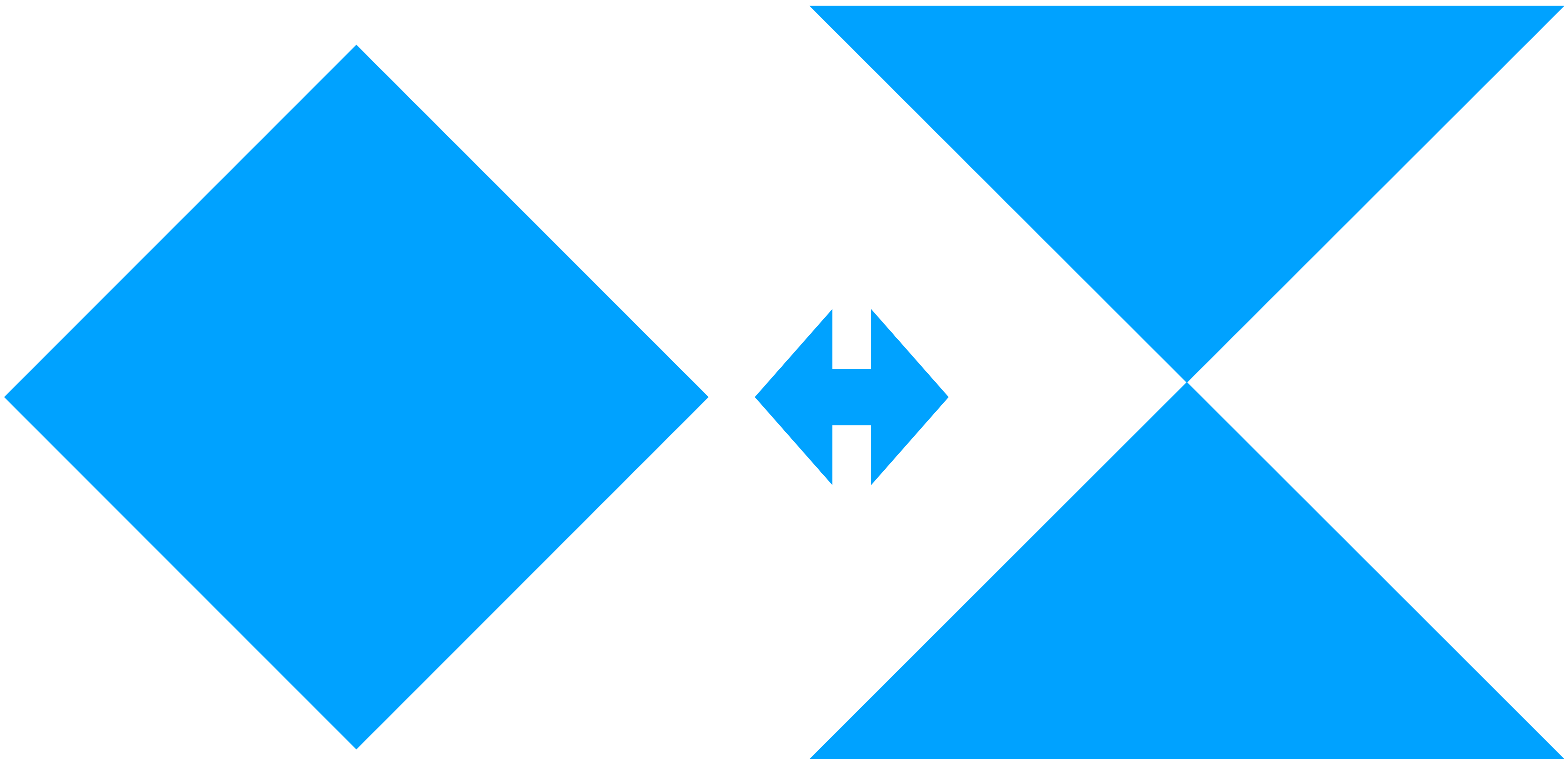
- BMSvdB generators spontaneously broken - low energy thms. but not represented by operators on Hilbert space.

The Conformal Boundary

- Fourier transform w.r.t. the null coordinate turns Penrose diamond into momentum light cone.



