

Determining Neutrino Properties from Supernova Neutrinos

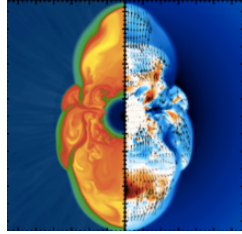
Kate Scholberg, Duke University
Solvay Workshop, Brussels, November 2017

OUTLINE

- Overview of neutrinos from supernovae
 - The signal
 - Detection
- Neutrino Physics
 - Absolute mass
 - Mass ordering
 - New physics?
- Summary

What can we learn from the next neutrino burst?

CORE COLLAPSE PHYSICS

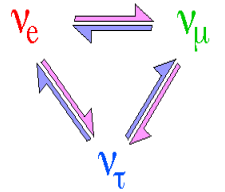


explosion mechanism
proto nstar cooling,
quark matter
black hole formation
accretion, SASI
nucleosynthesis
....

input from
photon (GW)
observations

from flavor,
energy, time
structure
of burst

input from
neutrino
experiments



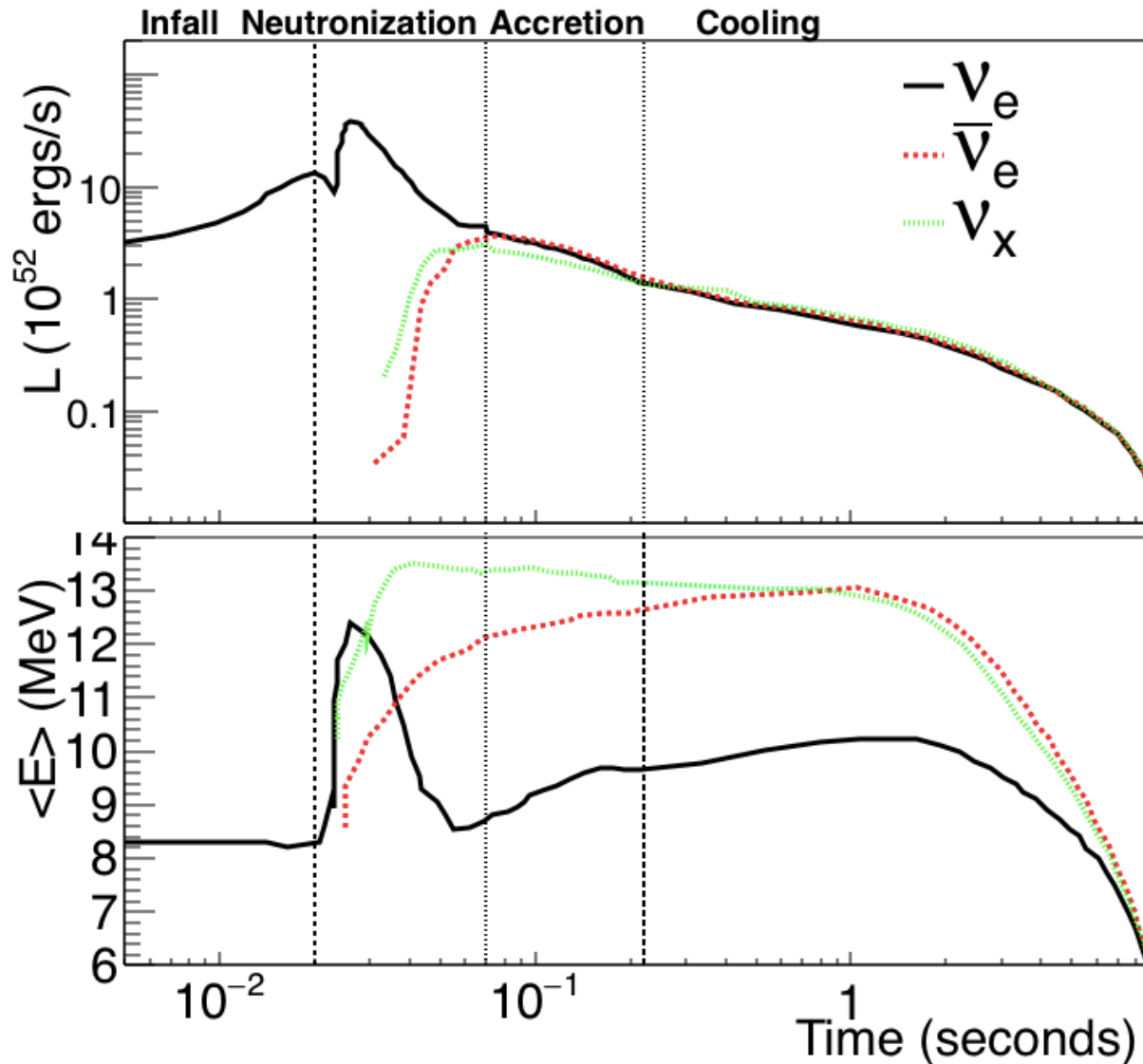
NEUTRINO and OTHER PARTICLE PHYSICS

ν absolute mass (not competitive)
 ν mixing from spectra:
flavor conversion in SN/Earth
(mass ordering)
other ν properties: sterile ν 's,
magnetic moment, ...
axions, extra dimensions,
FCNC, ...

+ EARLY ALERT

Expected neutrino luminosity and average energy vs time

Vast information in the *flavor-energy-time profile*

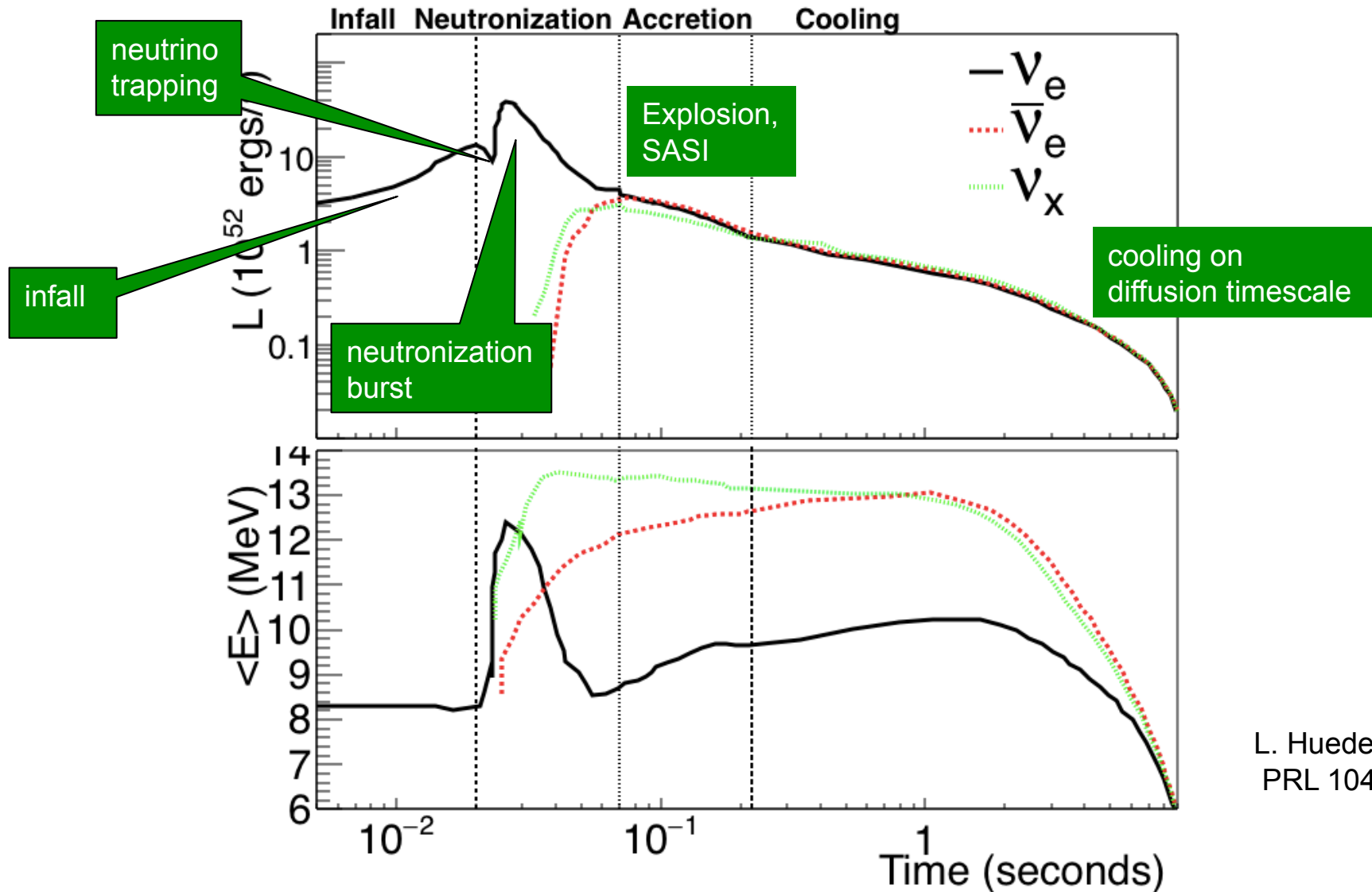


L. Huedepohl et al.,
PRL 104 251101

Generic feature:
(may or may not be robust) $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle$

Expected neutrino luminosity and average energy vs time

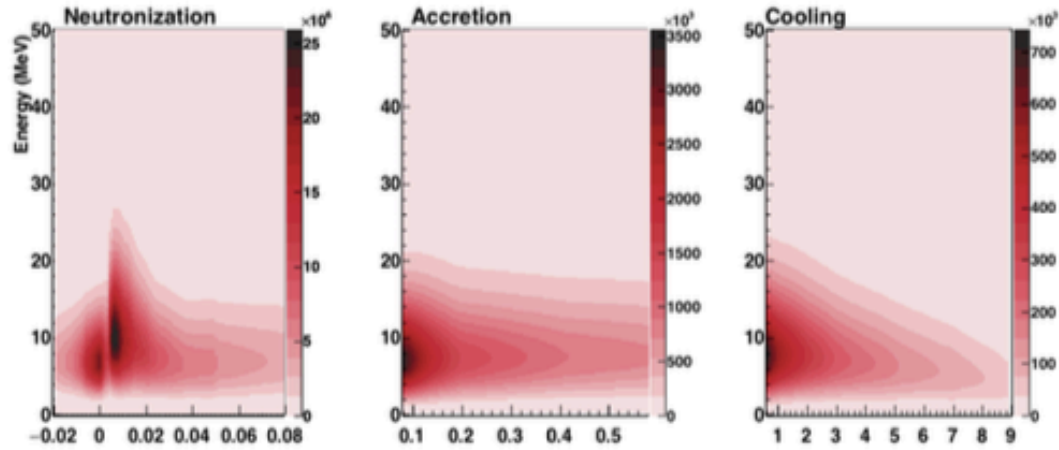
Vast information in the *flavor-energy-time profile*



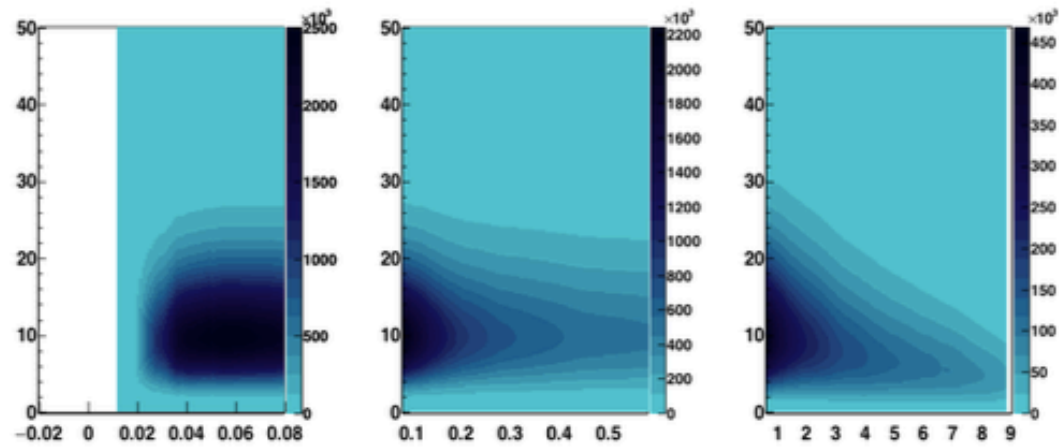
L. Huedepohl et al.,
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Generic feature: $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle$
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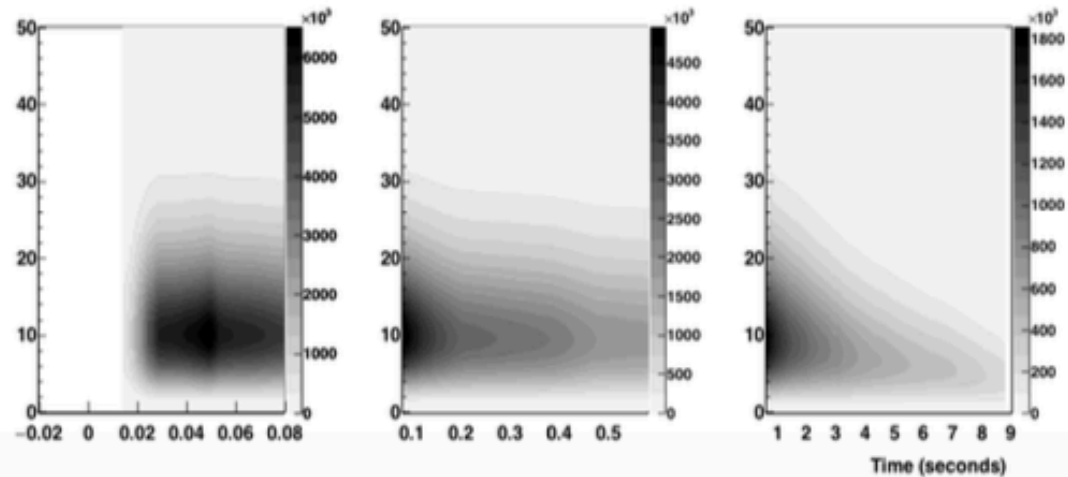
Fluxes as a function of time and energy



ν_e



$\bar{\nu}_e$



ν_x

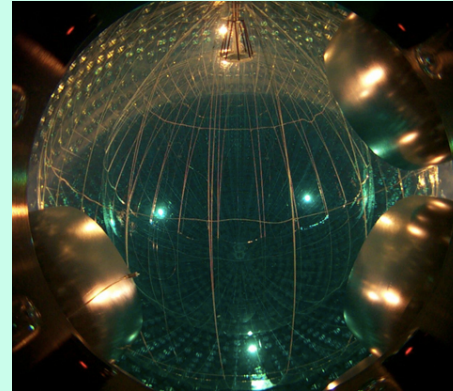
Supernova Neutrino Detectors

Water



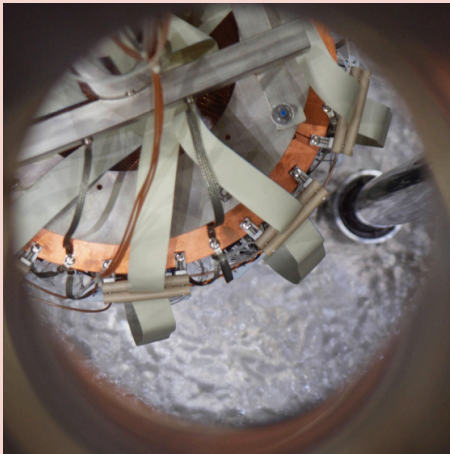
$$\bar{\nu}_e$$

Scintillator



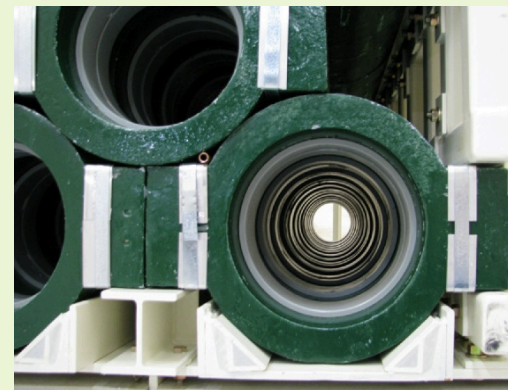
$$\bar{\nu}_e$$

Argon



$$\nu_e$$

Lead



$$\nu_e$$

+ some others (e.g. DM detectors)

Summary of supernova neutrino detectors

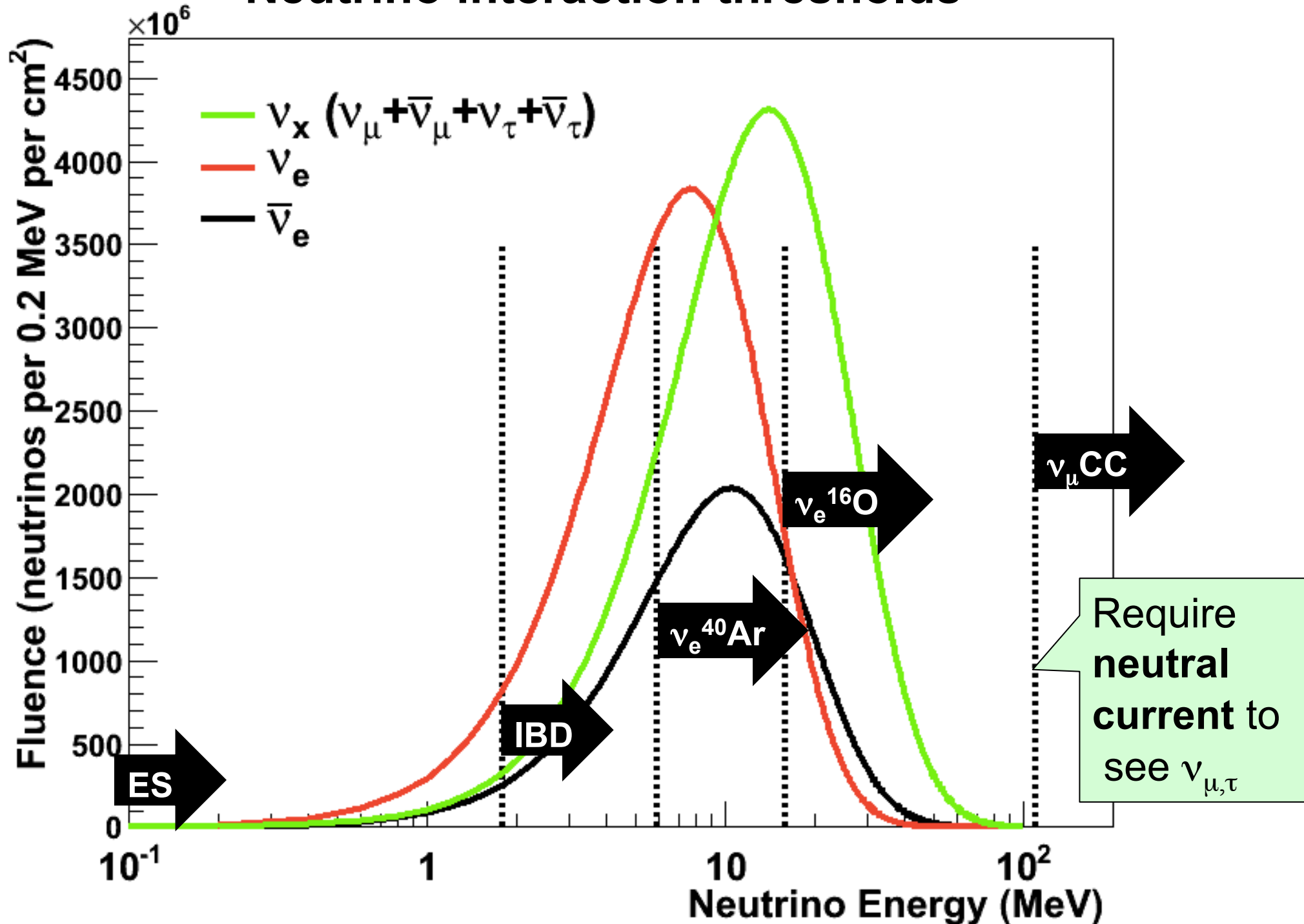
Galactic sensitivity

Detector	Type	Location	Mass (kton)	Events @ 10 kpc	Status
Super-K	Water	Japan	32	8000	Running
LVD	Scintillator	Italy	1	300	Running
KamLAND	Scintillator	Japan	1	300	Running
Borexino	Scintillator	Italy	0.3	100	Running
IceCube	Long string	South Pole	(600)	(10 ⁶)	Running
Baksan	Scintillator	Russia	0.33	50	Running
HALO	Lead	Canada	0.079	20	Running
Daya Bay	Scintillator	China	0.33	100	Running
NOvA	Scintillator	USA	15	3000	Running
MicroBooNE	Liquid argon	USA	0.17	17	Running
SNO+	Scintillator	Canada	1	300	Under construction
DUNE	Liquid argon	USA	40	3000	Future
Hyper-K	Water	Japan	540	110,000	Future
JUNO	Scintillator	China	20	6000	Future
PINGU	Long string	South pole	(600)	(10 ⁶)	Future

plus reactor experiments, DM experiments...

