

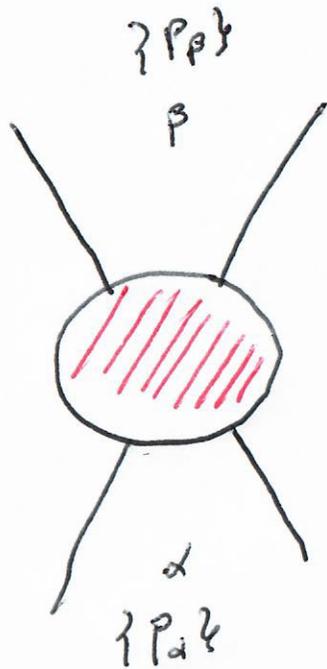
The scales of IR-physics

(A small journey through Van Neumann Spaces)

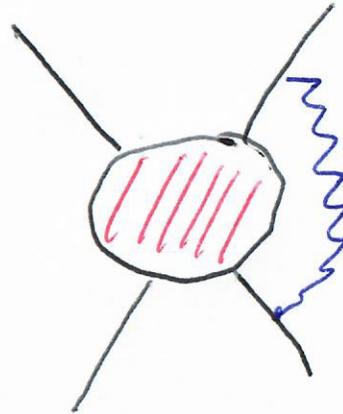
Work with: R. Letschka
S. Zell.

①

IR-Divergences



$$= \int^0_{dP}$$



After resummation

$$\approx \int^0_{dP} \left(\frac{\lambda}{\Lambda} \right)^{B_{dP}/2} \rightarrow 0$$

lim $\lambda \rightarrow 0$
If $B_{dP} \neq 0$

B_{dP} : Weinberg's IR-factor

QED :

$$\sum \rho_n \rho_m e^2 \beta_{nm}^{-1} \ln \left(\frac{1 + \beta_{nm}}{1 - \beta_{nm}} \right)$$

$$\beta_{nm} = \left[1 - \frac{m^4}{(p_n \cdot p_m)^2} \right]^{1/2}$$

Gravity :

$$\frac{G}{2\pi} \sum \rho_n \rho_m m_n m_m \frac{1 + \beta_{nm}^2}{\beta_{nm} (1 - \beta_{nm}^2)^{1/2}} \ln \left(\frac{1 + \beta_{nm}}{1 - \beta_{nm}} \right)$$

Some Preliminary Comments ⁽²⁾

i) The Key IR difference between QED and Gravity:

QED $\lim_{m_e \rightarrow 0} B_{\alpha\beta} \rightarrow \ln(m_e)$ collinear Divergences
(dipolar rad.)

Gravity good $\lim_{m \rightarrow 0}$ (quadrupolar)

ii) For some Kinematics (forward scattering)

$$B_{\alpha\beta} = 0$$

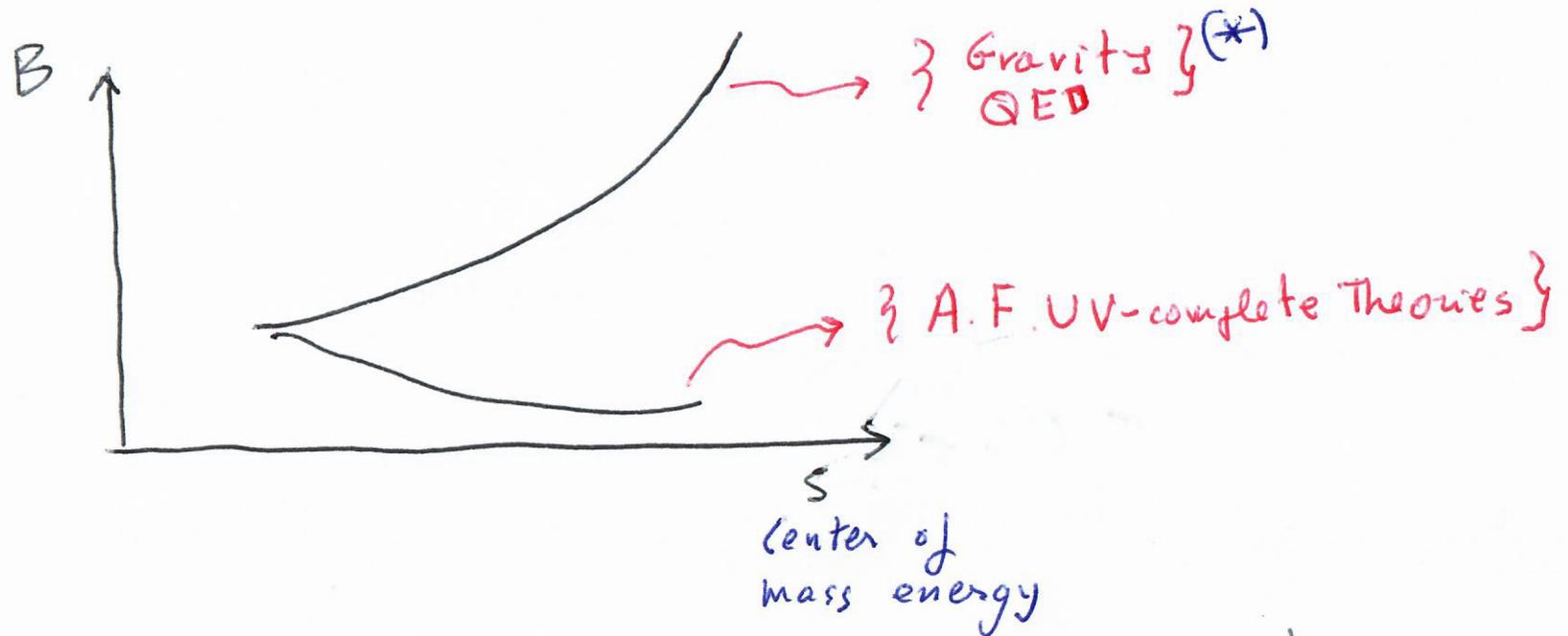
\Rightarrow For this Kinematics we get IR-finite amplitudes.

