Gravitational wave memory and gauge invariance

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Solvay workshop, Brussels

May 18, 2018

Talk outline

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- Perturbative and gauge invariant treatment of gravitational wave memory
- Electromagnetic analog of gravitational wave memory
- Gauge invariant variables in cosmology

- Memory and infrared divergences
- Conclusions

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- L. Bieri and D. Garfinkle, Phys. Rev. D **89**, 084039, (2014)
- L. Bieri, D. Garfinkle and S.T. Yau, Phys. Rev. D **94**, 064040 (2016)
- L. Bieri, D. Garfinkle and N. Yunes, Class. Quantum Grav. **34**, 215002 (2017).

Gravitational wave memory

- After the gravitational wave has passed the distance between the arms of the interferometer is different than before.
- Weak field slow motion: effect due to a change in the second time derivative of the quadrupole moment.
- Full nonlinear GR treatment: there is an additional effect due to the energy flux of gravitational waves (Christodoulou, PRL, **67**, 1486 (1991)) (based on 500 page Christodoulou and Klainerman proof).

Perturbative treatment of memory

- Intuitive and metric based approaches (Thorne, PRD **45**, 520 (1992), Wiseman and Will, PRD **44**, R2945 (1991))
- But these approaches have problems when the stress-energy can travel at the speed of light

Perturbation theory and gauge invariance

$F = F_0 + \delta F$

Under coordinate transformation along vector field k

δF changes to $\delta F + \mathcal{L}_k (F_0)$

So δF does not have invariant meaning, unless F_0 vanishes

This is analogous to gauge invariance in classical electrodynamics

- For a Minkowski background, the metric perturbation is not gauge invariant, but the Weyl tensor perturbation is.
- The Weyl tensor perturbation is also gauge invariant in deSitter and FLRW

Gauge invariant approach

- Do perturbation theory that is first order in the gravitational field with an electromagnetic field or other source of null stress-energy as the matter
- Use the electric and magnetic parts of the Weyl tensor as the basic variables.
- Expand all fields in powers of 1/r near null infinity
- Memory is second time integral of 1/r piece of electric part of the Weyl tensor.

Results

- There are two types of gravitational wave memory: one due to angular distribution of energy radiated to null infinity, the other due to a change in the E_{rr} component of the Weyl tensor.
- The effect is primarily due to the $\ell=2$ piece.

Electromagnetic analog of Gravitational wave memory

- Since the equations of linearized gravity are analogous to Maxwell’s equations, there should be electromagnetic analogs to our gravitational wave memory results.
- A test charge receives a kick proportional to integral of the electric field

- Allow charge to be radiated to null infinity (massless charged fields)
- Expand Maxwell’s equations in powers of 1/r near null infinity
- Find the integral of the 1/r piece of the electric field.

Results

- There are two types of memory: one due to the angular distribution of charge radiated to null infinity, the other due to the angular behavior of the change in the E_r component of the electric field.

Gauge invariant variables in cosmology

- The Weyl tensor vanishes in FLRW
- Memory in deSitter derived in similar way to Minkowski spacetime
- In FLRW the Weyl tensor is coupled to shear of the fluid, but decouples in the short wavelength limit.

Memory and infrared divergences

- Nonzero integral of E means change in A
- The Fourier transform of A is infrared divergent.
- Nonzero double integral of Weyl tensor means change in metric perturbation
- The Fourier transform of the metric is infrared divergent.

- After the electromagnetic wave has passed, E=0 and so A is pure gauge.
- So the change in A is given by a gauge transformation.
- After the wave has passed the Weyl tensor is zero but the metric perturbation is not (spacetime has changed from flat spacetime to a different flat spacetime)

- This change is described by a diffeomorphism compatible with asymptotic flatness: a BMS transformation.

Conclusions

- Perturbative and gauge invariant approach to gravitational memory indicates two types of memory: one due to stress-energy that gets to null infinity and one due to stress-energy that does not.
- There is an electromagnetic analog of gravitational wave memory

- Things look simpler in gauge invariant variables.
- Infrared divergences come about only from imposing a particular gauge and using momentum space.
- Gauge and BMS transformations come from describing things in terms of gauge dependent quantities.