Fourier Transform at Null Infinity

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- You can think of this as the spectrum of the BMSvdB generators.
- T.B.: This is the natural space on which asymptotic scattering theory lives.
- Lorentz invariance more transparent. Massive particles just require currents along the null cone as well as transverse to it.
- BMSvdB generators carry no quantum information apart from amount of momentum emitted at each angle.

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- $[\bar{Q}^{\pm\ i}_{\alpha}(P), Q^{\pm\ j}_{\beta}(P')]_+ = Z^{ij} \mathcal{M}_a(P, P')(\gamma^a)_{\alpha\beta} \delta(P \cdot P')$.

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- Probably version of CMHLS theorem that says algebras with higher helicity carrying generators have trivial scattering.

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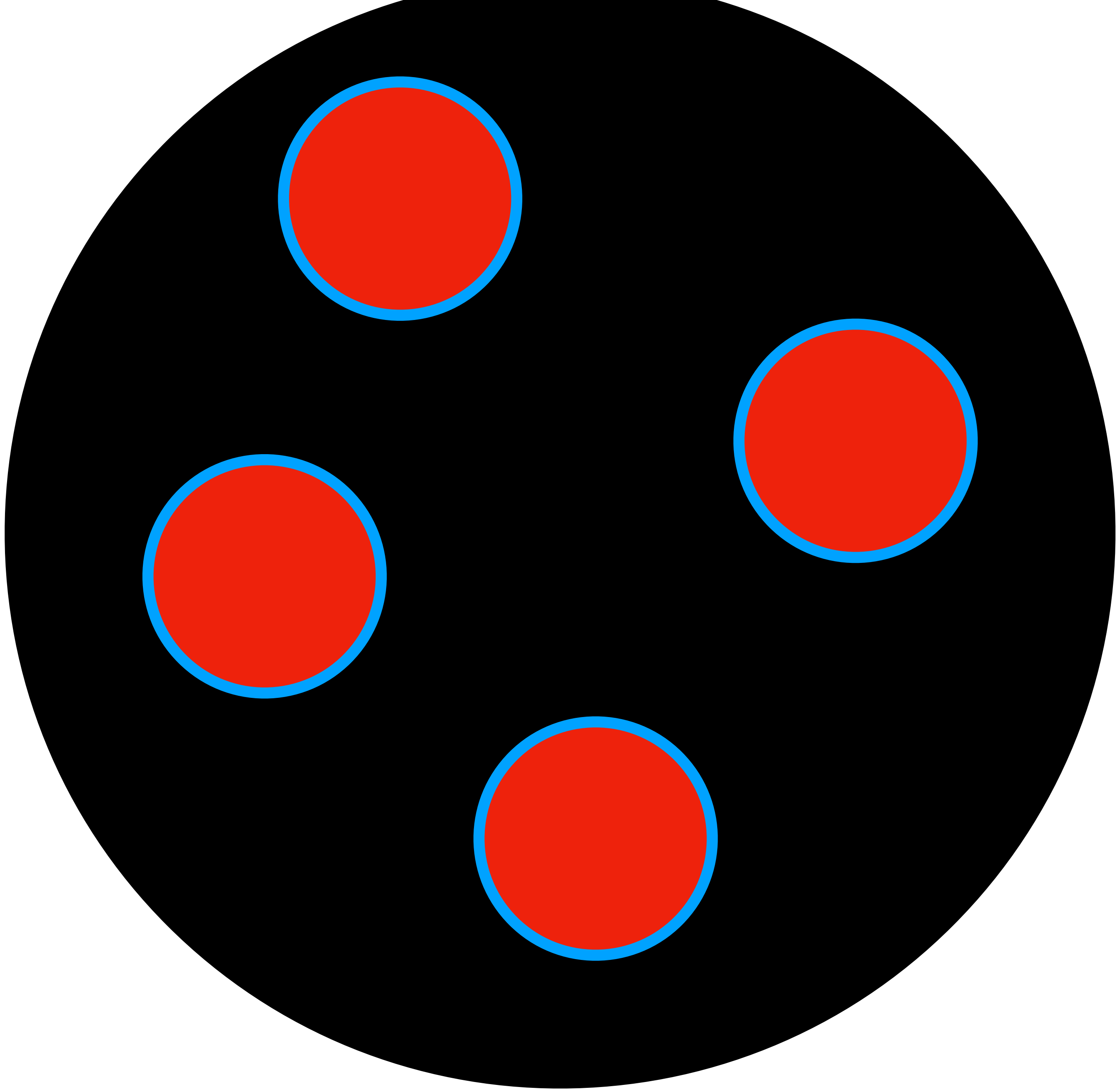
- This defines space of Exclusive Sterman- Weinberg jets.