Extragalactic

Neutrino interaction thresholds



Information on Neutrino Properties from Core Collapse

- Absolute Neutrino Mass
- Neutrino Mixing Parameters: Mass Ordering
- New Neutrino States?

A sampler...



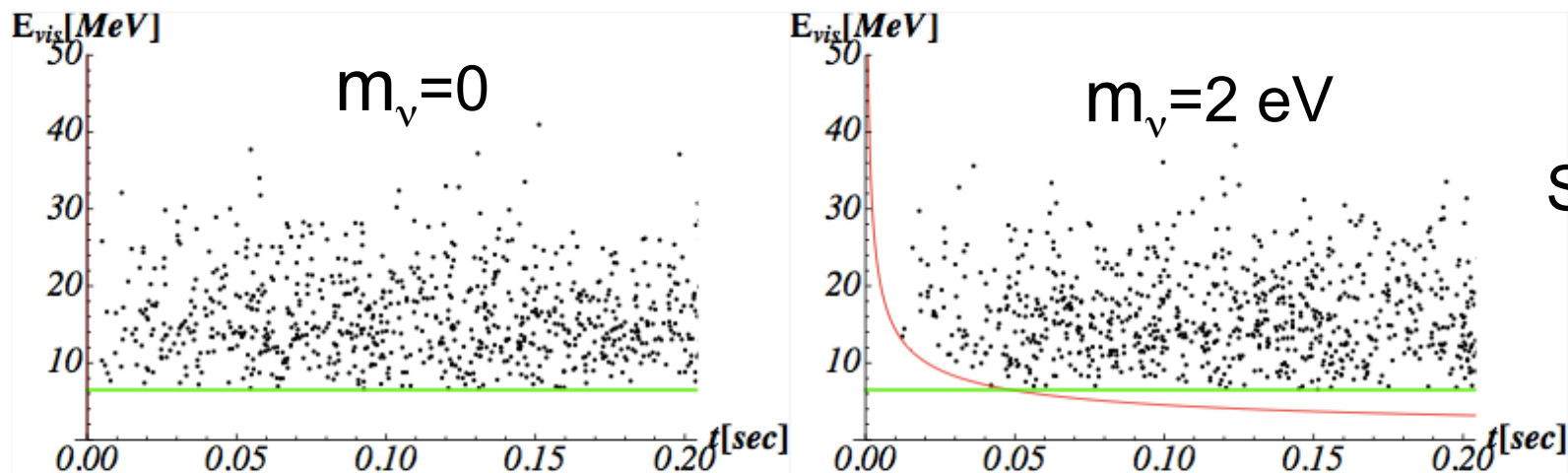
Neutrino Absolute Mass

Expect time of flight delay for massive neutrinos

$$\Delta t(m_\nu, E_\nu) \simeq 5.14 \text{ ms} \left(\frac{m_\nu}{\text{eV}} \right)^2 \left(\frac{10 \text{ MeV}}{E_\nu} \right)^2 \frac{D}{10 \text{ kpc}}$$

Look for:

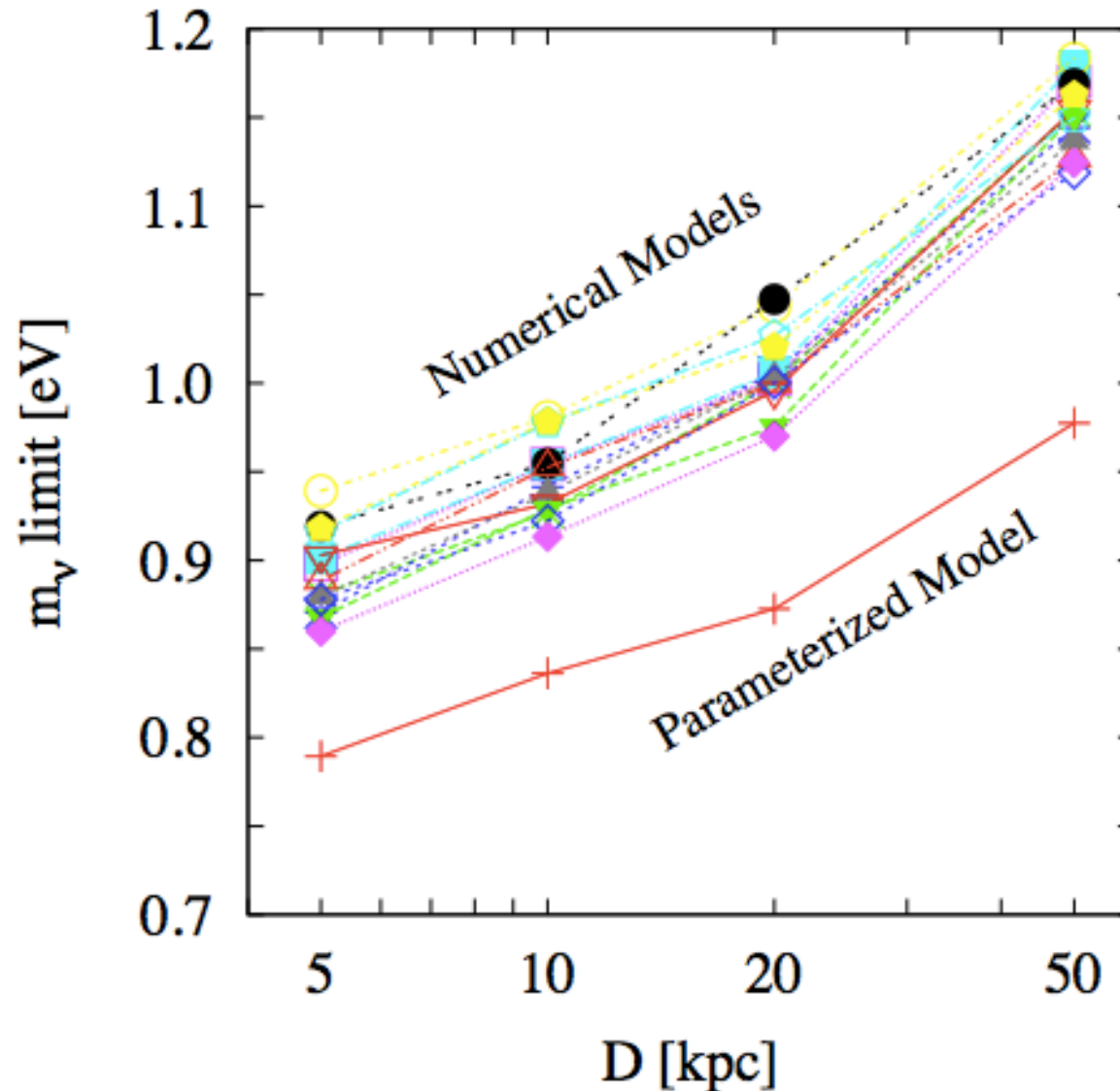
- ◆ energy-dependent time spread
- ◆ flavor-dependent delay



SK@10 kpc
 $\bar{\nu}_e$

A more recent study example

JUNO mass sensitivity (20 kton scintillator, low energy threshold)



J.-S. Lu et al.,
JCAP 1505, 044 (2015)

Future SN-based ν mass limits ~improvement over current laboratory limits, but not competitive w/next generation

Three-flavor neutrino mixing parameters

$$|\nu_f\rangle = \sum_{i=1}^N U_{fi}^* |\nu_i\rangle$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Parameters of Nature

3 masses

m_1, m_2, m_3
(2 mass differences
+ absolute scale)

3 mixing angles

$\theta_{23}, \theta_{12}, \theta_{13}$

1 CP phase

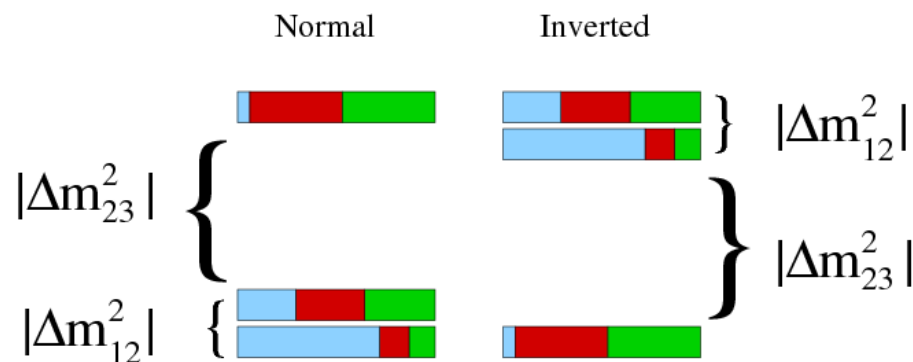
δ

(2 Majorana phases)

α_1, α_2

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s_{ij} \equiv \sin \theta_{ij}, c_{ij} \equiv \cos \theta_{ij}$$



signs of the
mass differences
matter

The three-flavor picture fits the data well

Global three-flavor fits to all data

	3σ range	<u>3σ knowledge</u>
$\sin^2 \theta_{12}$	0.271 \rightarrow 0.345	
$\theta_{12}/^\circ$	31.38 \rightarrow 35.99	$\sim 14\%$
$\sin^2 \theta_{23}$	0.385 \rightarrow 0.638	
$\theta_{23}/^\circ$	38.4 \rightarrow 53.0	$\sim 32\%$
$\sin^2 \theta_{13}$	0.01934 \rightarrow 0.02397	
$\theta_{13}/^\circ$	7.99 \rightarrow 8.91	$\sim 11\%$
$\delta_{\text{CP}}/^\circ$	0 \rightarrow 360	\sim no info
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	7.03 \rightarrow 8.09	$\sim 14\%$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$\left[\begin{array}{l} +2.407 \rightarrow +2.643 \\ -2.629 \rightarrow -2.405 \end{array} \right]$	$\sim 9\%$

What do we *not* know about the three-flavor paradigm?

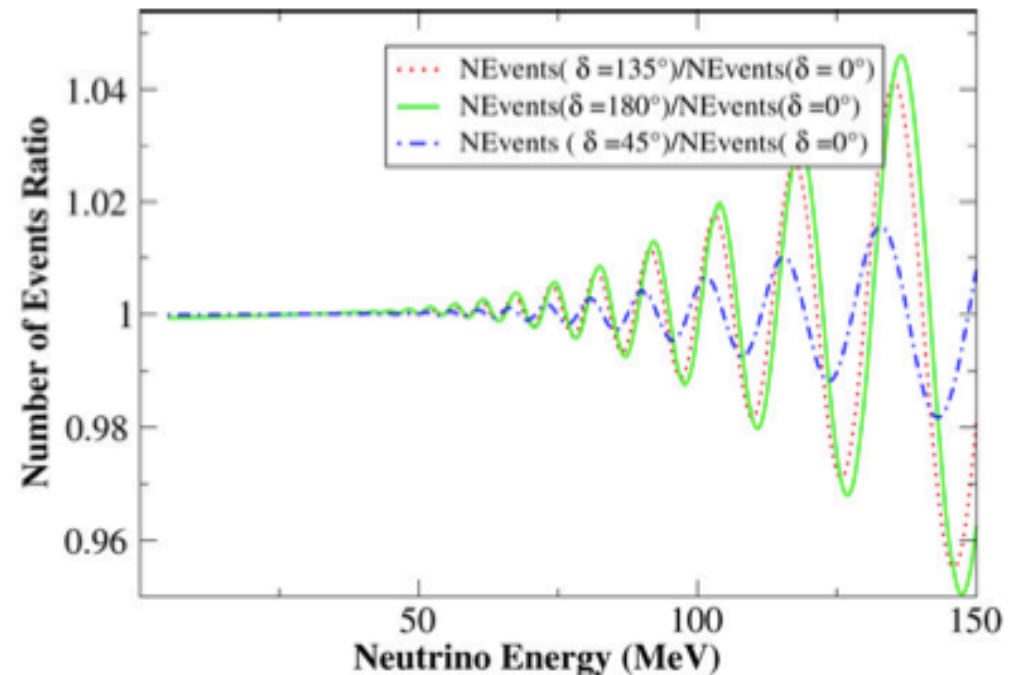
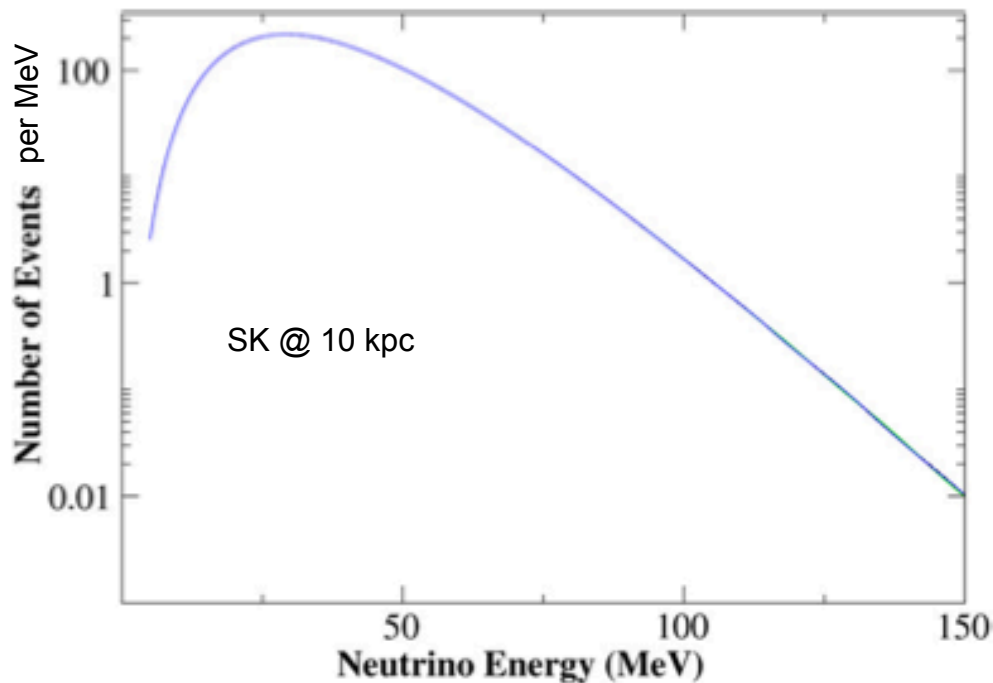
	3σ range	
$\sin^2 \theta_{12}$	0.271 \rightarrow 0.345	
$\theta_{12}/^\circ$	31.38 \rightarrow 35.99	
$\sin^2 \theta_{23}$	0.385 \rightarrow 0.638	Is θ_{23} non-negligibly greater or smaller than 45 deg?
$\theta_{23}/^\circ$	38.4 \rightarrow 53.0	
$\sin^2 \theta_{13}$	0.01934 \rightarrow 0.02397	
$\theta_{13}/^\circ$	7.99 \rightarrow 8.91	
$\delta_{CP}/^\circ$	0 \rightarrow 360	basically unknown
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	7.03 \rightarrow 8.09	
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$\left[\begin{array}{l} +2.407 \rightarrow +2.643 \\ -2.629 \rightarrow -2.405 \end{array} \right]$	sign of Δm^2 unknown (ordering of masses)

Can we learn about CP violation from a supernova?

Answer: maybe, but very hard...