③

Controversial claim (to be discussed later)

New QED (Gravity) symmetries $\longleftrightarrow B_{\alpha\beta} = 0$

iii) IR-classification of Q.F. Theories.



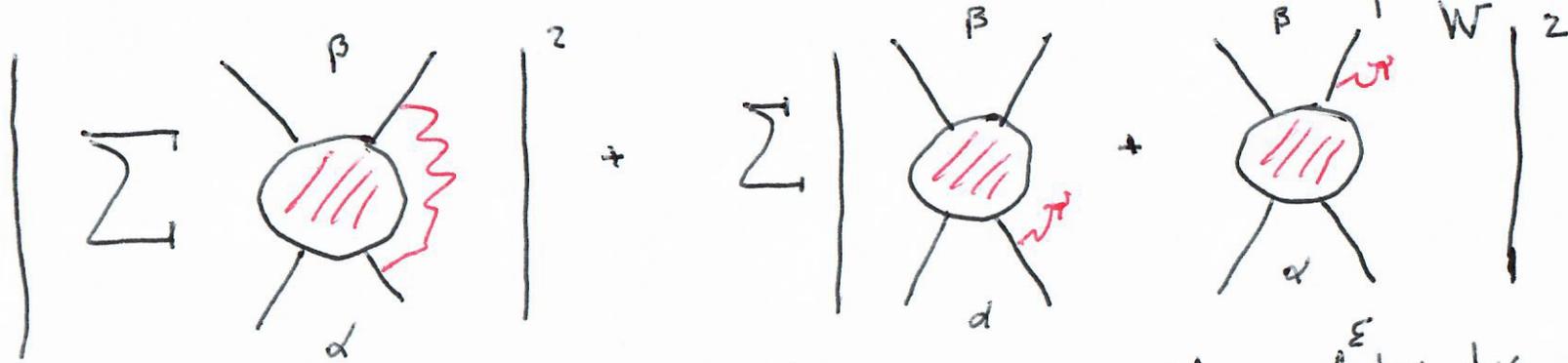
(*) Gravity: ultra planckian scattering with small impact parameter

$$B \sim G_N s$$

④

Standard Approach to IR-divergences

B.N
KLN-Theorem
Y.F.S
W²



(resummation)

$$\sum_n \frac{1}{n!} \int \frac{d^4k_1 \dots d^4k_n}{k_1 \dots k_n} (\tilde{B}_{\alpha\beta})^n$$

(inclusive)

↓

$$\left(\frac{\lambda}{\lambda}\right)^{B_{\alpha\beta}}$$

↖

$$\left(\frac{\epsilon}{\lambda}\right)^{\tilde{B}_{\alpha\beta}}$$

Weinberg's soft theorem

↔ $B = \tilde{B}$

$$\sigma_{\text{inclusive}} \sim \sigma_0 \left(\frac{\epsilon}{\lambda}\right)^{B_{\alpha\beta}}$$

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Comment:

In A.F theories $\beta(s) \rightarrow 0$
 $s \rightarrow \infty$

IR-effects at h. energy are small.