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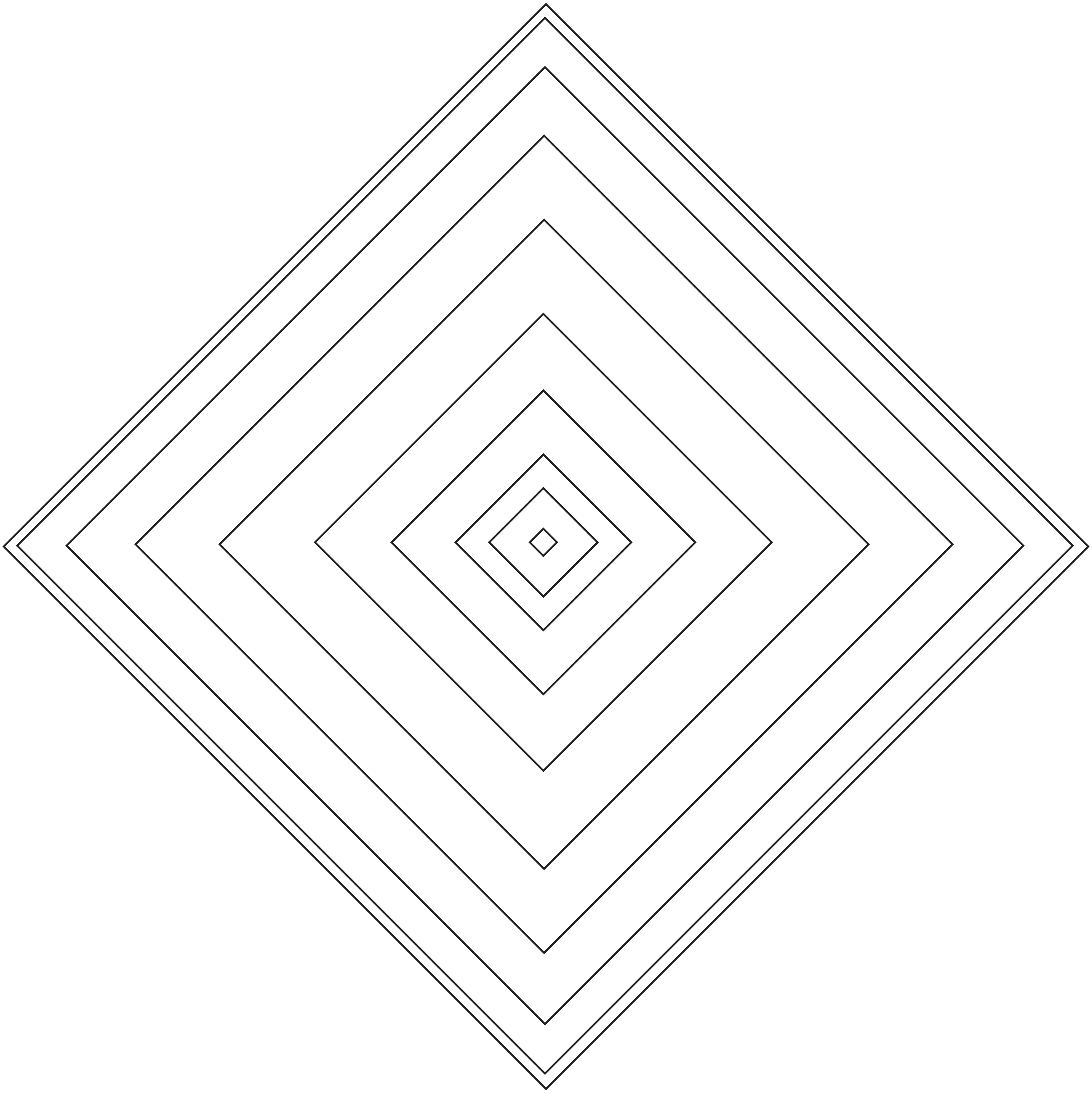
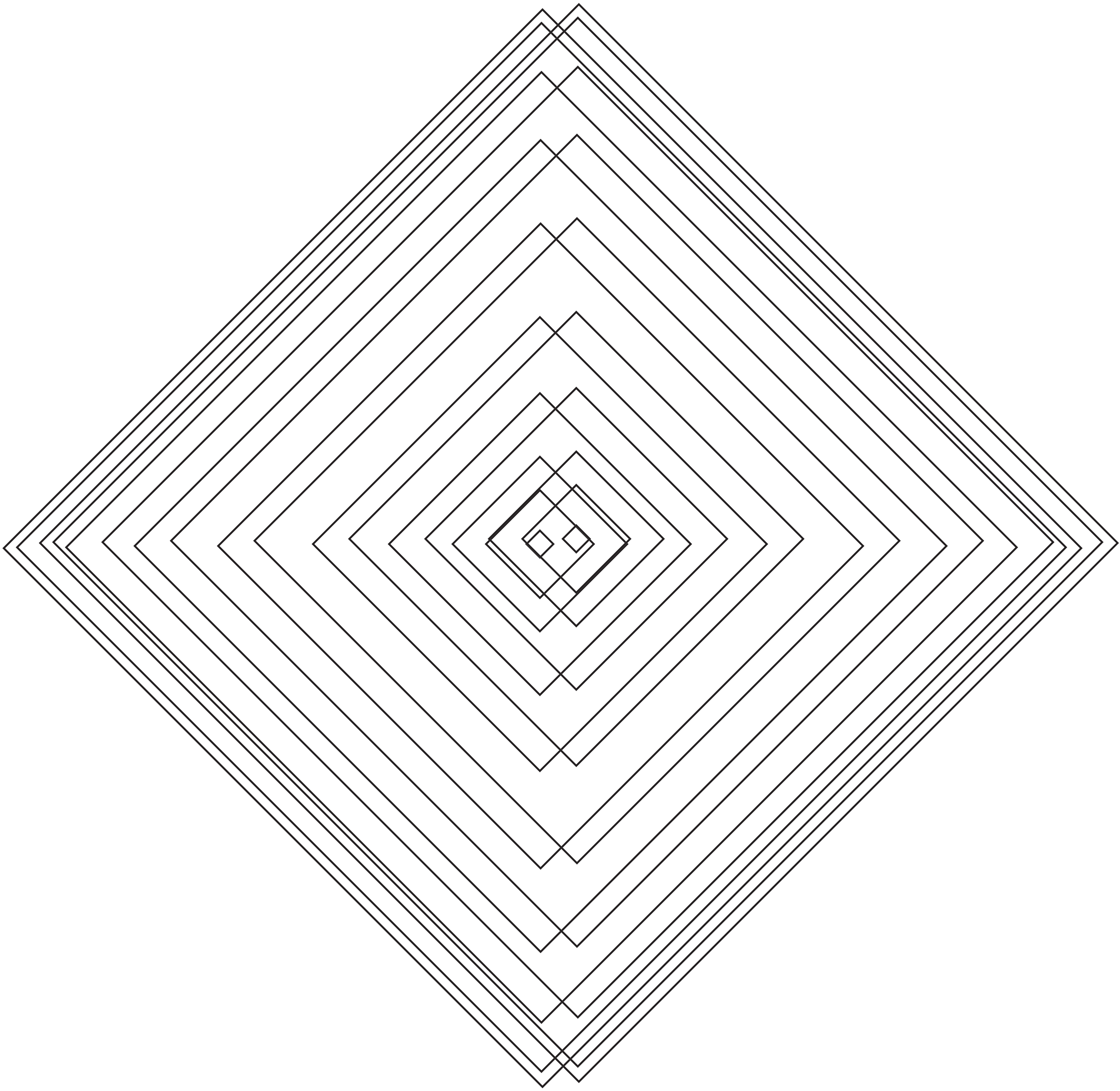
- Relativity is restored by insisting QM systems along different trajectories agree on overlaps of causal diamonds (entanglement spectra of density matrix for overlap subsystem are the same).
- Specification of causal diamonds and overlaps along a sufficiently rich set of trajectories fixes causal structure of space-time (which DOES NOT fluctuate).