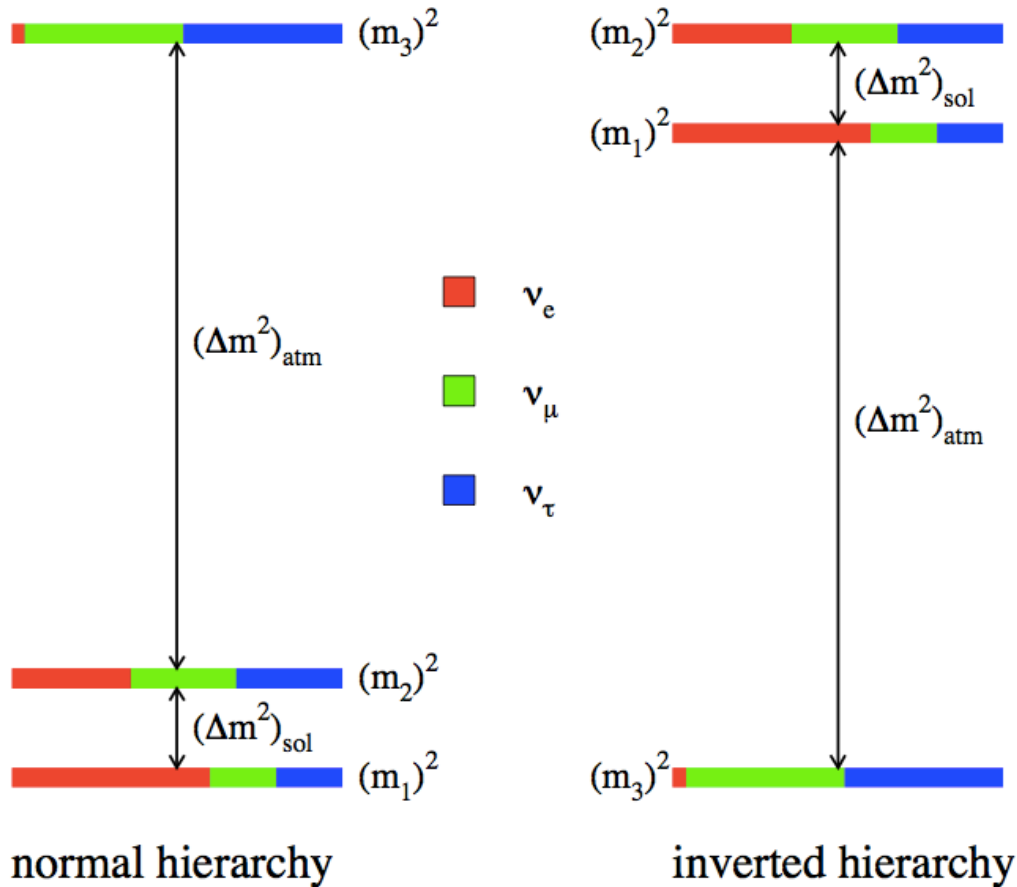
- Effect of non-zero δ is mainly $\mu\tau$ mixing... unobservable...
- However if ν_μ and ν_τ fluxes differ at neutrinosphere (FCNC?), get small effects on electron flavor, but in high energy tail where rate is low

A.B. Balantakin, J. Gava and C. Volpe,
Phys. Lett. B 662, 396 (2008)



Next on the list to go after experimentally: mass ordering (hierarchy)

(sign of Δm^2_{32})



$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

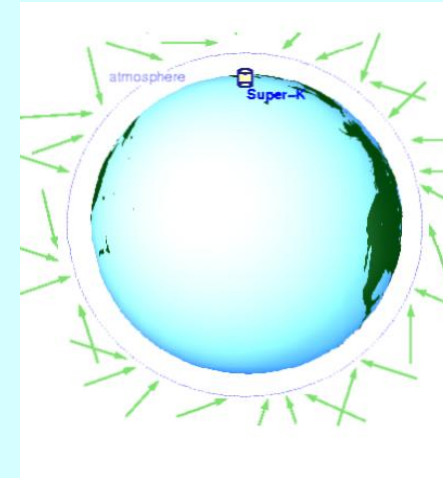
Four of the possible ways to get MO



Long-baseline beams



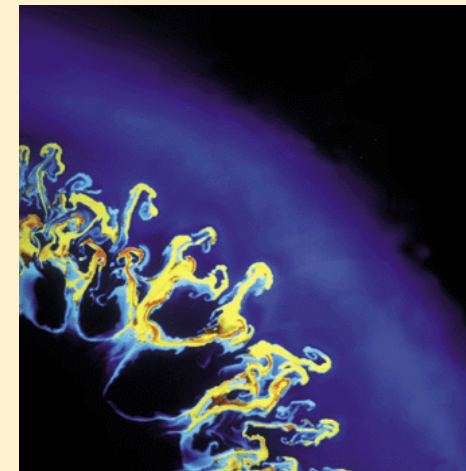
Atmospheric neutrinos



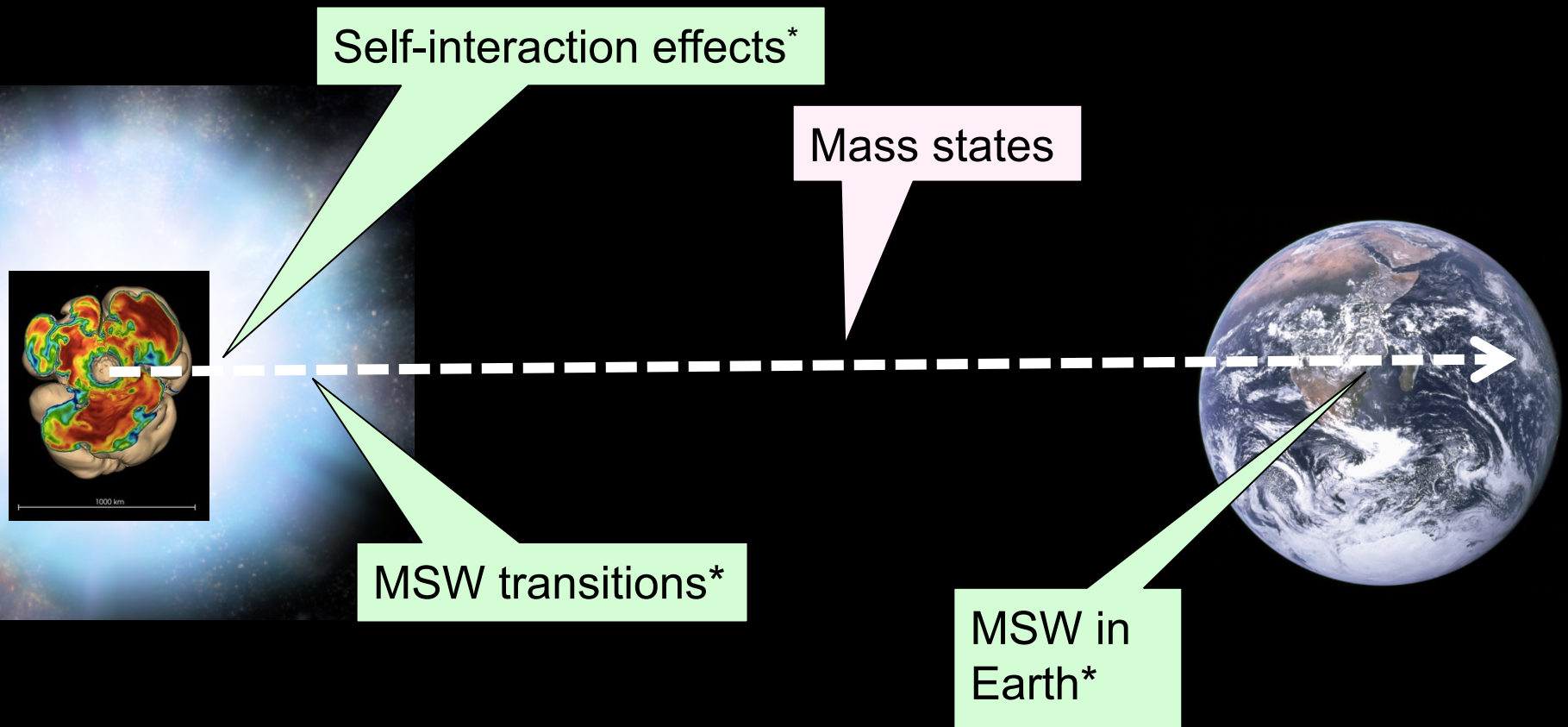
Reactors



Supernovae



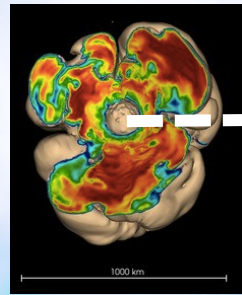
Neutrino Mixing for Supernova Neutrinos



* All of these depend on
MO to some extent
... **multiple signatures** of MO
(although some model-dependence)

Not to scale!

Neutrino Mixing in the Supernova Itself



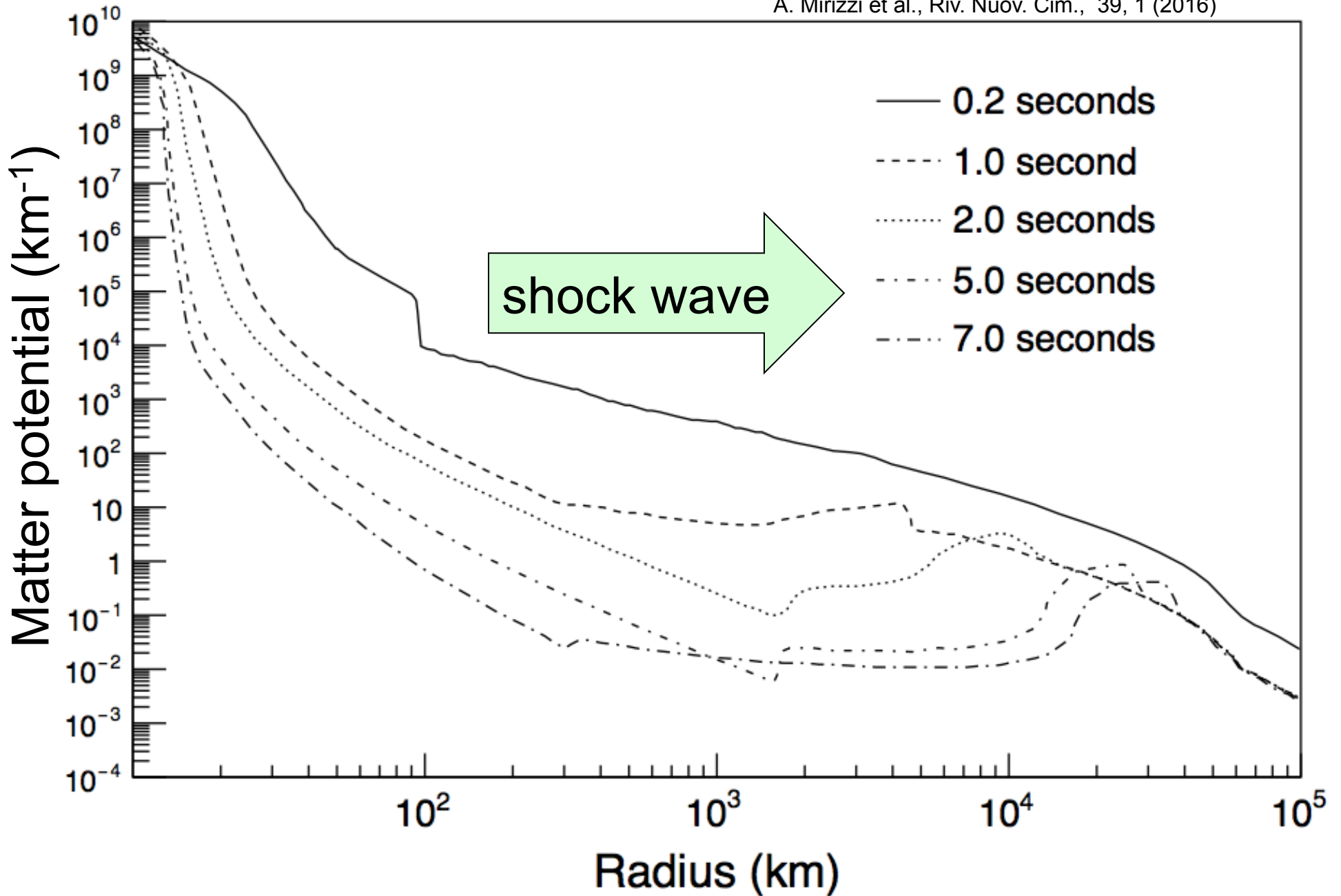
Self-interaction effects

MSW transitions



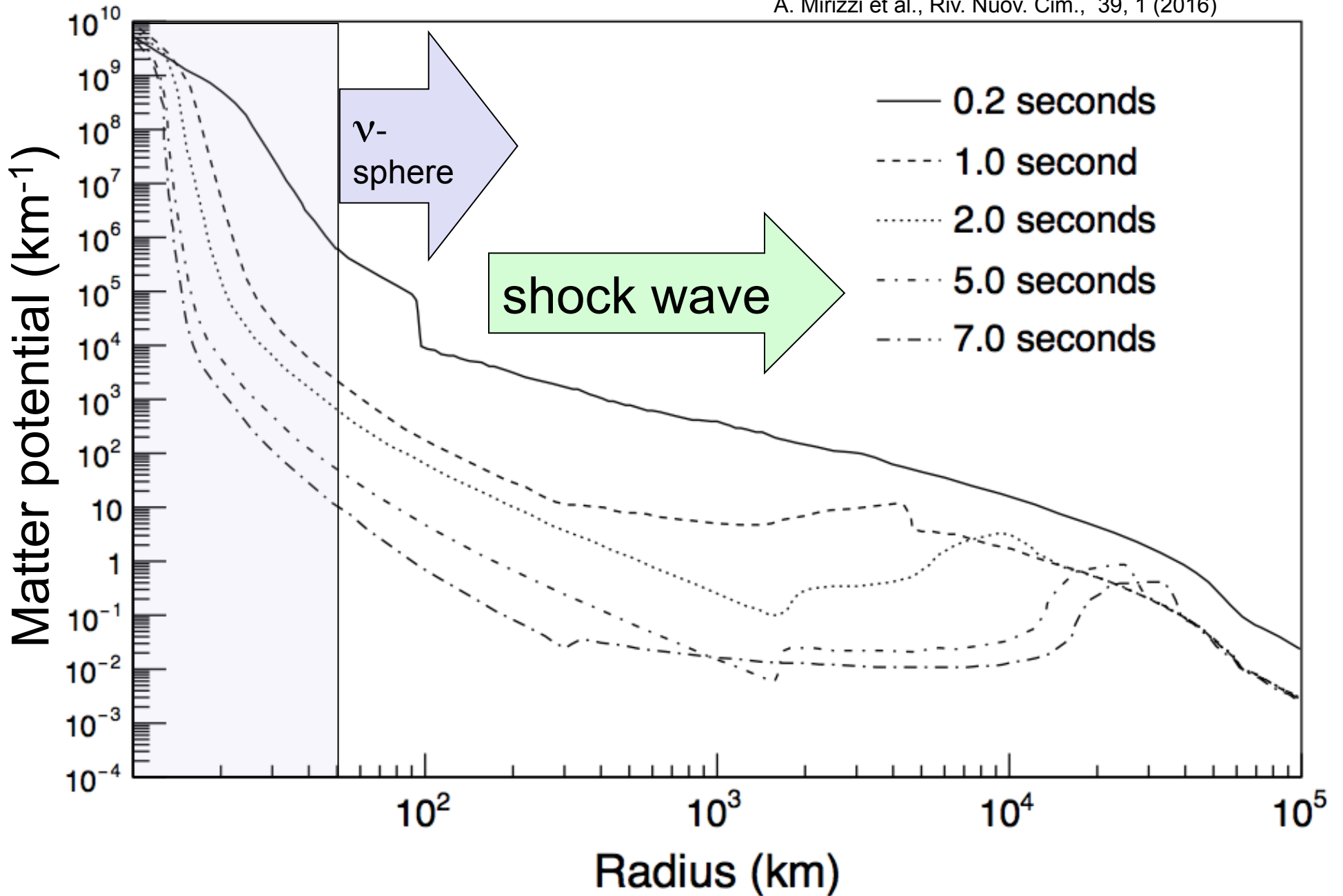
Matter potential (\propto density) in a supernova vs time

A. Mirizzi et al., Riv. Nuov. Cim., 39, 1 (2016)



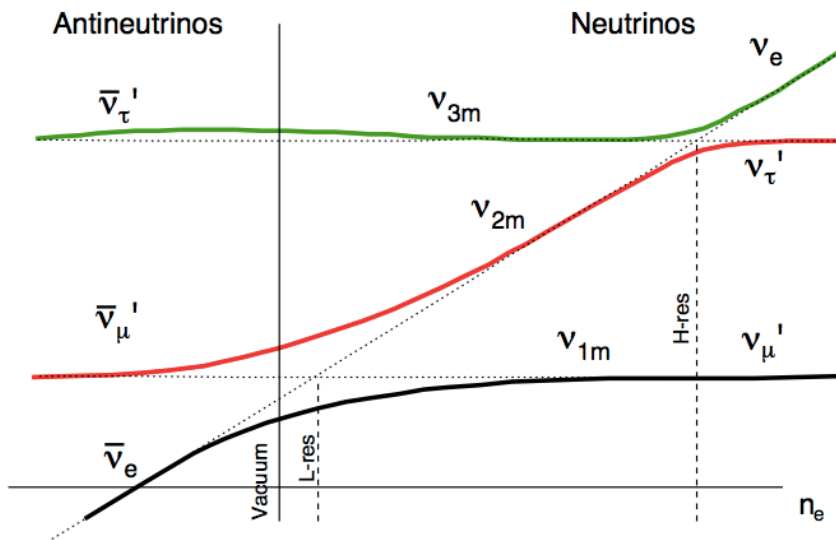
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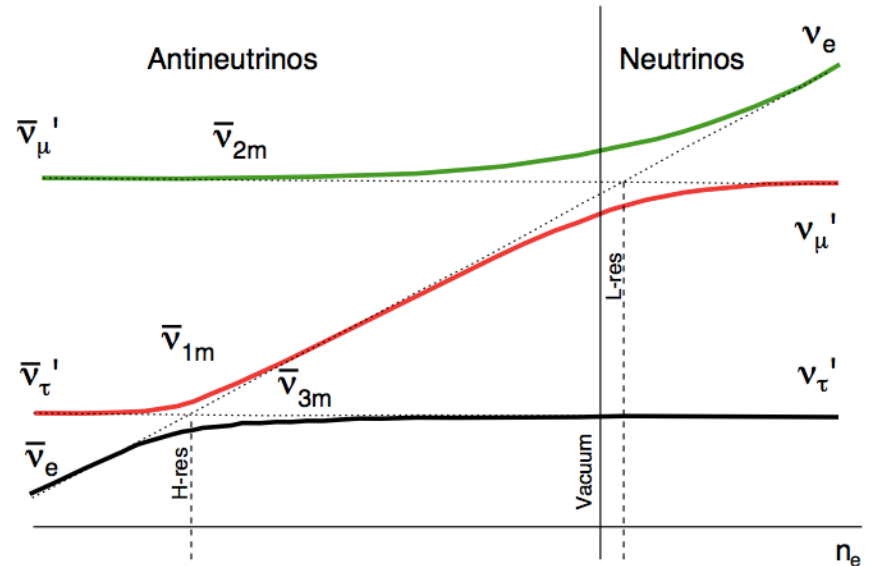