Weinberg Sterman $e^+ e^- \rightarrow 2 \text{ jets}$

In case $\beta(s) \rightarrow \infty$
 $s \rightarrow \infty$

IR-effects act as a suppression factor $\left(\frac{\epsilon}{\Lambda}\right)^B$ $\epsilon \ll \Lambda$
at high energies

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Coherent States: The root of IR-problem

$$\int |\alpha\rangle = \sum_{\beta} \int_{\alpha\beta} |e_{\beta}\rangle \otimes \sum_n \frac{1}{n!} \int_{\lambda}^E d\kappa_1 \dots d\kappa_n S_{\alpha\beta}(\kappa_1) \dots S_{\alpha\beta}(\kappa_n) a^{\dagger}(\kappa_1) \dots a^{\dagger}(\kappa_n) |0\rangle =$$

$|e_{\beta}\rangle \otimes |0\rangle$
photon vacuum

$$= \sum_{\beta} S_{\alpha\beta}^0 e^{\int_{\lambda}^E S_{\alpha\beta}(\kappa) a^{\dagger}(\kappa) d\kappa} |0\rangle \otimes |e_{\beta}\rangle$$

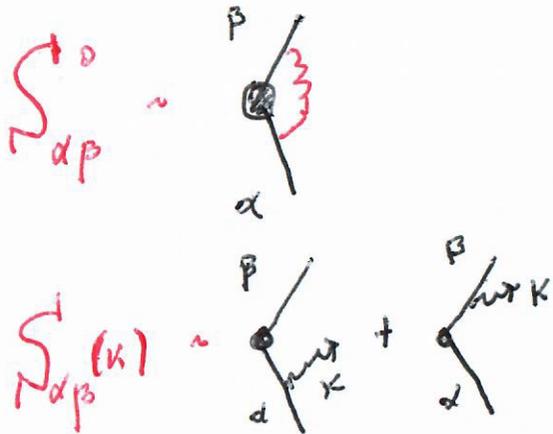
coh. state

$$= \int_{\beta} S_{\alpha\beta}^0 e^{\frac{1}{2} \int^E |S_{\alpha\beta}|^2} |D(\alpha, \beta)\rangle \otimes |e_{\beta}\rangle$$

$$= \int_{\beta} F_{\alpha\beta} |e_{\beta}\rangle \otimes |D(\alpha, \beta)\rangle$$

IR-finite

normalized coherent state.



⑦
Problem: The coherent state $|D(\alpha, \beta)\rangle$ is ill defined in the $\lambda \rightarrow 0$ limit.

It contains ∞ # of $k=0$ photons
i.e. orthogonal to all states in "standard" photon
Fock space.

In general long range forces (QED or Gravity) \Rightarrow
Coherent states with finite energy

BUT
 ∞ # of modes.

To deal with these states we need to use

Von Neumann definition of ∞ -tensor products.

⑧

Von Neumann Space

(used in IR-problem in:

- Wightman Schweser
- Kibble
- Faddeev Kulish

H_k - harmonic oscillator $\omega(k) = k$

$$\bigotimes_k H_k$$

$$a_k^+ a_k$$

$$p_k = \frac{1}{i\sqrt{2}} (a_k - a_k^+)$$

$$q_k = \frac{1}{\sqrt{2}} (a_k + a_k^+)$$

Weyl operator algebra:

$$\left[U(\sigma(k)) = e^{i \sum_k \sigma(k) p_k} \quad V(\tau(k)) = e^{i \sum_k \tau(k) q_k} \right]$$

$\sum_k \sigma(k)$ and $\sum_k \tau(k)$ finite.

$$H_{VN} = \bigotimes_{[k]} H_{[\alpha]}$$

unitary representations of the Weyl algebra

different inequivalent representations

(9)

Spectral decomposition:

$$\frac{1}{2} (p_k^2 + q_k^2 - 1) = \sum_n P_k^{(n)} \quad \sum P_k^{(n)} = 1$$

↪ projector on space with n-quanta of momentum k

V.N.-sequence

$\alpha(k)$ where $\sum \alpha(k)$ can be divergent.

$$E_{\alpha(k)} = \prod P_k^{\alpha(k)}$$

Equivalence of VN-sequences:

$$\alpha(k) \sim \beta(k)$$

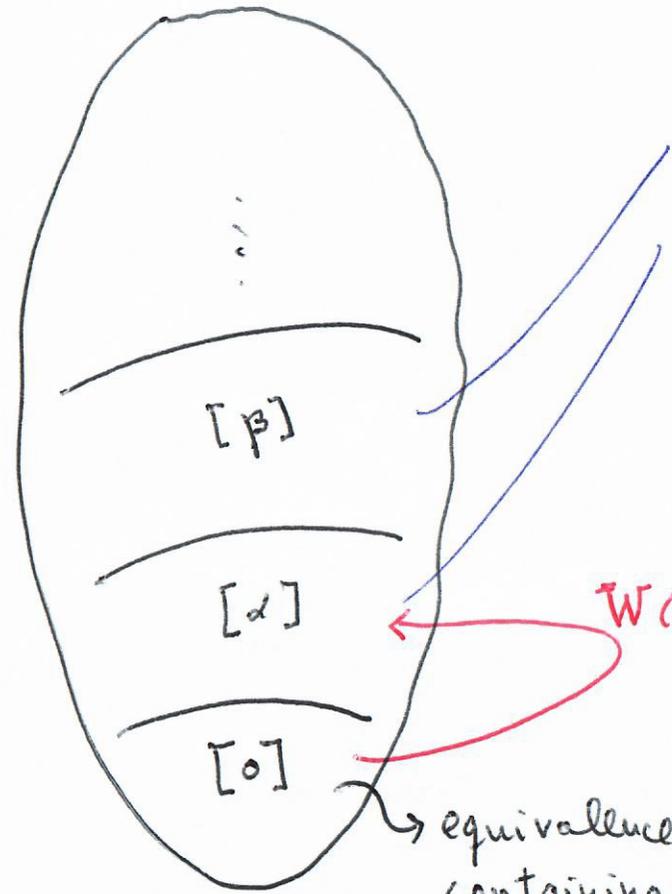
if $\sum_k (\alpha(k) - \beta(k))$ is convergent.

Denote $[\alpha]$ the equivalence class.

projector: $M_{[\alpha]} = \sum_{P \in [\alpha]} E_{P(k)} \longrightarrow H_{[\alpha]}$

each $H_{[\alpha]}$ defines a repr of Weyl algebra.

These spaces contain "inequivalent" ∞ # of quanta



$$W(\alpha) : H_{[0]} \longrightarrow H_{[\alpha]}$$

$$W(\alpha) = e^{-1/2 \sum_k |\alpha(k)|^2} e^{-\sum_k \alpha(k) a_k^\dagger} e^{\sum_k \alpha^*(k) a_k}$$

$W(\alpha)$

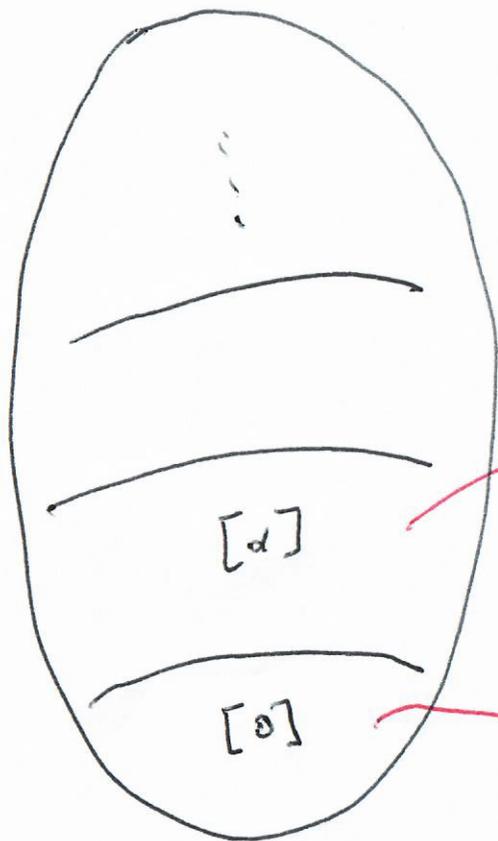
equivalence class containing $d(k)=0$
 Standard Fock space with $|0\rangle$ vacuum.

$$H_{VN}$$

||

$$\otimes H_{[\alpha]}$$

Comment: Inequivalent quantum vacua ?



H_{VN}

$|\alpha\rangle$ zero energy state in class $[\alpha]$?

"LGT"

