BRUSSELS

- Thank the organisers
- Work with Sasha Haco, Stephen Hawking, Andy Strominger.

Can soft hair account for black hole entropy —

why do believe in black hole entropy

(i) 2nd law $dA \geq 0$

(ii) $dM = \kappa dA/8\pi + \Phi dQ + \Omega dJ$ together with $T = \kappa/2\pi$

$\Rightarrow S = A/4$

(iii) The Euclidean calculation of Gibbons + Hawking.

Here the horizon has been turned into an S^2

Effectively tracing over anything inside the horizon.

Will come back to other approaches at the end

Lets look at the covariant phase space

(Peierls, Ashtekar + ..., Witten + ..., Zuckerman, Wald + ...)

Start from the action

$$I = \frac{1}{16\pi}\int_M R + \frac{1}{8\pi}\int_{\partial M} K$$

Variation $\delta I = \int_M (\text{Einstein})^{ab} h_{ab} + \int_M \nabla_a \theta^a$

$g_{ab} \to g_{ab} + h_{ab}$

integration by parts $\rightarrow \int \theta^a d\Sigma_a$

This of this as $\sum_i p_i \delta q_i$

θ^a is the presymplectic potential

Explicitly $\theta^a = \frac{1}{16\pi}\left(\nabla_b h^{ab} - \nabla^a h\right) = \theta^a(g,h)$

Next construct the presymplectic current

$$\omega^a = \delta' \theta^a(g,h) - \delta\theta^a(g,h')$$
$$= \omega^a(g,h,h')$$

[Think $\delta p_i \wedge \delta q_i$]

This is a 2-form on the phase space labelled by tangent vectors h, which will obey the linearised Einstein equations.

The symplectic form is then

$$\Omega(g,h,h') = \int_{\Sigma} \omega_a \, d\Sigma^a \quad \Leftarrow \text{ Think } \Omega \sim \sum_i dp_i \wedge dq_i$$

But if g obeys Einstein and h, h' obey linearised this is independent of Σ and because h' is pure gauge then

$$= \int_{\partial\Sigma} S_{ab} \, dS^{ab}$$

Suppose that $h'_{ab} = \nabla_a \xi'_b + \nabla_b \xi'_a$ = pure gauge

then

$$Q_{\xi'}(g,h) = \int_{\partial\Sigma} S_{ab} \, dS^{ab}$$ is the difference in charge enclosed in Σ, between g and $g+h$ and conjugate to ξ.

This should depend on h and not on how one gets to h, in other words, this 1-form on phase space should be exact. Q might therefore need correction.

Diffeomorphism invariance requires $L_{\xi_1} L_{\xi_2} - L_{\xi_2} L_{\xi_1} = L_{[\xi_1, \xi_2]}$ which is reflected in requirement

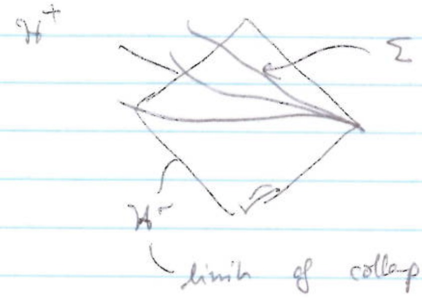
$$Q_\xi Q_{\xi'} - Q_{\xi'} Q_\xi = Q_{[\xi,\xi']} + \underbrace{K_{\xi,\xi'}}_{\text{Co-cycle}}$$

This will be a boundary integral, and unless it vanishes there will be a diffeomorphism anomaly. Note this is classical and comes from the Chevalley – Eilenberg cohomology

Explicitly

$$K = \frac{1}{16\pi} \int dS^{ab} \Big[4 \nabla_c \xi_a \nabla^c \xi'_b - 2 \nabla_c \xi^c \left(\nabla^a \xi'^b \right) + 2 \nabla_c \xi'^c \left(\nabla^a \xi^b \right) - 2 R^{abcd} \xi_c \xi'_d \Big]$$

Now think about an observer in the domain of outer communication



In each case, $\mathcal{H}^+$ is the boundary of what such an observer can face.

So the question is does K vanish.

Lets look at K on $\mathcal{H}$.

Easiest to do on Kruskal co-ordinates, lets do this in Kerr –

$$ds^2 = \rho_+^2\, dU dV + \rho_+^2\, d\theta^2 + \frac{(r_+^2+a^2)^2 \sin^2\theta}{\rho_+^2}\, d\phi^2$$

Horizon at $r_\pm = m \pm \sqrt{m^2-a^2}$

$$+ \left(U dV - V dU\right) \frac{a \sin^2\theta}{2 r_+ \rho_+^2} \left(3 r_+^4 + a^2 r_+^2 + a^2 r_+^2 \cos^2\theta - a^4 \cos^2\theta\right)$$

$$+ \; O(2)$$

$V > 0$
$U < 0$

[Carter, Boyer, Lindquist]

$$T_R = \frac{r_+ - r_-}{4\pi a}$$

$$T_L = \frac{r_+ + r_-}{4\pi a}$$

Choose

$$\xi^U_{(n)} = -\left(2\pi T_R h_n + h_n'\right) U / T_L + T_R$$

$$\xi^V_{(n)} = \left(2\pi T_L h_n - h_n'\right) V / T_L + T_R$$

$$\xi^\phi_{(n)} = 2\pi T_R h_n + h_n' / 2\pi (T_L + T_R)$$

$$\xi^\theta_{(n)} = 0$$

$$h_n = e^{in(\phi + \ln V / 2\pi T_R)}$$

$n = 0$ — linear combination of the Killing vectors

These are horizon super-rotations

This vector field obeys

$$[h_n, h_m] = (n-m)\, h_{n+m} \qquad \text{ie Witt algebra}$$

But

$$K^{(R)}_{n,m} = 2J \frac{T_R}{T_L + T_R} - i n^3 \delta_{n+m,0}$$

Although this was worked out at the bifurcation surface, it is OK all the way up $\mathcal{H}^+$

There is a second vector field with $T_L \leftrightarrow T_R$, $\tilde{\zeta}_{(n)}$ that gives

$$K^{(L)} = 2J \frac{T_L}{T_L + T_R} - i n^3 \delta_{n+m,0}$$

so the charge algebra is anomalous.