MSW Transitions in Supernova Matter

Normal Ordering



Inverted Ordering

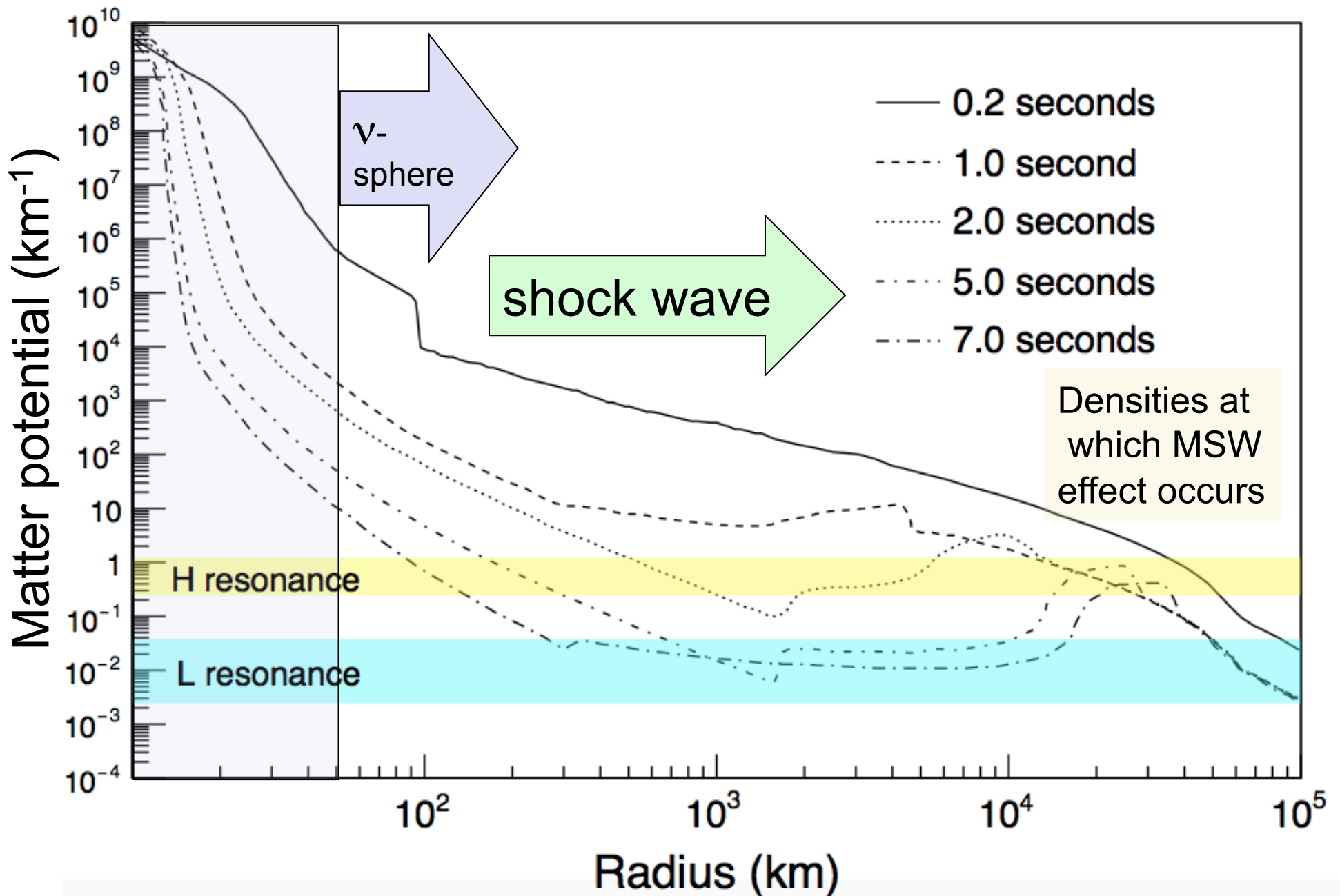


A. Mirizzi et al., Riv. Nuov. Cim., 39, 1 (2016), G. Raffelt, Proc. Int. Sch. Phys. Ferml, 182, 61 (2012)

$$P_{ee} \simeq \begin{cases} \sin^2 \theta_{12} P_H & (\nu, \text{NH}), \\ \cos^2 \theta_{12} & (\bar{\nu}, \text{NH}), \\ \sin^2 \theta_{12} & (\nu, \text{IH}), \\ \cos^2 \theta_{12} P_H & (\bar{\nu}, \text{IH}). \end{cases}$$

- **Mass-ordering-dependent** transition probability for neutrinos and antineutrinos
- Can be adiabatic, or non-adiabatic at a shock front

Matter potential (\propto density) in a supernova vs time



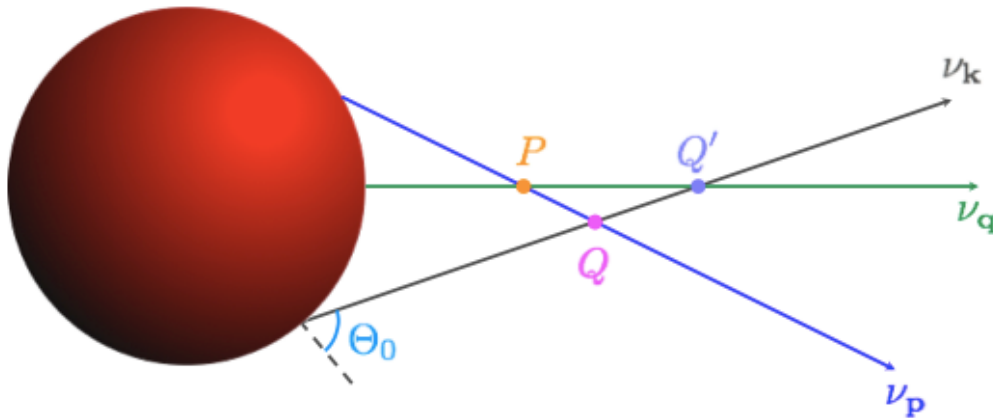
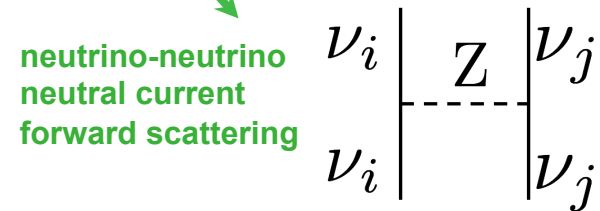
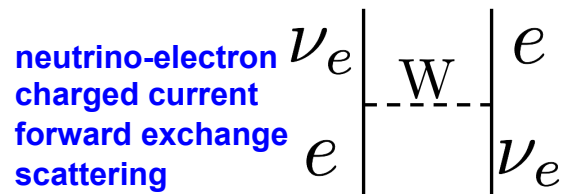
MSW effects *may turn on and off* as the shock propagates

And another effect: “self-interaction effects”

In the proto-neutron star the neutrino density is so high that **neutrino-neutrino interactions** matter

$$\psi_{\nu,i} = \begin{bmatrix} \text{amplitude to be } \nu_e \\ \text{amplitude to be } \nu_{\mu,\tau} \end{bmatrix} \quad \text{From G. Fuller}$$

$$i \frac{\partial}{\partial t} \psi_{\nu,i} = (\mathcal{H}_{\text{vac},i} + \mathcal{H}_{e,i} + \mathcal{H}_{\nu\nu,i}) \psi_{\nu,i}$$

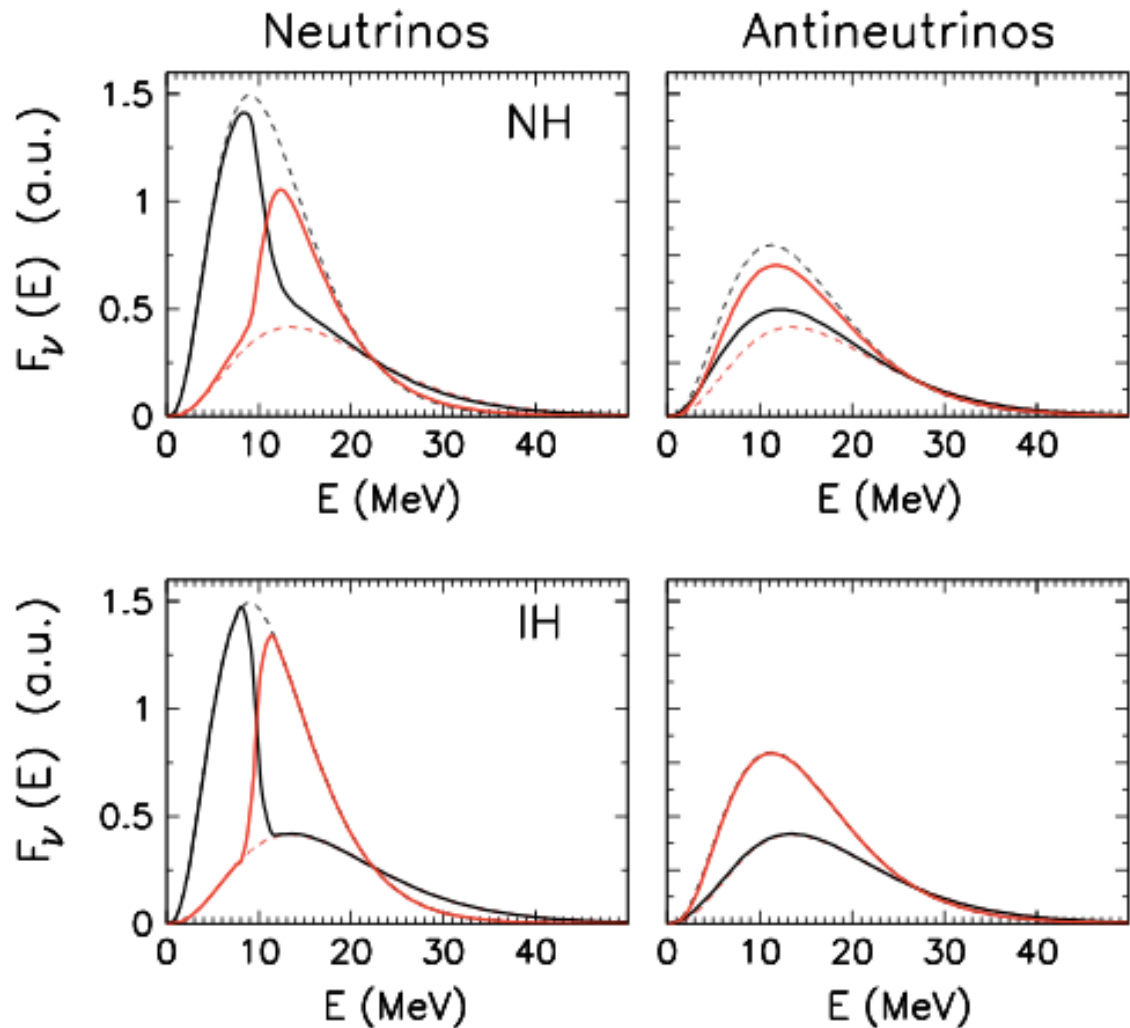


Anisotropic, nonlinear
quantum coupling of all
neutrino flavor evolution
histories:
“collective effects”

Must solve many **millions** of coupled, nonlinear partial differential equations!!

“The physics is addictive” -- G. Raffelt

A consequence: spectral “swaps” or “splits”



Dashed: no osc

Red: ν_x

Black: ν_e

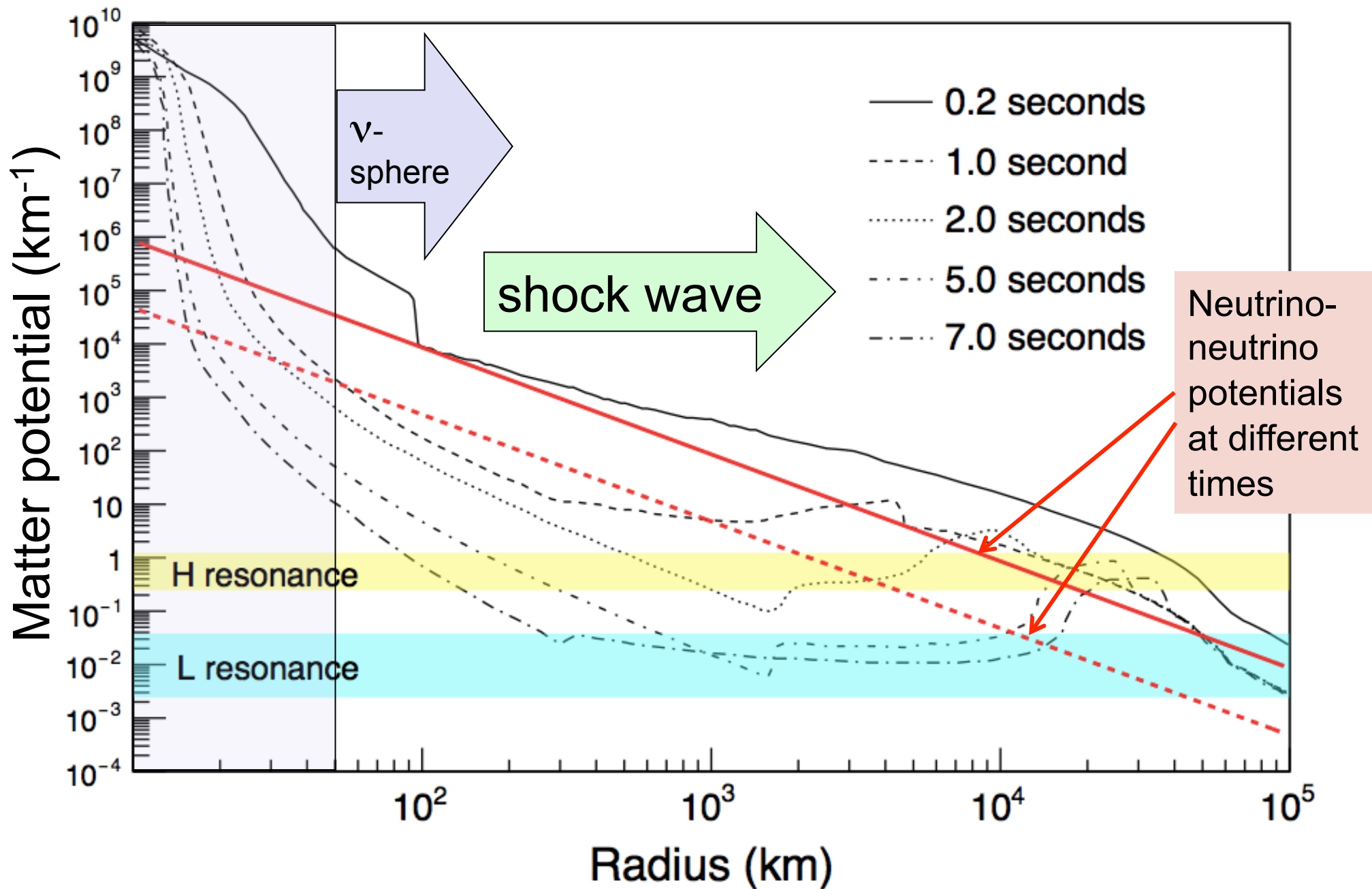
Can get
spectral flavor
conversion
above or
below specific
energy
thresholds

A. Mirizzi et al., Riv. Nuov. Cim., 39, 1 (2016) , S. Chakraborty and A. Mirizzi, PRD 90, 033004 (2014)

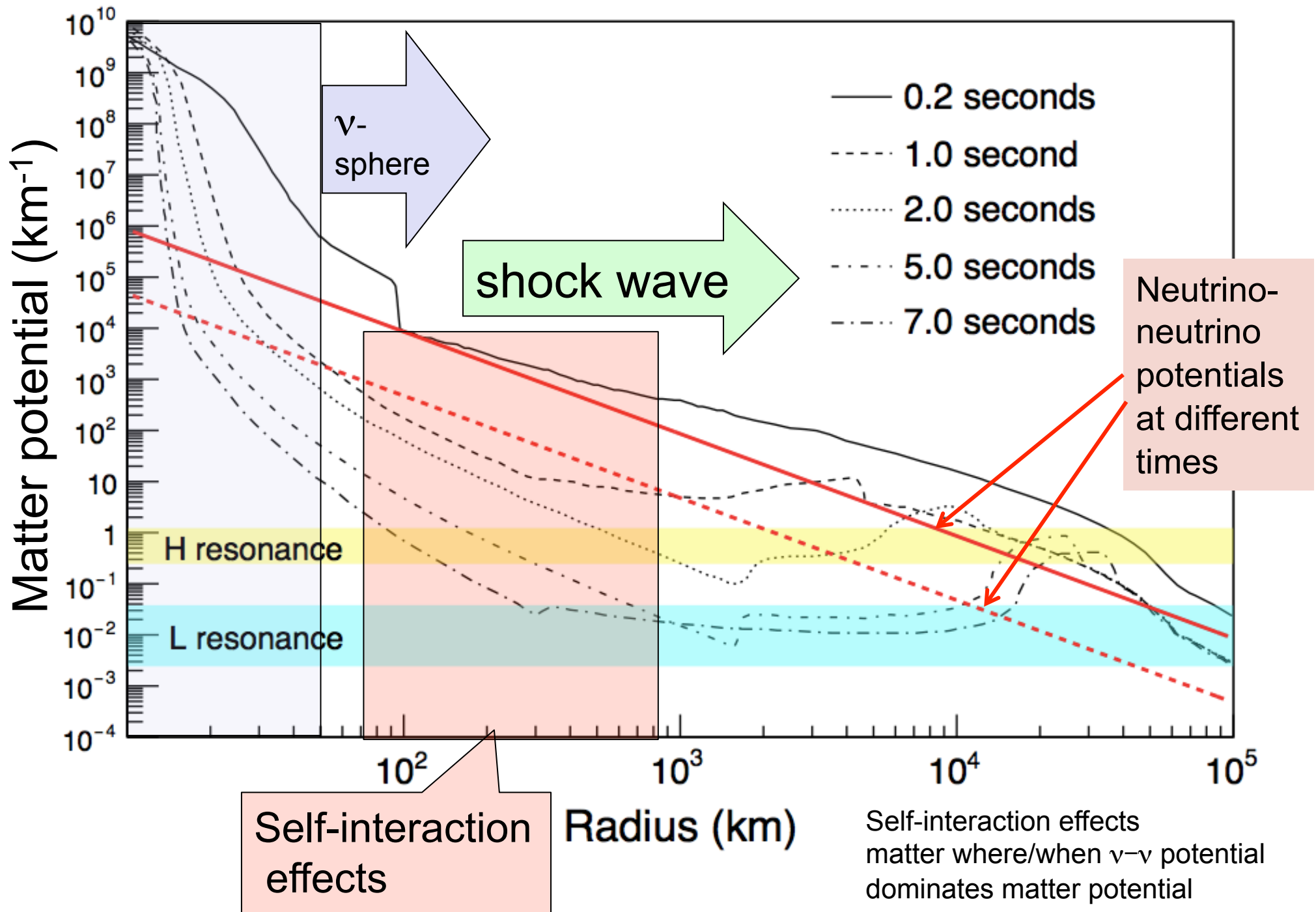
Initial fluxes $F_{\nu_e}^0 : F_{\bar{\nu}_e}^0 : F_{\nu_x}^0 = 2.40 : 1.60 : 1.0$