$|0\rangle$ zero energy state in class $[0]$

Before addressing this question let us discuss

Von Neumann weak equivalence and **TIME EVOLUTION**

$$\alpha(k) \approx \beta(k) \quad \text{weak equivalence}$$

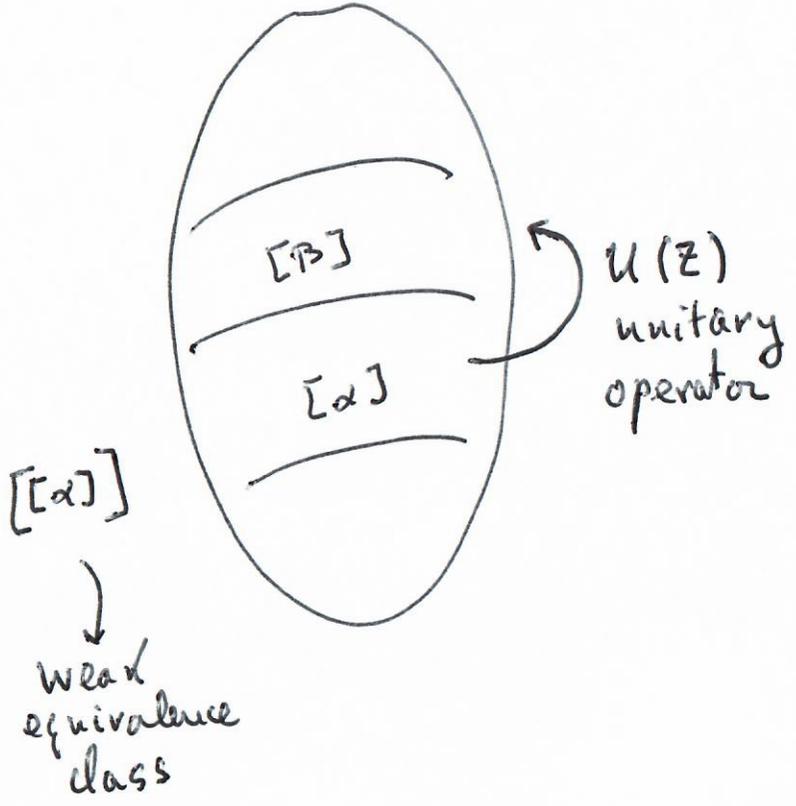
if $\exists Z(k)$ phases such that

$$\alpha(k) \sim \beta(k) Z(k) \quad \text{equivalence}$$

Simplest example of phase is time evolution.

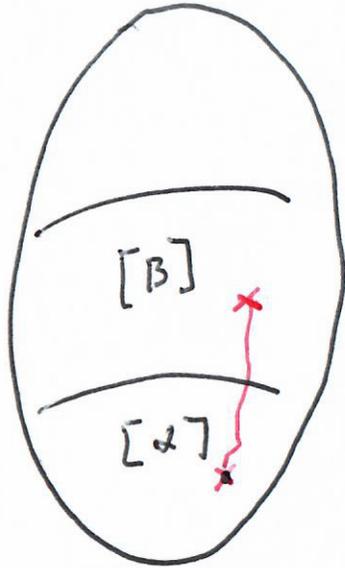
$$W(\alpha; t) |0\rangle = e^{-\frac{1}{2} \int |\alpha_k|^2} e^{\int \alpha_k a_k^\dagger} e^{\int \alpha_k^\dagger a_k} |0\rangle$$

$$\alpha_k(t) = \alpha(k) e^{i\epsilon(k)t} \rightarrow \sim (\alpha(k))^2 \omega(k)$$



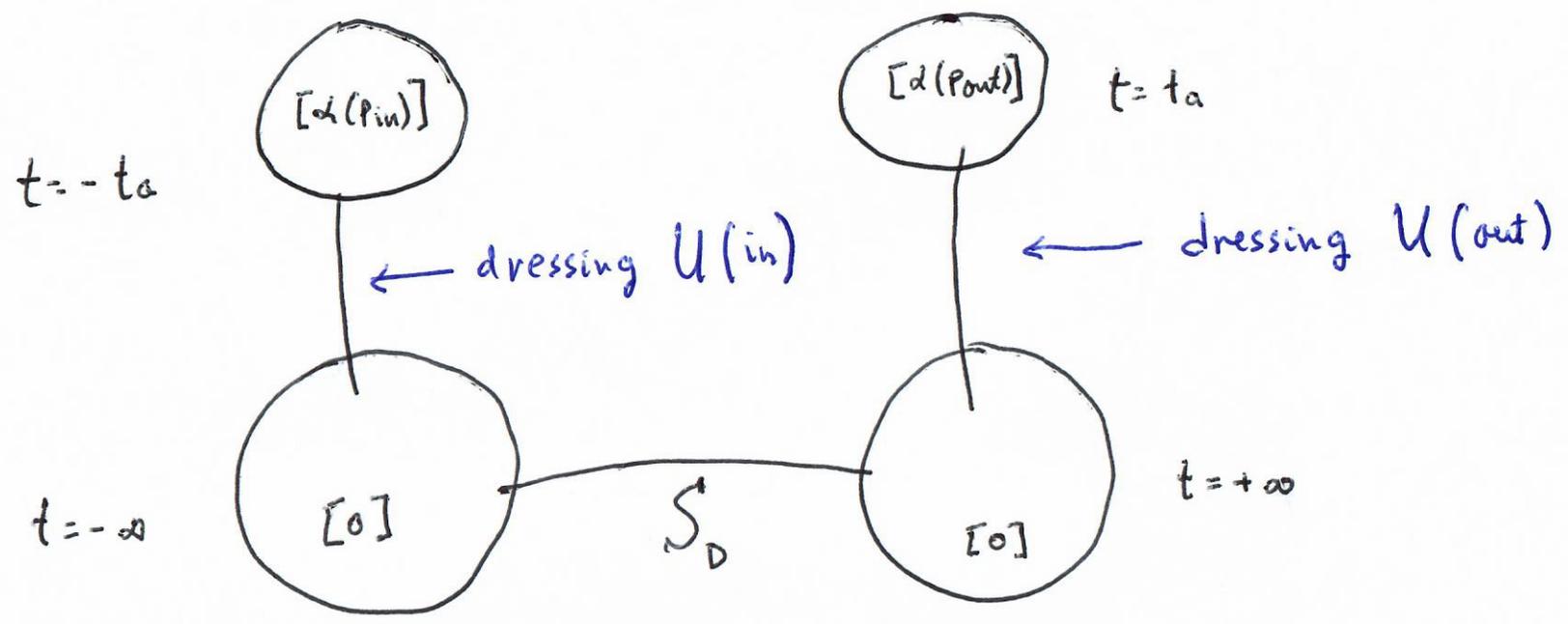
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Time Evolution in VN-space