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- Time dependence of Hamiltonian required by insisting slices remain within the past (*e.g.* Milne).

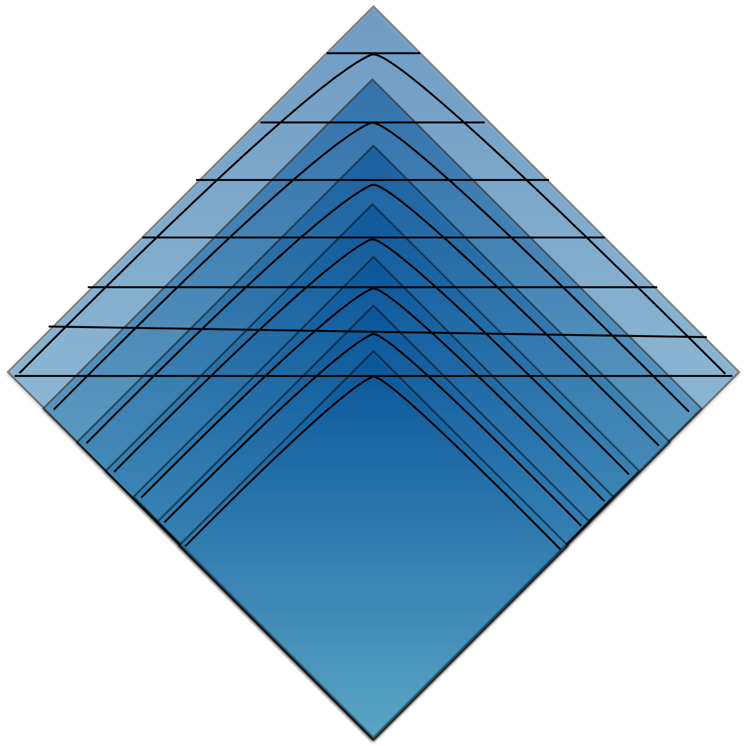

Figure 1: Horizontal slices are FRW, hyperbolic slices are HST.

causal diamond. The HST model has a clear description of both particle and black hole states, and the transitions between them. Section 3. is a review of the EHI cosmology and its approximate $SL(2)$ symmetry. The FRW description of this system is a good description in the limit of large causal diamonds and the real system has no singularity. Section 4. is the core of this paper. It describes the inflationary model, which we believe is relevant to the universe we observe. We derive bounds on the maximum temperature of the universe, which are related to the values of inflationary parameters. This model makes it very explicit that one *must* choose low entropy initial conditions in order to have local excitations in the universe. Further constraints come from insisting that the local excitations are more complex than a few large black holes or the radiation from their decay. We call this excuse for the low entropy initial conditions a *topikès-thropic* explanation, from the Greek word *topikès* for *local*. We show that more refined versions of this argument put an upper bound on the reheat temperature of the universe in the HST model in terms of parameters characterizing the inflationary era. We also argue that in this framework the number of e-folds is essentially given by an upper bound we announced some time ago [9]. In this section we also give a brief review of observational signatures of this model. A more comprehensive paper about the predictions for two and three point functions of fluctuations will appear shortly [**?**].

Section 4. also contains brief comments about baryogenesis in the HST model. Our bound on the reheat temperature allows many conventional low energy mechanisms for baryogenesis, but rules out high scale leptogenesis. We also point out the possibility of producing the baryon asymmetry during the era of black hole decay by applying anthropic arguments

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- Fast Scrambler because invariant under fuzzy approximation to group of volume preserving transformations of the sphere.

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- In some small diamond $[-n, n]$ number of small incoming and out going blocks and their sizes, will change.
- If $\sum E_i \sim n$, the final state will be "generic". This is the formation of large (size n) meta-stable excitation. It will decay into a state with a less energetic excitation plus a jet with probability $e^{-cnE_{jet}}$.

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- Details of this need a lot of work.

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- Consistency conditions for trajectories in relative motion not yet solved.

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- Special $SO(3)$ subgroup picked out by early time evolution, but is forgotten after no localized excitations are left.