- Depend on flavor flux ratio
- Can be suppressed by matter density
- Time-dependent, also affected by shock propagation

Matter potential (\propto density) in a supernova vs time



Matter potential (\propto density) in a supernova vs time

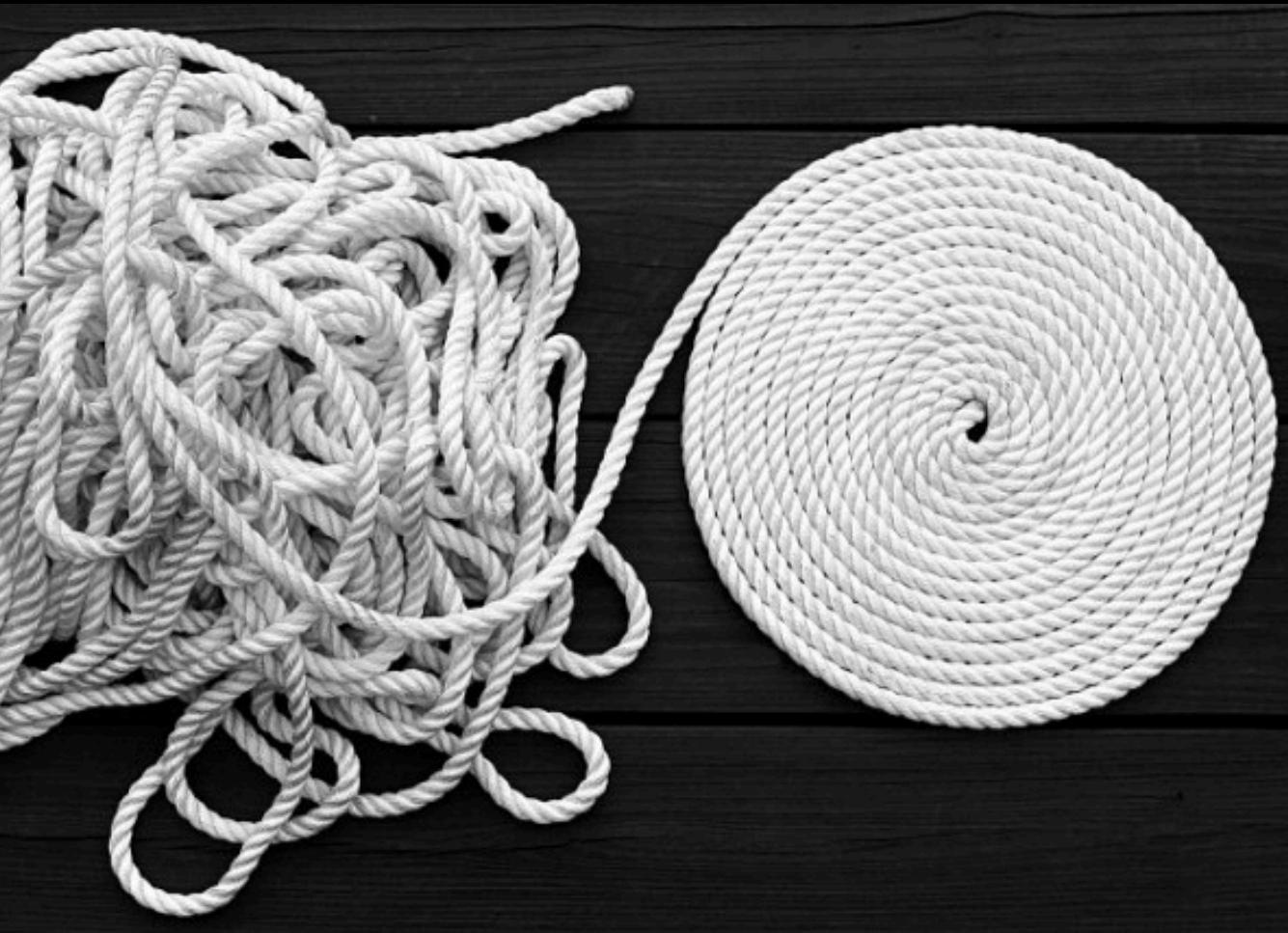


Both MSW and collective effects are complicated... depend on details of the initial fluxes, matter density profile, turbulence, shock wave propagation...

MSW is well understood,
but self-interaction effects are
still under study...

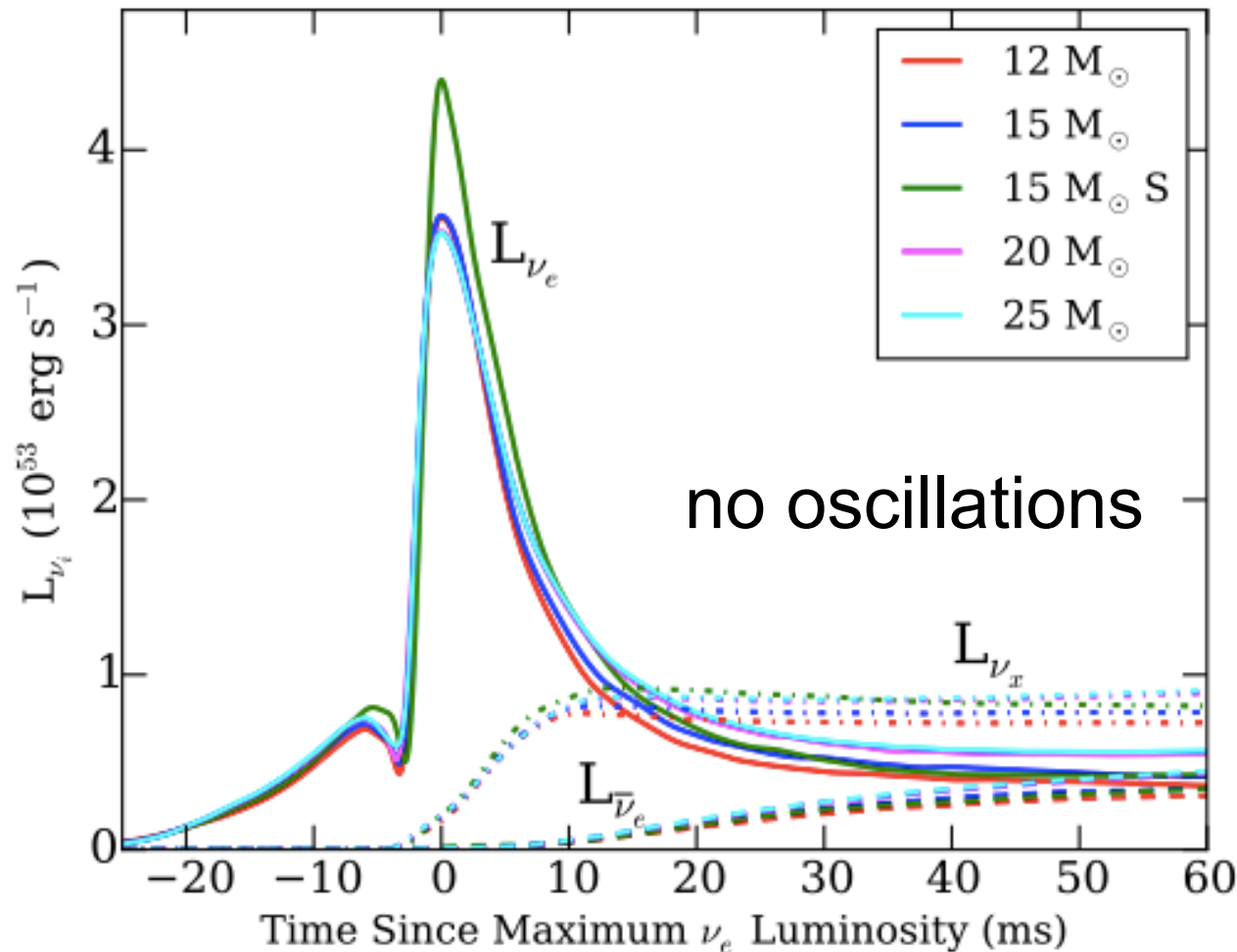


Both MSW and collective effects are complicated... depend on details of the initial fluxes, matter density profile, turbulence, shock wave propagation... MSW is well understood, but self-interaction effects are still under study...



Challenge for theorists is to find **robust, model-independent observables...** challenge for experimentalists is to understand and optimize observability

An example of a robust MO signature: **the neutronization burst**



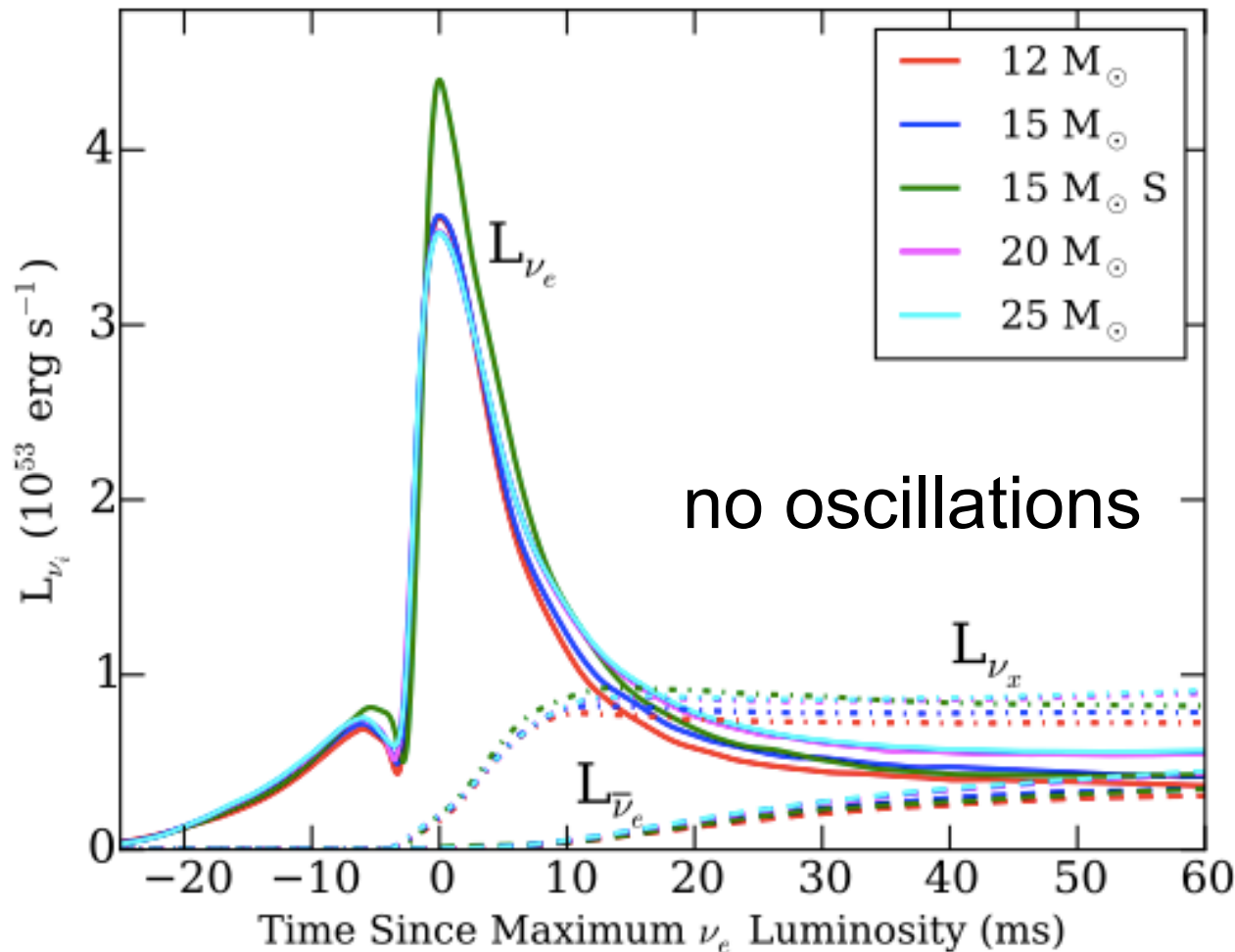
J. Wallace et al., Ap.J., 817, 182 (2016)

- almost a “standard candle”, ~independent of model
- strongly dominated by **electron flavor**
- ~no collective effects; ***MSW flavor transitions only***

$$\text{NMO: } F_{\nu_e} = F_{\nu_x}^0$$

$$\text{IMO: } F_{\nu_e} = \sin^2 \theta_{12} F_{\nu_e}^0 + \cos^2 \theta_{12} F_{\nu_x}^0$$

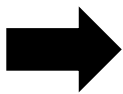
An example of a robust MO signature: **the neutronization burst**



J. Wallace et al., Ap.J., 817, 182 (2016)

~no collective effects; MSW oscillations only

$$\begin{aligned} \text{NMO: } F_{\nu_e} &= F_{\nu_x}^0 & \rightarrow & \nu_e \text{ strongly suppressed, since } \sim \text{no } \nu_x \\ \text{IMO: } F_{\nu_e} &= \sin^2 \theta_{12} F_{\nu_e}^0 + \cos^2 \theta_{12} F_{\nu_x}^0 & \rightarrow & \nu_e \text{ suppressed by } \sin^2 \theta_{12} \sim 0.31 \end{aligned}$$



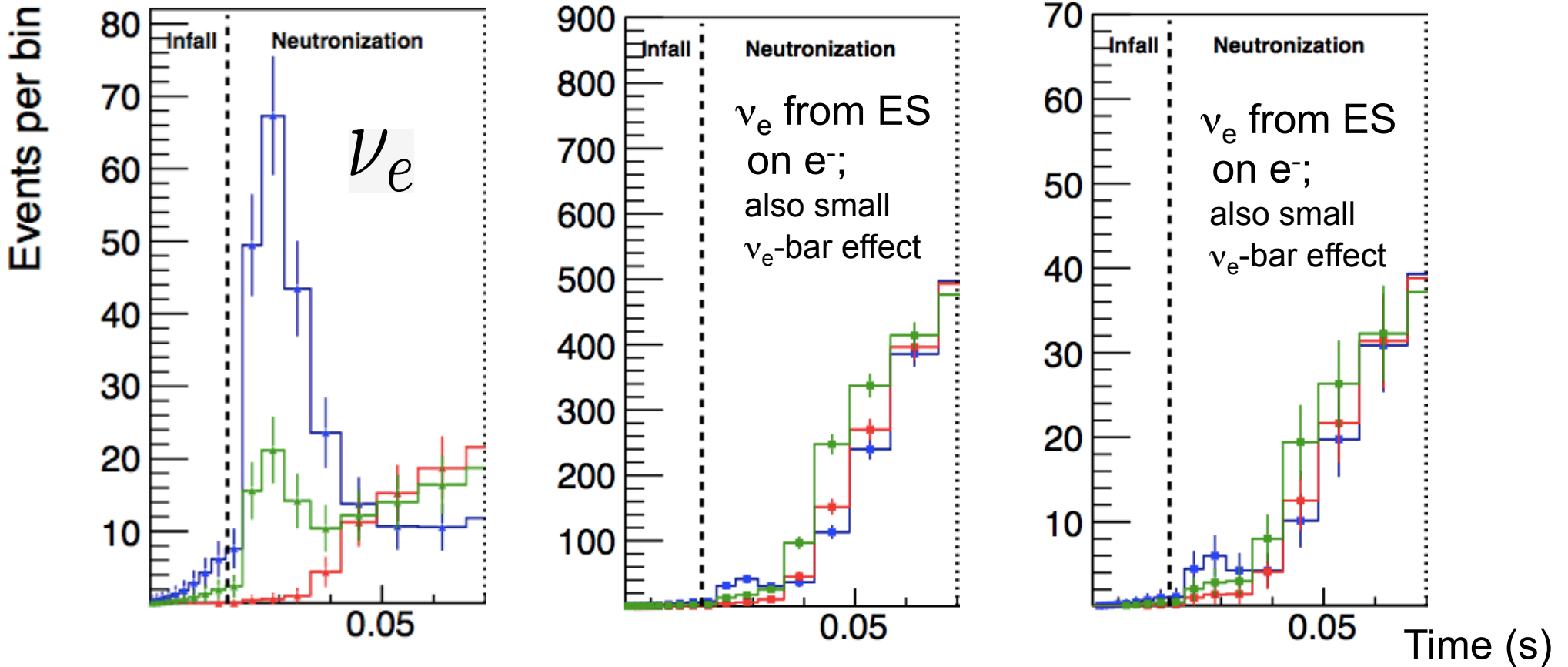
suppression for IMO,
stronger suppression for NMO

An example of a robust MO signature: the neutronization burst

40 kton LAr

374 kton water

20 kton scint



NMO: $F_{\nu_e} = F_{\nu_x}^0$

IMO: $F_{\nu_e} = \sin^2 \theta_{12} F_{\nu_e}^0 + \cos^2 \theta_{12} F_{\nu_x}^0$

NMO: $F_{\bar{\nu}_e} = \cos^2 \theta_{12} F_{\bar{\nu}_e}^0 + \sin^2 \theta_{12} F_{\bar{\nu}_x}^0$