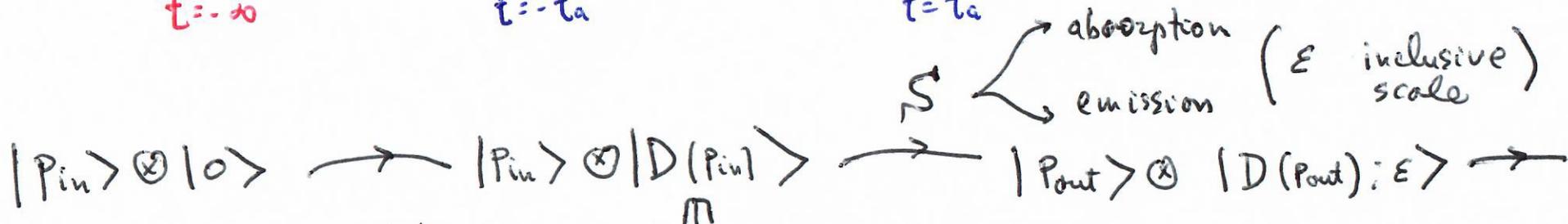
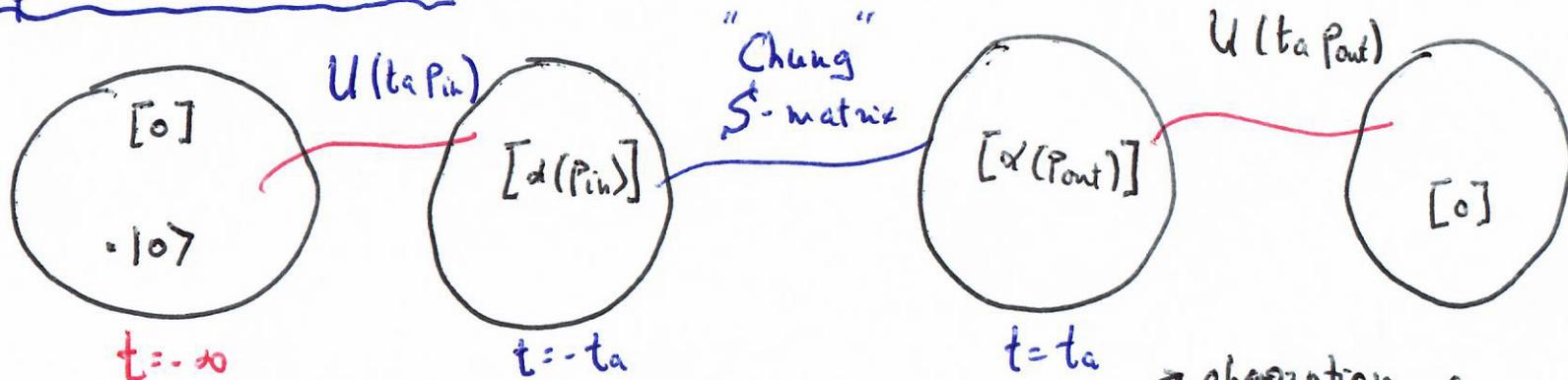
A necessary condition to change equivalence class through unitary evolution is ∞ amount of time.