Now look at black hole scattering (Press, Teukolsky + ...)

Absorption probability for particles of energy δE, δJ and angular momentum

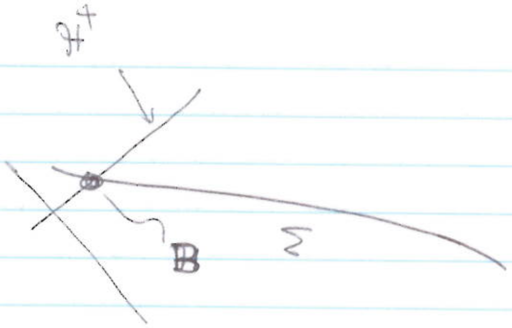
$$\sim \left| \Gamma\left(1 + \frac{i\omega_L}{2\pi T_L}\right)\right|^2 \left| \Gamma\left(1 + \frac{i\omega_R}{2\pi T_L}\right)\right|^2$$

where
$$\omega_L = 2M^3/J \;\; \delta E$$
$$\omega_R = 2M^3/J \;\; \delta E - \delta J$$

These are the energies conjugate to $\tilde{\zeta}_{(0)}$ and $\zeta_{(0)}$ respectively

This is a relation that appears elsewhere in physics

This is the absorption probability for a pair of 2-d conformal field theories with energy ω_L at temperature T_L and ω_R, T_R



If observers on Σ regard $\mathcal{H}$ as a boundary then they will have to cancel the anomaly. They will ascribe to B a CFT that cancels the anomaly and describes the properties of the black hole

~~Black~~ Black hole horizons then need to have such a holographic picture to account for observations of the black hole.

CFT's are ~~..~~ described by their central charge amongst other things. Explicitly

$$K^{(L)} + K^{(R)} \Rightarrow 2J = \frac{c_L + c_R}{12} \Rightarrow c_L + c_R = 24J$$

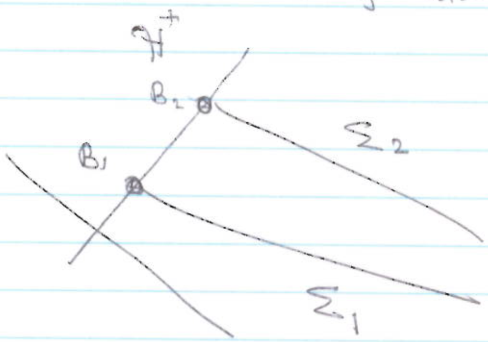
but $c_L = 12J \frac{T_L}{T_L+T_R}$ $c_R = 12J \frac{T_R}{T_L+T_R}$

Note that this cannot be continuously varied as required by the Zamolodikov thm J = integer (or odd half-integer)

The problem here is that $c_{L,R}$ do not obey that property so this picture is incomplete. Generally believed that $c_L = c_R$ else T not conserved.

$\nabla_i T^{ij} \sim c_L - c_R$

Not certain of that follows



There could be evolution between B_1 and B_2 and so there should be something on the horizon. In Kerr

$$\frac{1}{2}\epsilon_{abcd} R^{ab}{}_{ef} R^{cdef} \neq 0$$

$\Rightarrow$ On $\mathcal{H}$ one could have a Chern-Simons gravity term

Γ is in the algebra of $SO_{3,1}$

connection on the boundary

$$I_{CS} = \beta \int \mathrm{Tr}\left(\Gamma d\Gamma + \tfrac{2}{3}\Gamma^3\right) \qquad \text{(Kraus + Larsen)}$$

on the horizon. β gives rise to a contribution to the central charges on the boundary of

$$c_L - c_R = 96\pi\beta$$

Thus the choice $\beta = \dfrac{J}{4}\,\dfrac{T_L - T_R}{T_L + T_R}$ $\Leftarrow$ This comes from requiring the energy-momentum tensor ~~[illegible]~~ non-conservation to cancel.

will give $c_L = c_R = 12J$.

This allow the possibility for evolution as stuff passing through $\mathcal{H}$ will couple to the CS and change s.e. things.

Cardy: The entropy of a CFT at temperature T is

$$S = \frac{\pi^2}{3} c T$$

so in our case $S_{\text{Horizon CFTs}} = \dfrac{\pi^2}{3}\left(c_L T_L + c_R T_R\right)$

$$= \tfrac{1}{4}A$$

by plugging in our previous formulae

Relation to other works :

1. Strominger + Vafa : ......

The took configurations with same quantum numbers as extreme (or near extreme) and found the CFT of the corresponding string theory. Agreement here.

2. Wald

Entropy is a Noether charge and so one found the area of A_+, Conjugate $\partial_t + \Omega_+ \partial_\phi$

Extended by Castro + Rodrigues to speculate there was entropy associated with the inner horizon $\partial_t + \Omega_- \partial_\phi$

$S_\pm = CFT_L \pm CFT_R$

but this is in complete agreement with our computation.

3. Ashtekar

Branched polymer description of the horizon but some of these are known to be essentially described by CFTs (Cardy, Fisher, Parisi, Brydges)