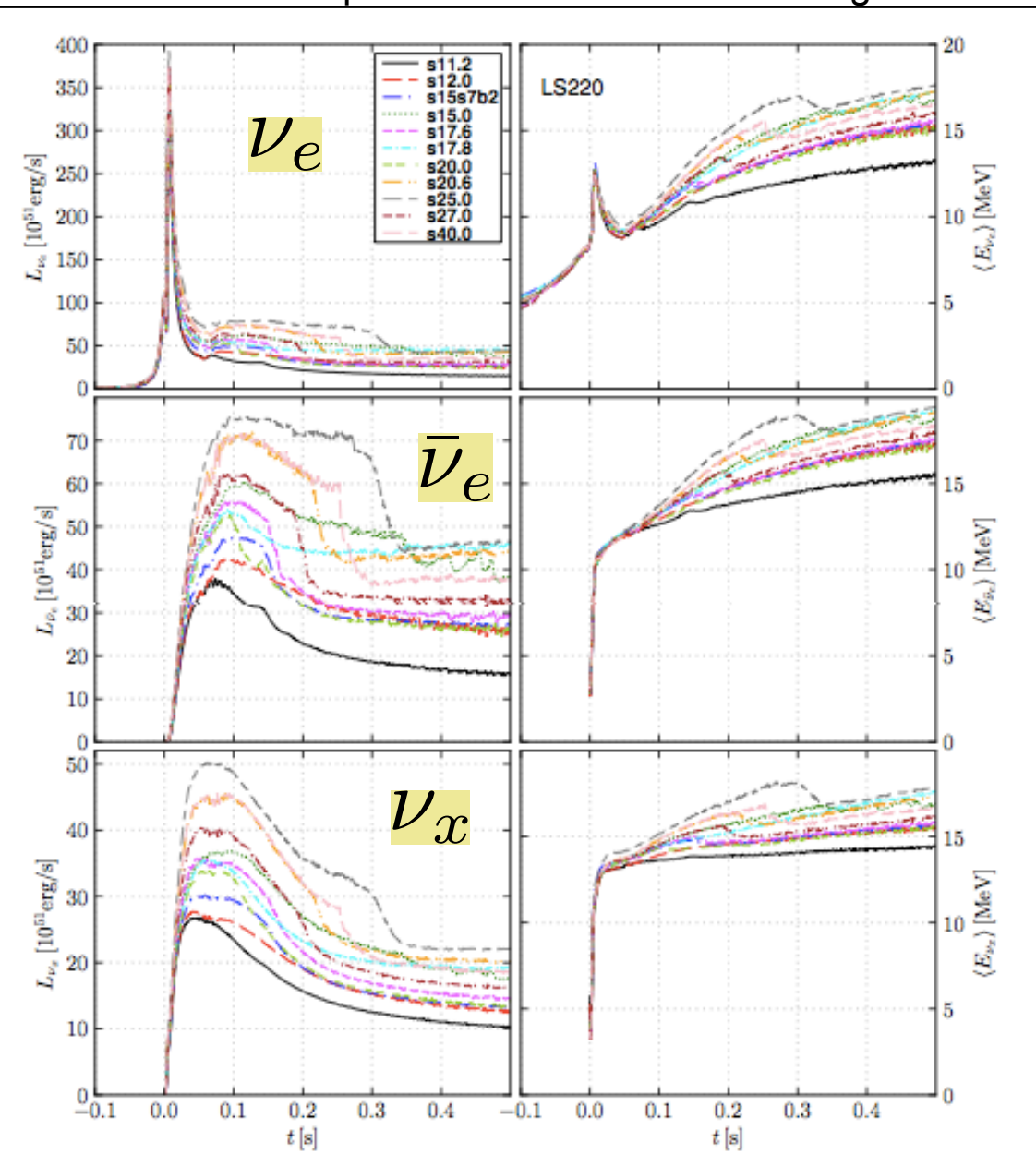
IMO: $F_{\bar{\nu}_e} = F_{\bar{\nu}_x}^0$

- ◆— No oscillations
- ◆— Normal ordering
- ◆— Inverted ordering

suppression for IMO,
stronger suppression for NMO

Another somewhat robust example: **early time profile**

Different lines represent different 1D “Garching” models



➔ Still MSW-dominated (maybe); ν_e -bar, ν_x -bar turning on and fairly consistent behavior between models

MSW for ν_e -bar :

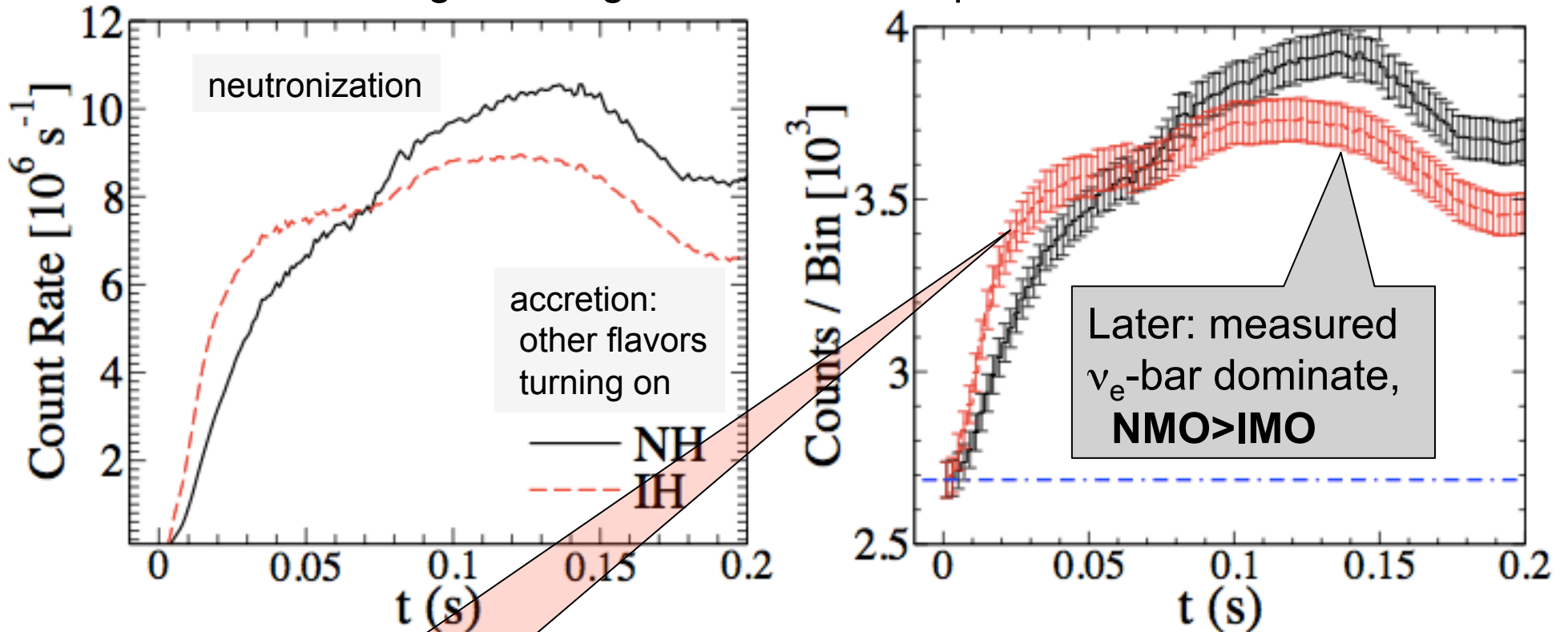
NMO: $F_{\bar{\nu}_e} = \cos^2 \theta_{12} F_{\bar{\nu}_e}^0 + \sin^2 \theta_{12} F_{\bar{\nu}_x}^0$
 IMO: $F_{\bar{\nu}_e} = F_{\bar{\nu}_x}^0$

NMO ➔ ν_e -bar mostly non-oscillated
 IMO ➔ ν_e -bar represents original ν_x -bar flux, which is lower during accretion, so will be suppressed

Another somewhat robust example: **early time profile**

Still MSW-dominated; $\bar{\nu}_e$ and $\bar{\nu}_x$ turning on

IceCube signal: integrated Cherenkov photons



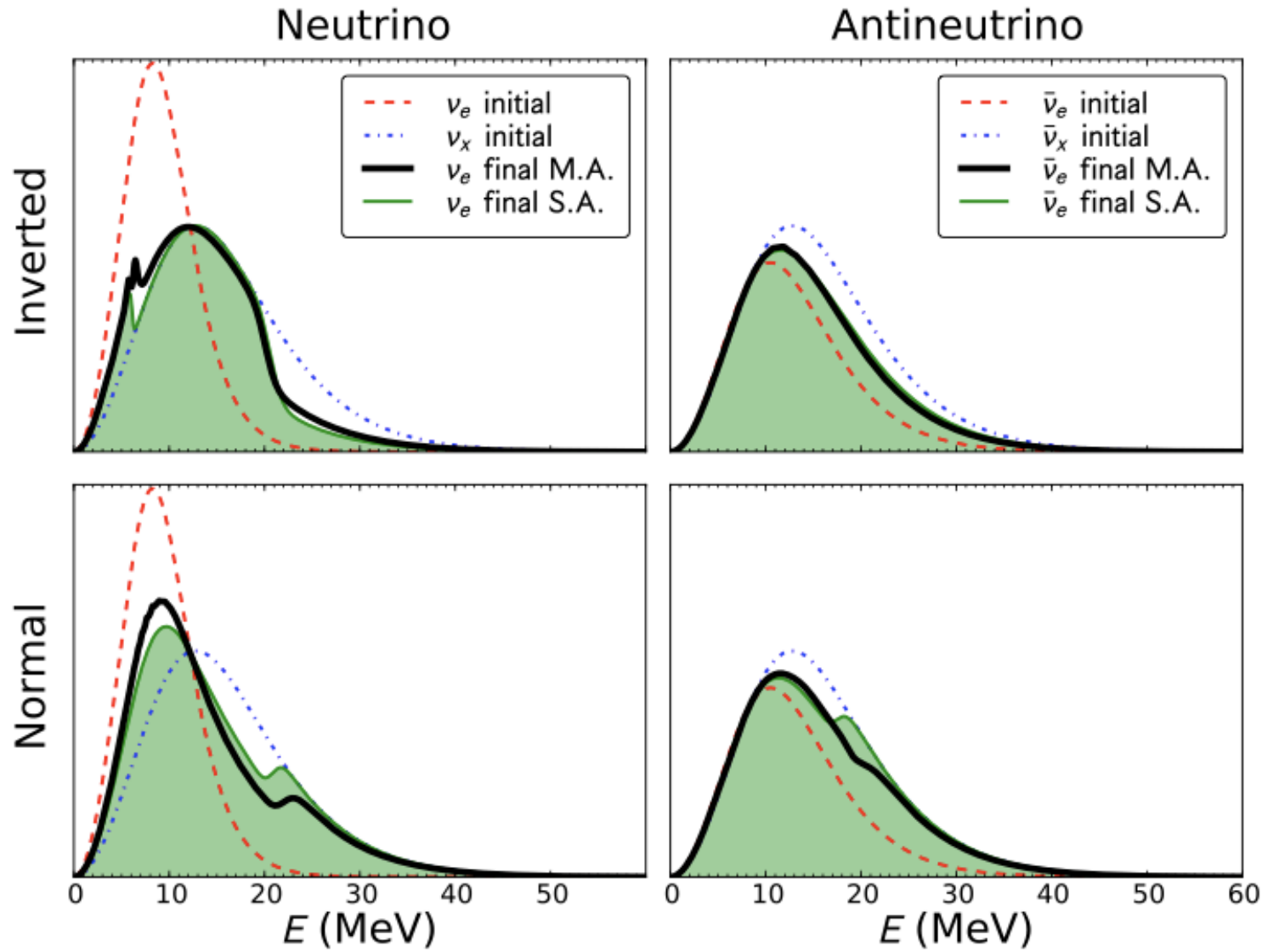
A. Mirizzi et al., Riv. Nuov. Cim., 39, 1 (2016), Serpico et al., PRD 85, 085031 (2012)

Early: measured $\bar{\nu}_e$ dominate, **IMO > NMO**

NMO \rightarrow $\bar{\nu}_e$ strongly suppressed, since \sim no $\bar{\nu}_x$
 IMO \rightarrow $\bar{\nu}_e$ suppressed by $\sin^2\theta_{12} \sim 0.3$

NMO \rightarrow $\bar{\nu}_e$ -bar mostly non-oscillated
 IMO \rightarrow $\bar{\nu}_e$ -bar represents original $\bar{\nu}_x$ -bar flux, which is lower during accretion

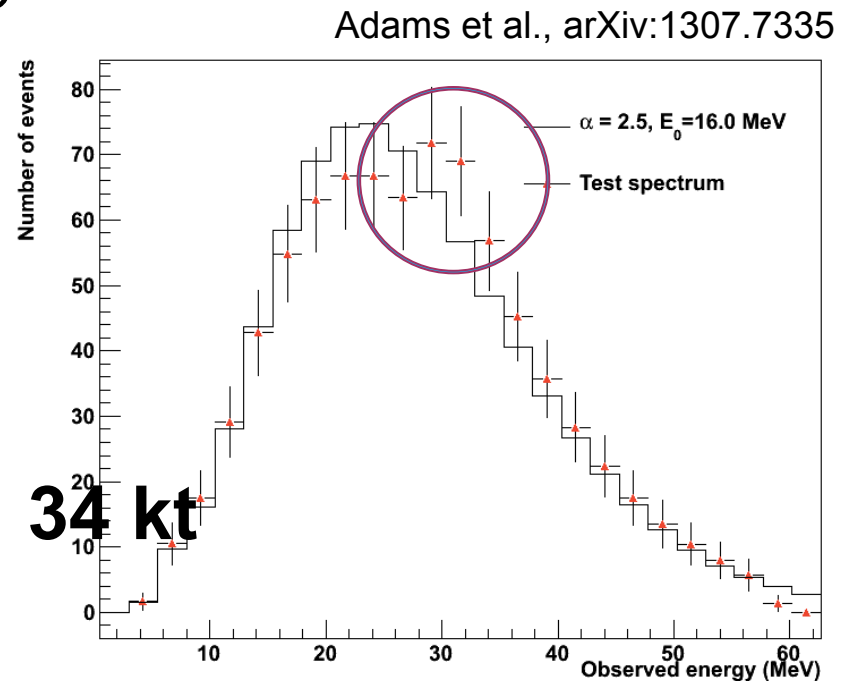
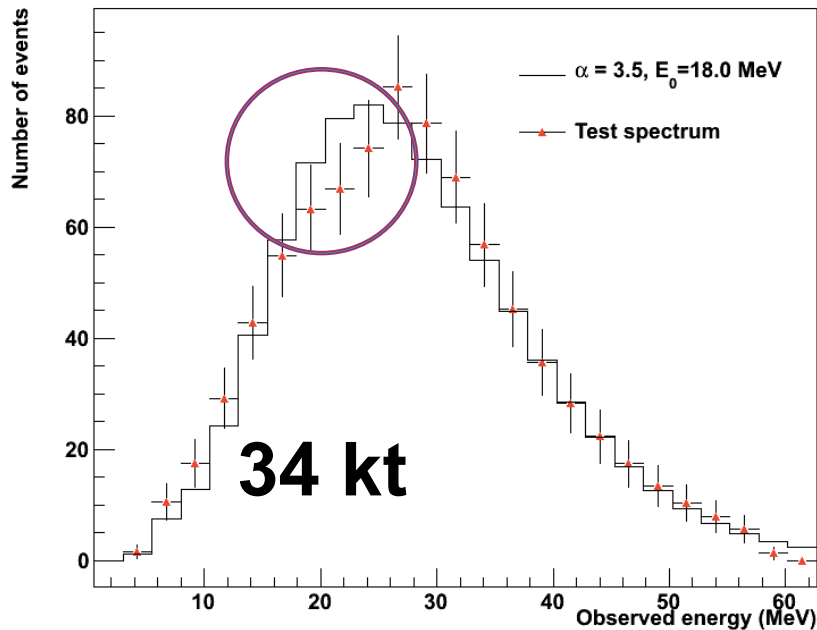
Other examples: **spectral swaps from self-interaction**



Distinctive spectral swap features depend on neutrino mass hierarchy, for neutrinos vs antineutrinos

Time-dependent shock-wave-induced effects

Snapshots at ~ 1 second intervals (1 s integration), 34-kt argon for cooling phase w/ shock, NMO

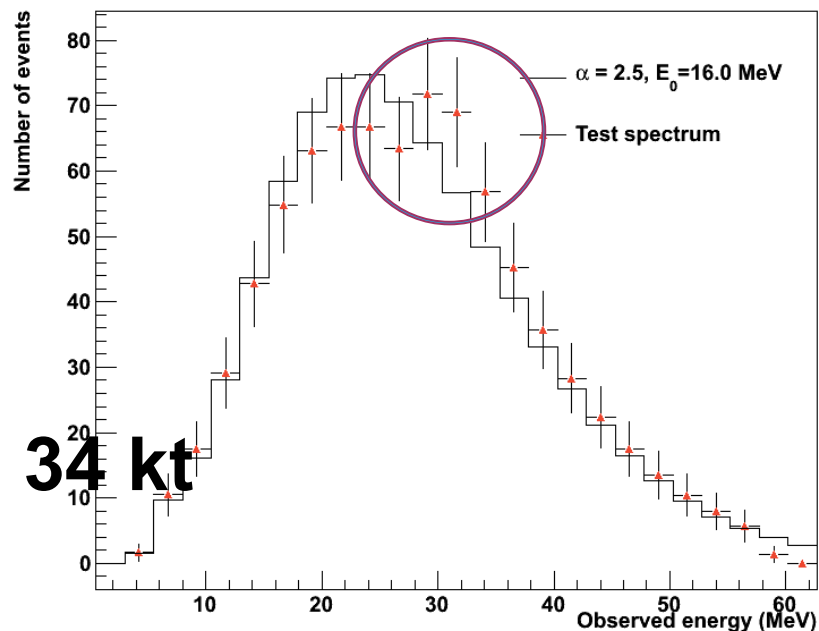
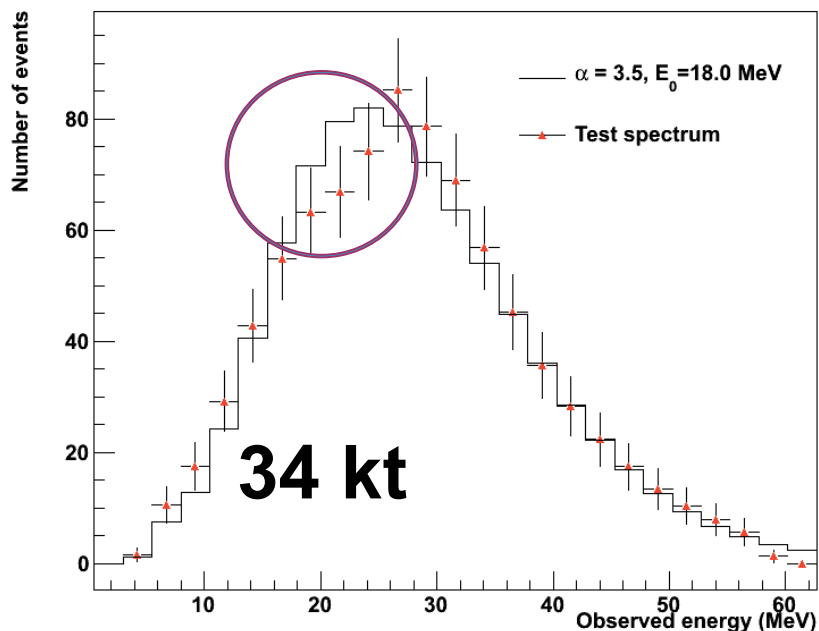