The S - matrix



dressing takes ∞ time and
changes equivalence class

Equivalent version:

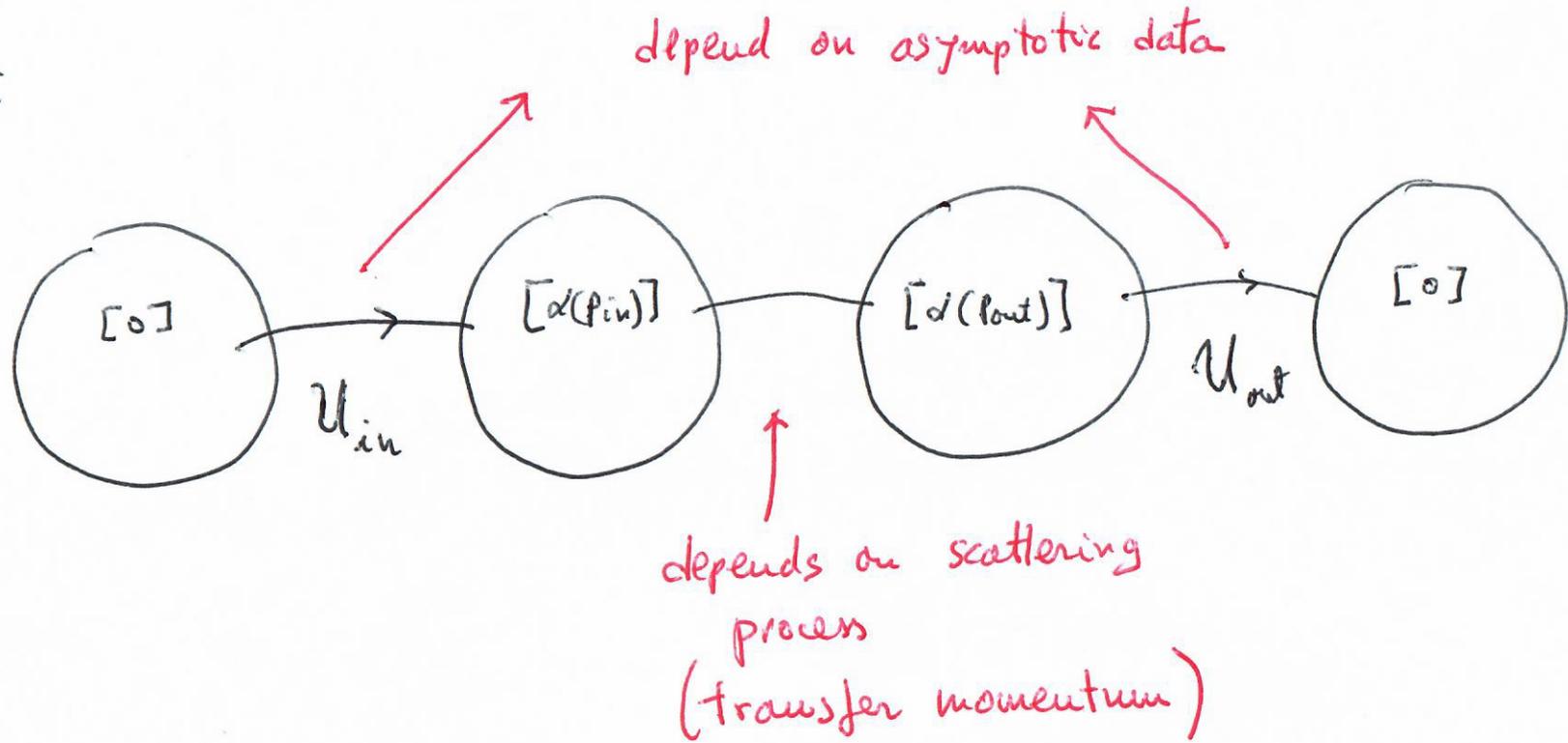


$|P_{out}\rangle \otimes |\tilde{\epsilon}\rangle$
 ↑
 radiated soft photons.

Two scales in the problem:

$t_a \sim$ "dressing scale" $r \sim \frac{\hbar}{t_a}$
 $\epsilon \sim$ inclusive radiative scale,

Note:

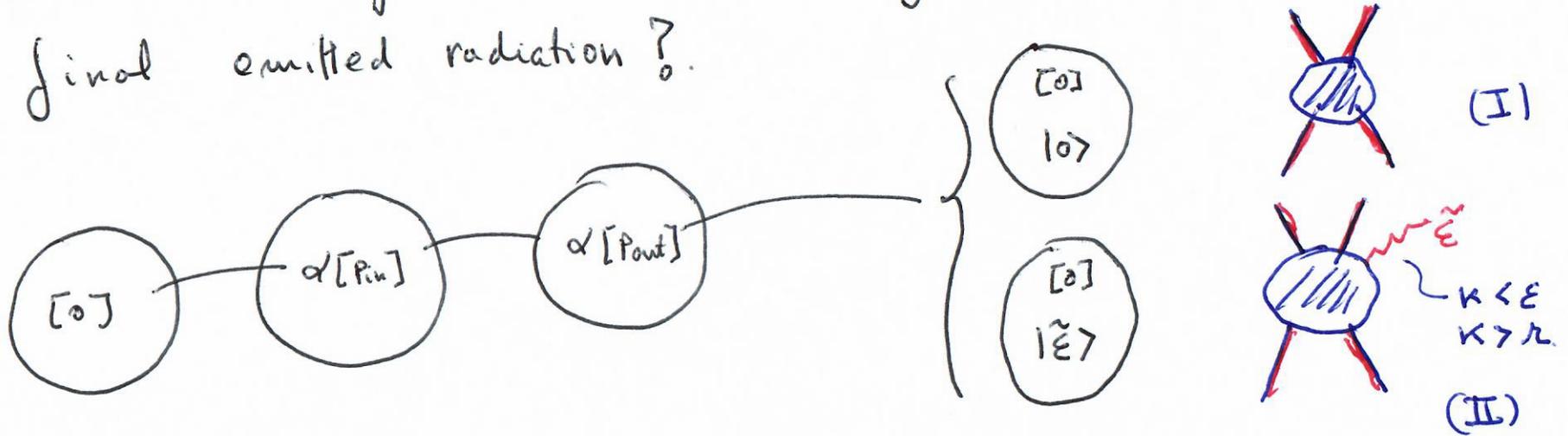


Note: In trivial scattering i.e. $B_{in, out} = 0$

$$U(in) \circ U(out) = \mathbb{1}_d$$

Quantum Coherence.

How much information is lost if we trace over final emitted radiation?



In case (I) full coherence

In case (II) we trace in $\mathcal{F}_r[0]$ over emitted radiation with $k < E$ (E IR-resolution scale)

Photons in $r < k < E$ are not entangled to hard modes.

$$S \approx \ln\left(\frac{E}{r}\right)$$

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You could try to trace over dressing. (C. Ch. N. Smeroff)

To do that is equivalent to cancel the contribution to the S-matrix of the asymptotic dynamics i.e. is only possible if $B_{in\ out} = 0$

The case of massless QED

It is well known (LN, W) that in this case for any process with exchange of momentum

$$B_{in\ out} \sim -\ln(me)$$

$$\approx \left(\frac{E}{\Lambda} \right)^{-\ln(me)} \rightarrow 0$$

From V.N point of view

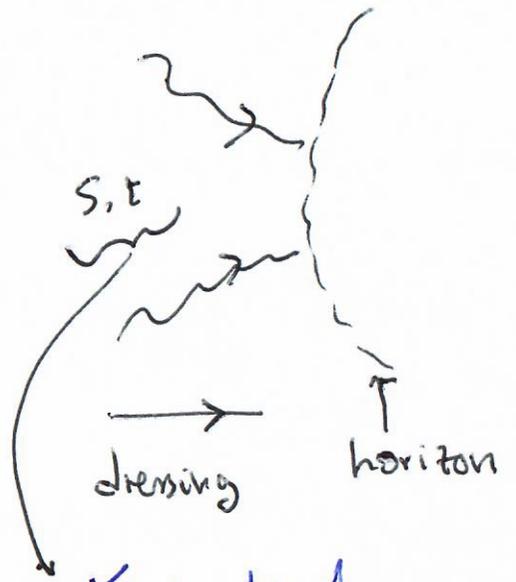
sequences: $\alpha(k) \sim -\frac{\ln(me)}{|k|} \Rightarrow$ formal $\infty \neq$ BUT also ∞ Energy.

\Rightarrow only trivial scattering $B_{in\ out} = 0$ is allowed.

Soft hair:

$[\alpha(s,t)]$

VN equivalence class
created by dressing
($-\infty$, horizon)



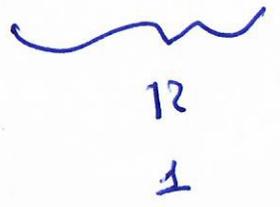
BH as coherent state:

$|\langle 2 | BH \rangle|^2 \approx$

Kinematical conditions where you expect BH-formation

$\sum_n |\Gamma_{2 \rightarrow n}|^2 \left(\frac{\epsilon}{\lambda}\right)^B \sim \left(\frac{\epsilon}{\lambda}\right)^N \int_1^\infty \left(\frac{N}{n^2}\right)^n dn$

$B \sim G_N S \equiv N$



~~Thank~~

You