For NMO (*not* for IMO), “non-thermal” features clearly visible, and change as shock moves through the SN

10 kpc spectra from A. Friedland/JJ Cherry/H. Duan
smeared w/ SNOwGLoBES response w/collective effects
Black line: best fit to pinched thermal spectrum

Time-dependent shock-wave-induced effects

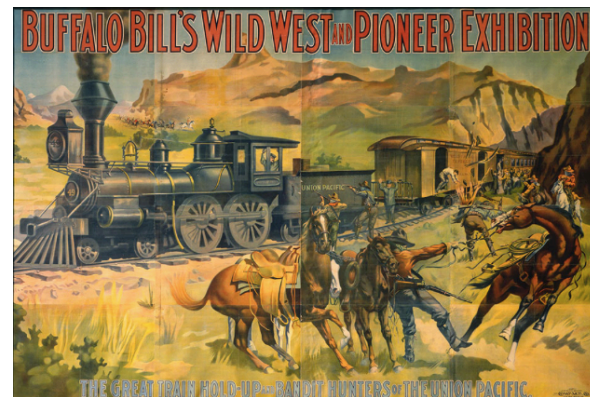
Snapshots at ~ 1 second intervals (1 s integration), 34-kt argon for cooling phase w/ shock, NMO



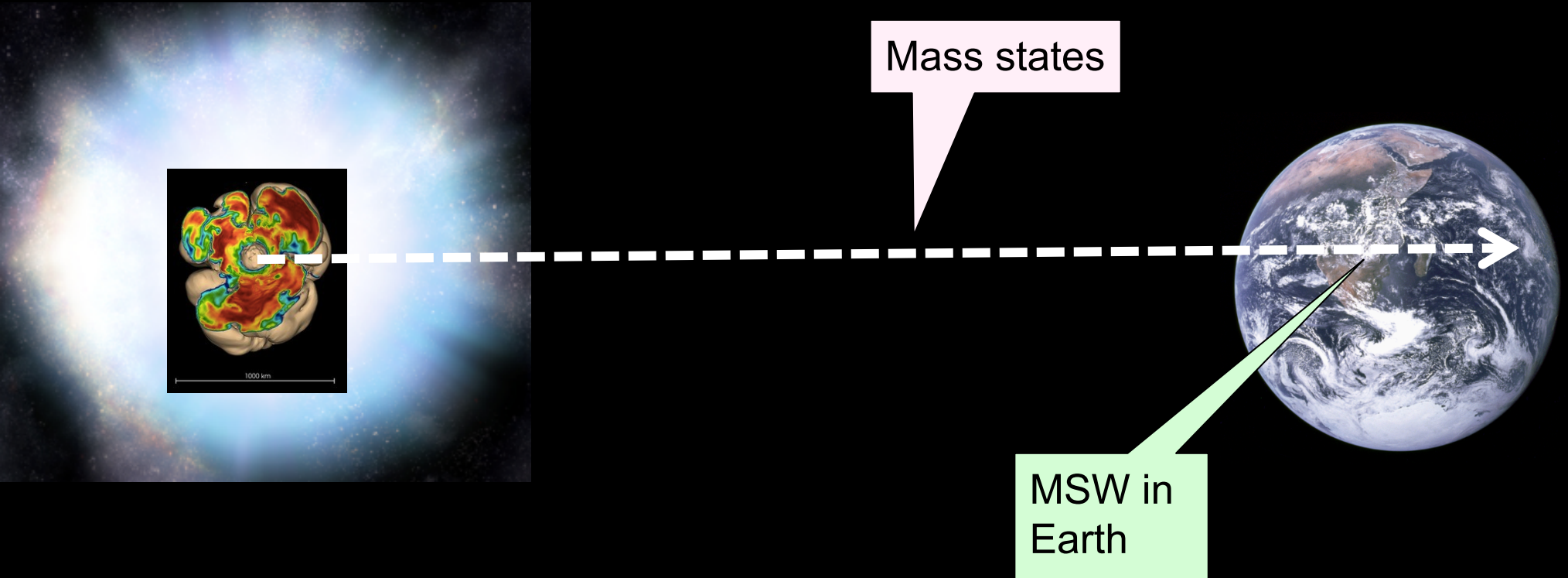
For NMO (*not* for IMO), “non-thermal” features clearly visible, and change as shock moves through the SN

10 kpc spectra from A. Friedland/JJ Cherry/H. Duan
smeared w/ SNOwGLoBES response w/collective effects
Black line: best fit to pinched thermal spectrum

Warning: collective effect signatures
are still a bit of a Wild West;
more theory work in progress

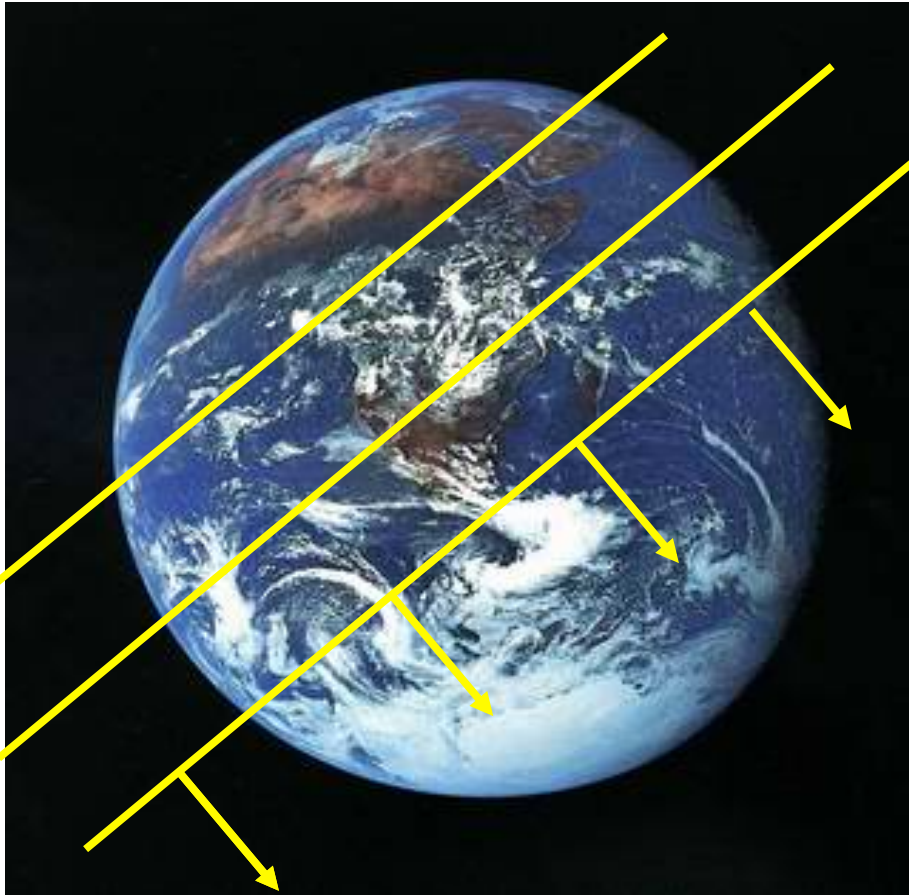


Neutrino Mixing in the Earth



- Well-understood, and supernova-model-independent!
- Alas, a small effect...
- Requires Earth shadowing

Matter-induced oscillations in the Earth



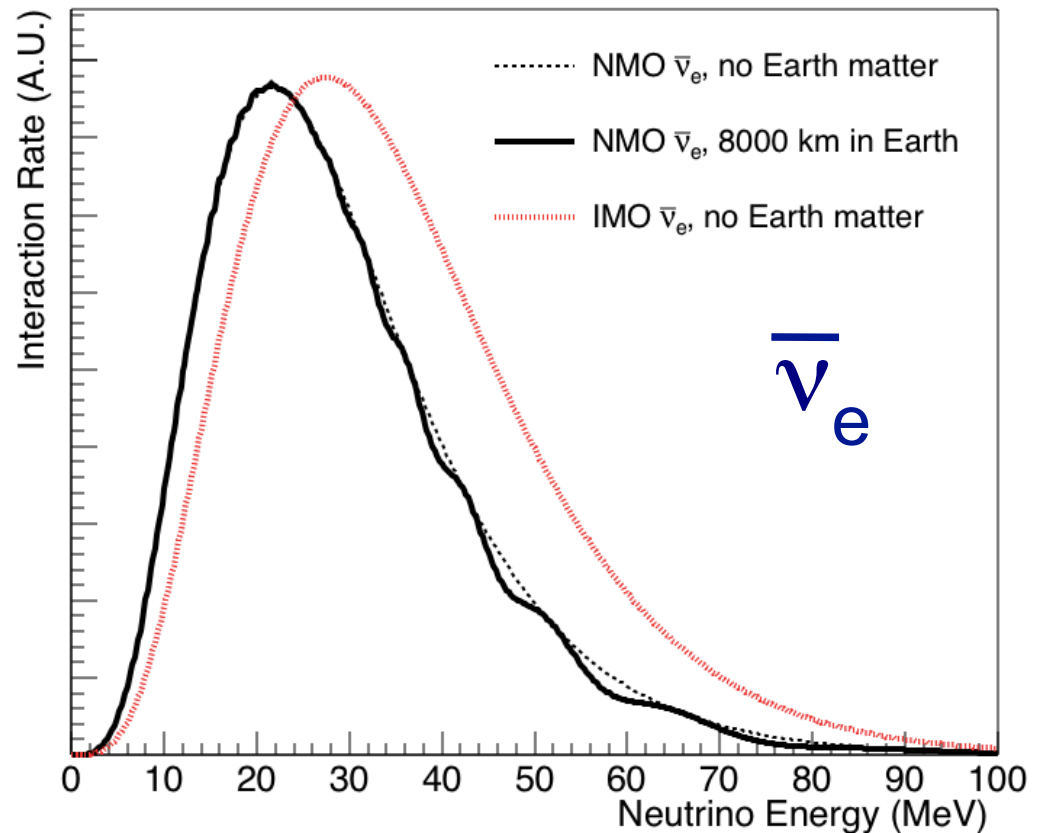
Requires very good energy resolution to resolve wiggles

$$\text{NMO: } F_{\bar{\nu}_e}^\oplus = (1 - \bar{P}_{2e})F_{\bar{\nu}_e}^0 + \bar{P}_{2e}F_{\bar{\nu}_x}^0 \quad \text{and} \quad F_{\nu_e}^\oplus = F_{\nu_x}^0$$

$$\text{IMO: } F_{\bar{\nu}_e}^\oplus = F_{\bar{\nu}_x}^0 \quad \text{and} \quad F_{\nu_e}^\oplus = (1 - P_{2e})F_{\nu_e}^0 + P_{2e}F_{\nu_x}^0$$

$$P_{2e} = \sin^2 \theta_{12} + \sin 2\theta_{12}^m \sin(2\theta_{12}^m - 2\theta_{12}) \sin^2 \left(\frac{\delta m^2 \sin 2\theta_{12}}{4E \sin 2\theta_{12}^m} L \right)$$

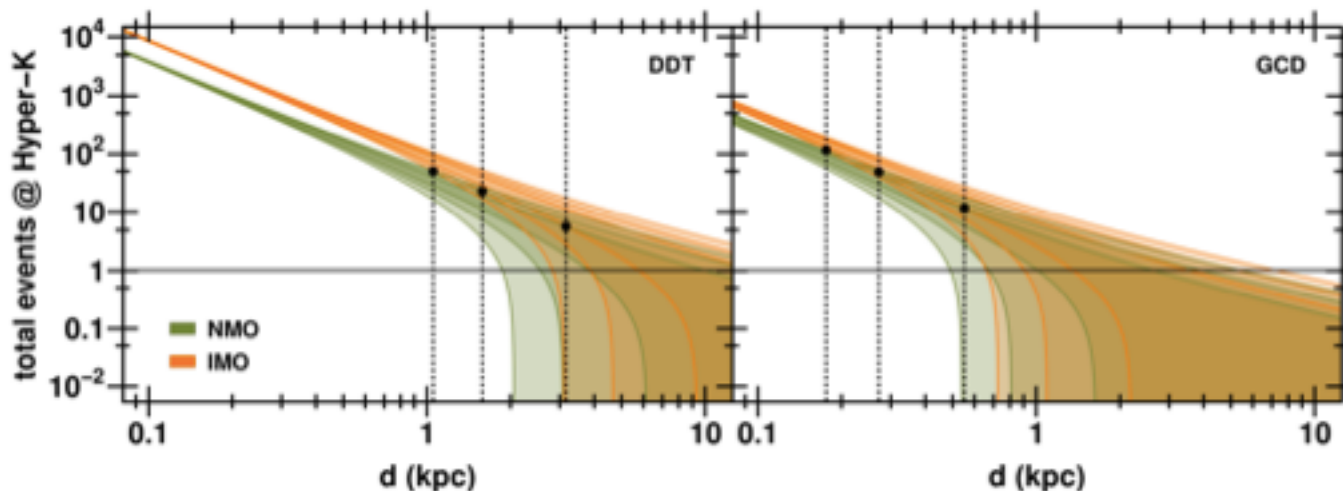
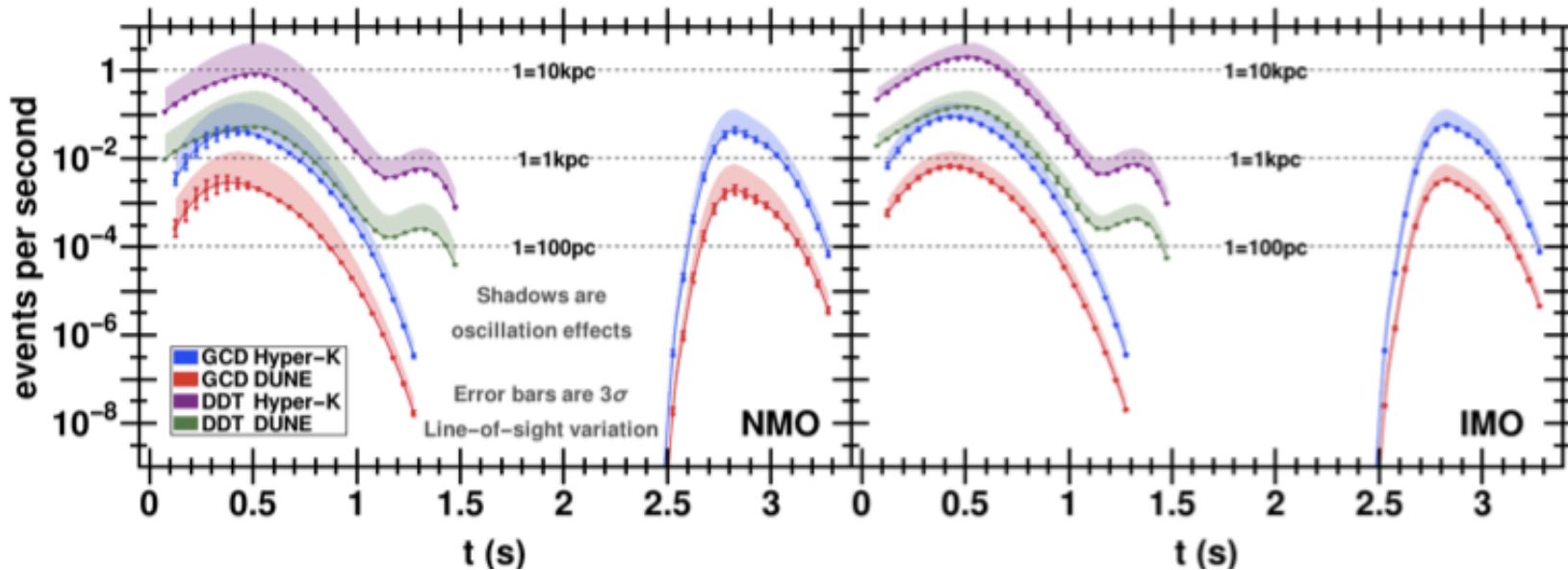
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A long shot: Type Ia Supernovae

- Thermonuclear mechanism (specific mechanism unknown)
- MSW oscillations only (ν density too low for collective)
- Very low flux, but observable within ~ 1 kpc for next-generation expts

W. Wright et al., PRD95 043006 (2017), arXiv:1609.07403

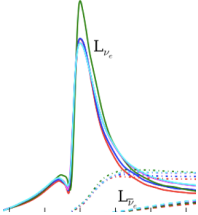


If mechanism is known,
w/HK can discriminate
MO @ 1σ for $d < 3.17$ kpc
for DDT model,
 $d < 0.55$ kpc for GCD

Need to be lucky!

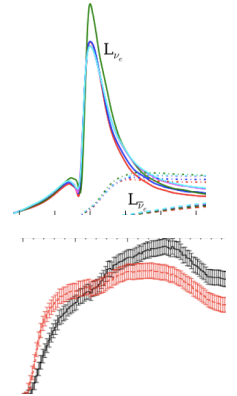
Summary Table for Supernova MO Signatures

	Normal	Inverted	Robustness	Observability
Neutronization burst	Very suppressed	Suppressed	Excellent	Good, need ν_e (HK, DUNE,...)



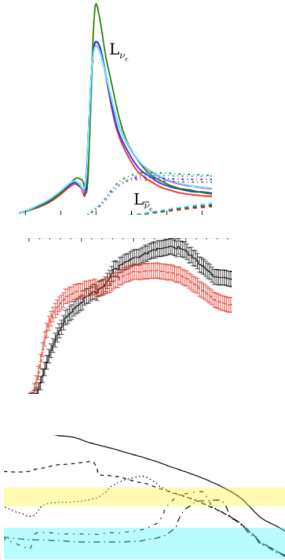
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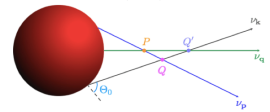
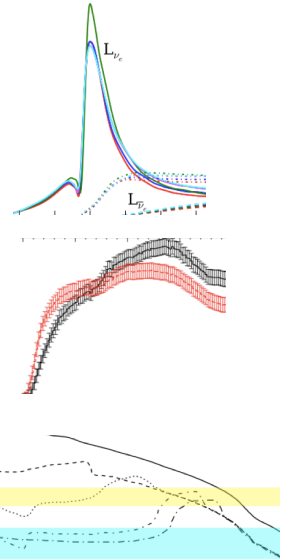
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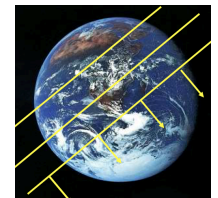
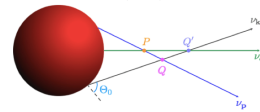
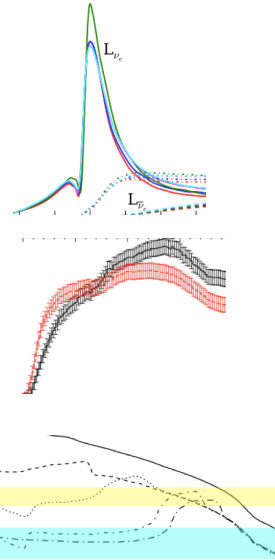
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Self-interaction effects	Multiple time- and energy-dependent signatures		Yee-haw	Good, want multiple (all...)



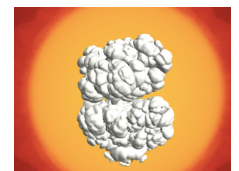
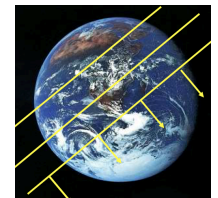
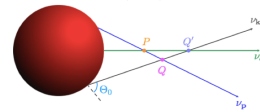
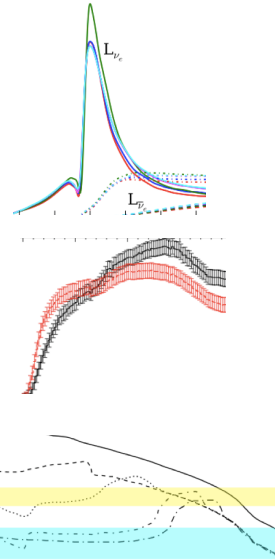
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Earth Matter	Wiggles in anti- ν_e	Wiggles in ν_e	Excellent	Hard, need energy resolution, stats (JUNO,...)



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Self-interaction effects	Multiple time- and energy-dependent signatures		Yee-haw	Good, want multiple (all...)
Earth Matter	Wiggles in anti- ν_e	Wiggles in ν_e	Excellent	Hard, need energy resolution, stats (JUNO,...)
Type Ia	Lower flux	Higher flux	Quite	Hard, need stats+luck (HK, DUNE,...)



**For supernova neutrinos, the more
the merrier!**

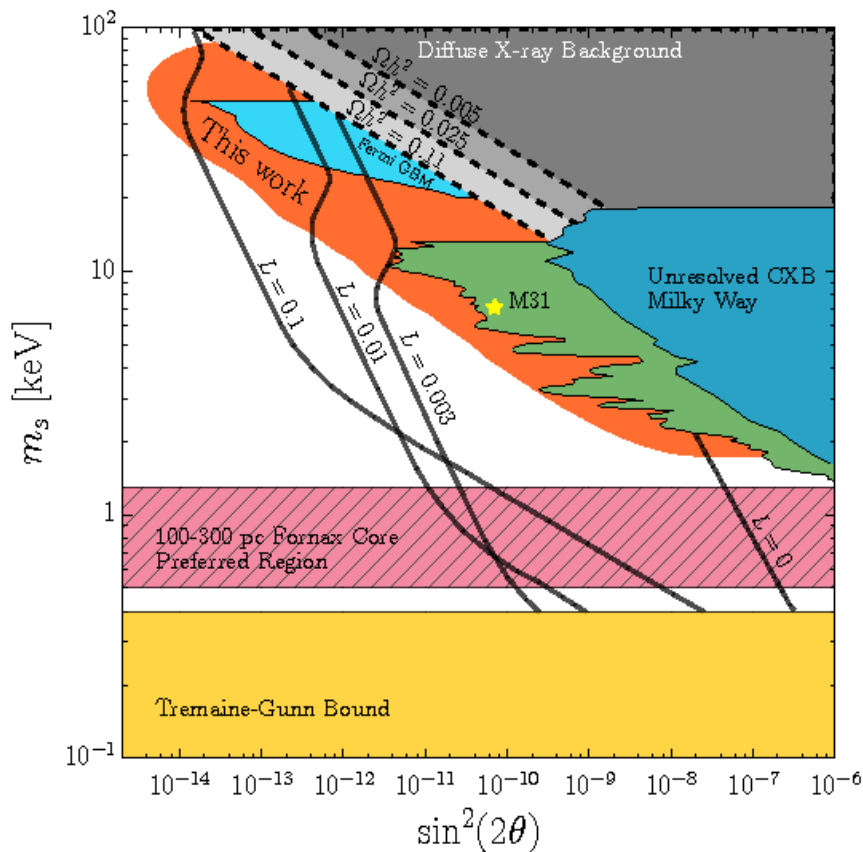


New Neutrino States or Interactions?

Sterile neutrinos, non-standard ν interactions, other exotica...



An even wilder West...
can have complicated
effects on flavor time-evolution



But some robust bounds
from the “energy leakage”
argument

Limits on \sim keV sterile neutrinos

C. A. Argüelles, et al. arXiv:1605.00654 [hep-ph]

Summary

A nearby supernova will bring information much information about neutrinos as well as core-collapse physics (in a virtuous circle)

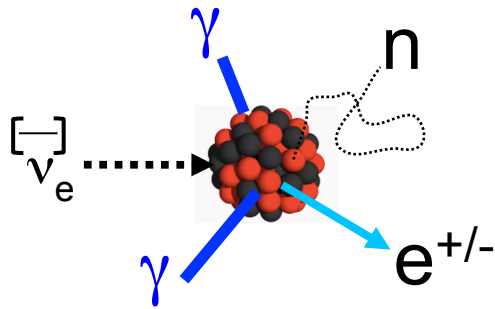
- ✧ Absolute mass: not competitive with near-future laboratory measurements, but should not be forgotten
- ✧ **Mass ordering:** several approaches, some still under theoretical study, but some robust
- ✧ Information on BSM physics also possible... maybe surprises...

Need energy, flavor, time structure...
all detectors bring something to the table

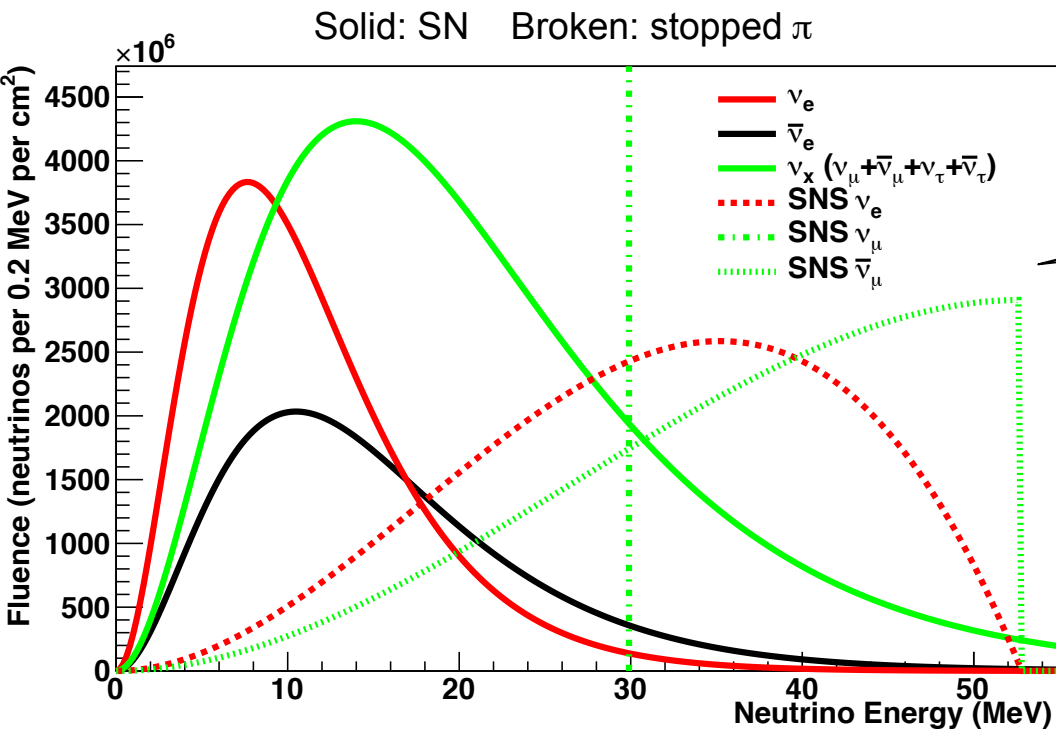


Extras/backups

`\begin{aside}`



Interactions with nuclei
(cross sections & products)
very poorly understood...
sparse theory & experiment
(*only* measurements at better
than ~50% level are for ^{12}C)

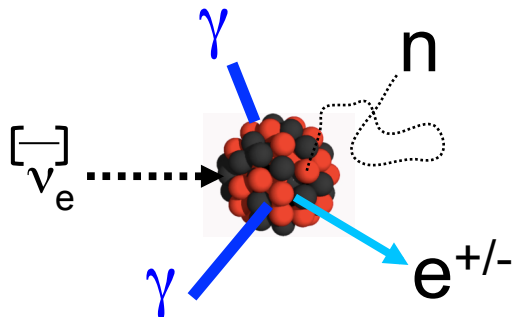
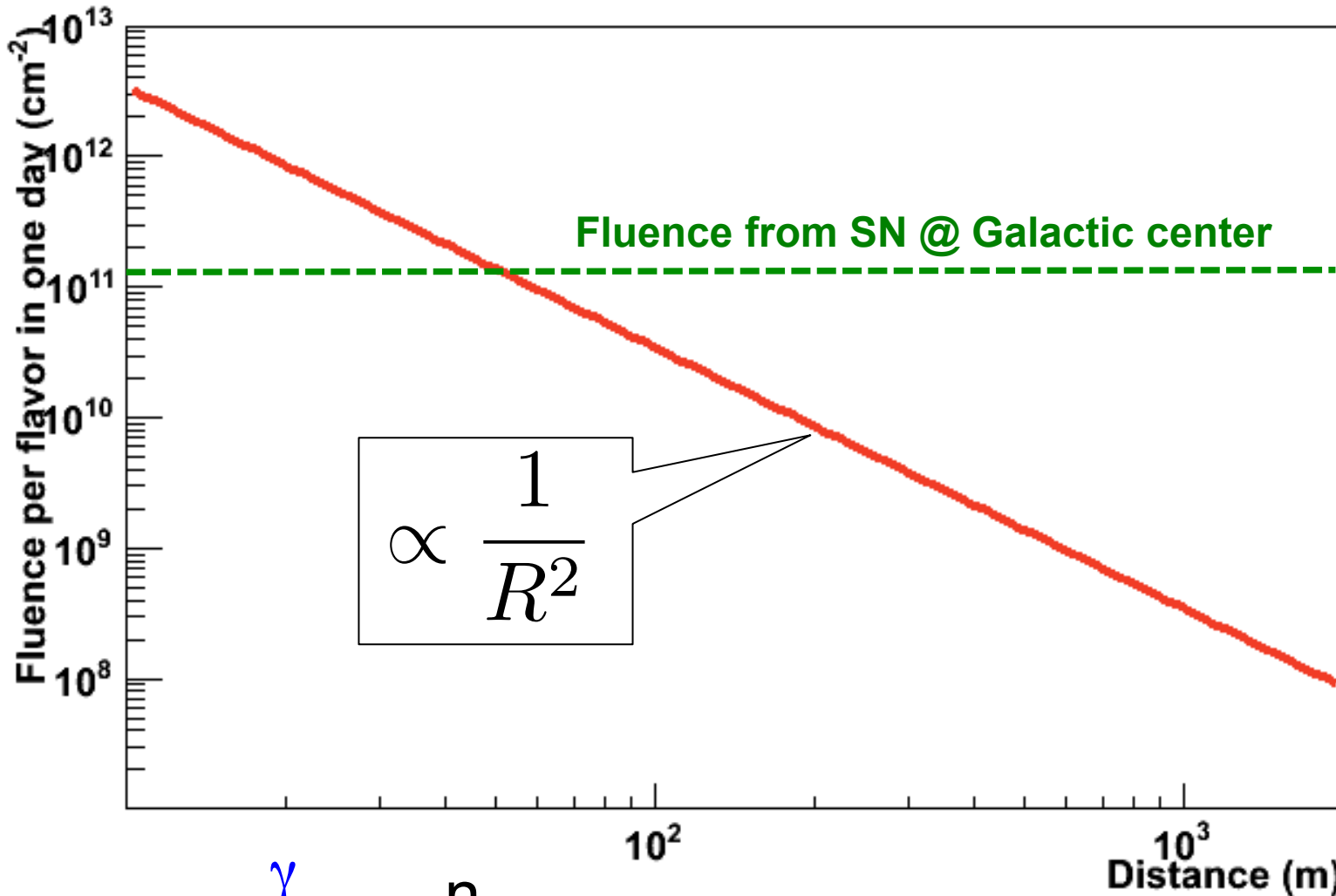


Neutrinos from pion decay at rest have spectrum overlapping with SN ν spectrum, e.g., at ORNL Spallation Neutron Source



Fluence at ~50 m from the stopped pion source amounts to ~ a supernova a day!

(or 0.2 microsupernovae per pulse, 60 Hz of pulses)



This is an excellent opportunity to study poorly understood neutrino-nucleus interactions in the supernova energy range

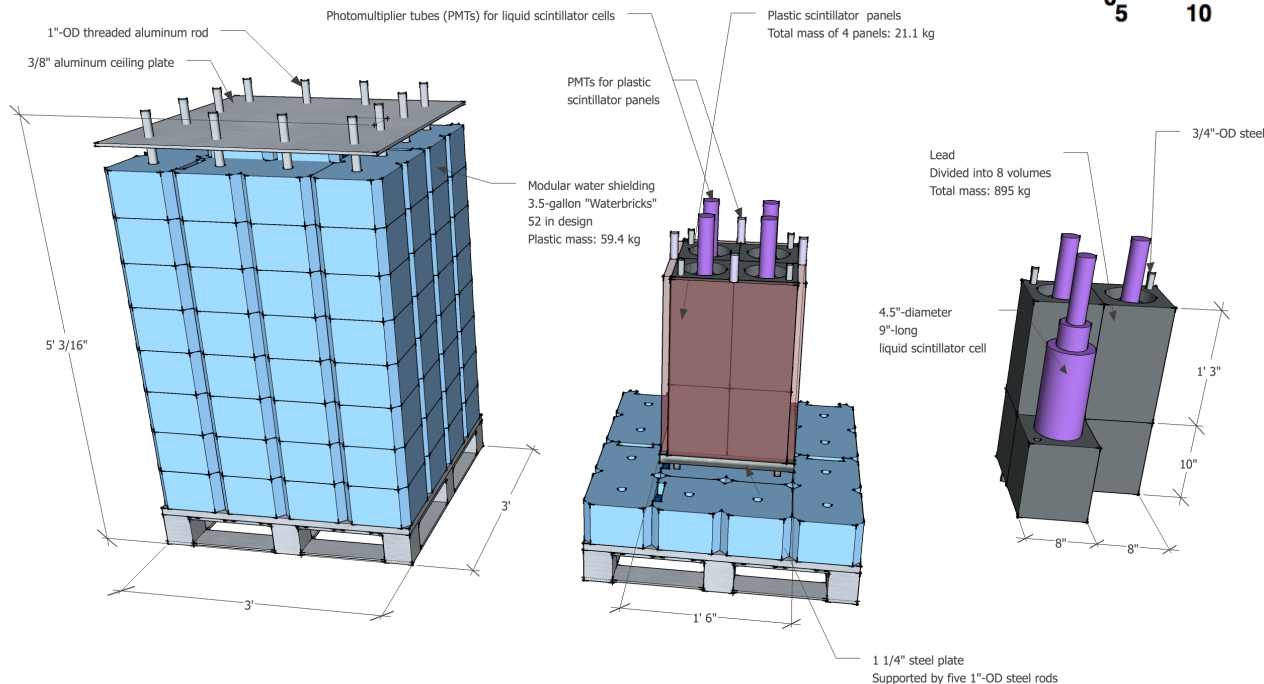
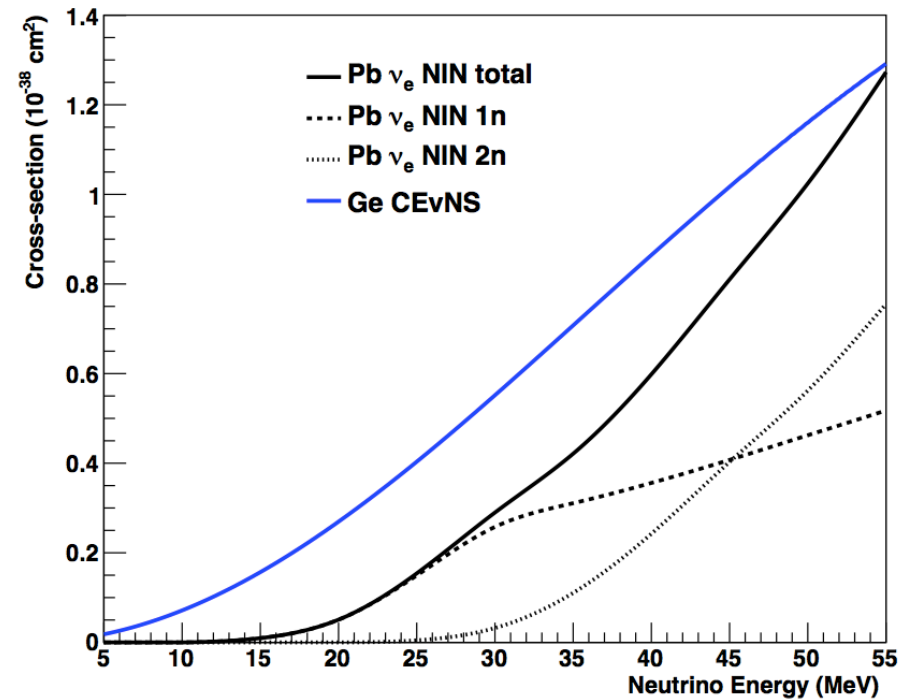
Currently measuring *neutrino-induced neutrons* in lead, (iron, copper), ...



↓
1n, 2n emission



↓
1n, 2n, γ emission



\end{aside}