

# Neutrinoless $\beta\beta$ decay matrix elements: present and future

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東京大学  
THE UNIVERSITY OF TOKYO

# Nuclear physics and neutrinoless $\beta\beta$ decay

Neutrinos, dark matter studied in experiments using nuclei

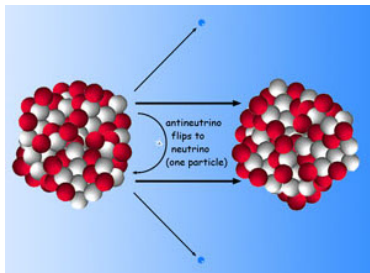
Nuclear matrix elements depend on nuclear structure crucial to anticipate reach and fully exploit experiments

$$0\nu\beta\beta \text{ decay: } \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto |M^{0\nu\beta\beta}|^2 m_{\beta\beta}^2$$

$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{d\mathbf{q}^2} \propto \left| \sum_i c_i \zeta_i \mathcal{F}_i \right|^2$$

$M^{0\nu\beta\beta}$ : Nuclear matrix element

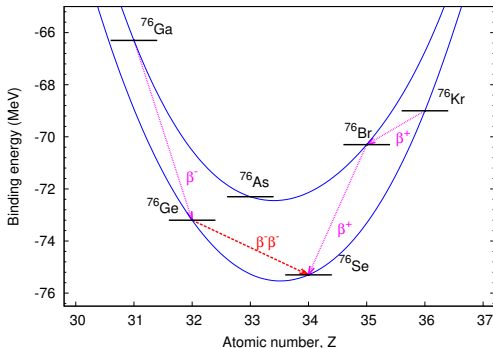
$\mathcal{F}_i$ : Nuclear structure factor



# Neutrinoless $\beta\beta$ decay

Lepton-number violation, Majorana nature of neutrinos

Second order process only observable in rare cases with  $\beta$ -decay energetically forbidden or hindered by  $\Delta J$

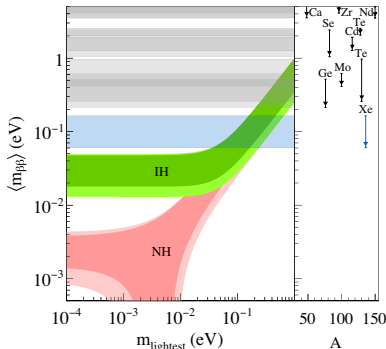
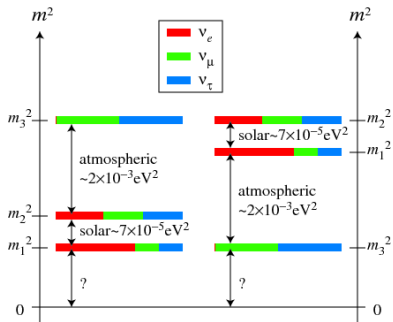


Best limit:  $^{76}\text{Ge}$  (GERDA),  $^{130}\text{Te}$  (CUORE),  $^{136}\text{Xe}$  (EXO, KamLAND-Zen)

# Next generation experiments: inverted hierarchy

The decay lifetime is  $T_{1/2}^{0\nu\beta\beta} (0^+ \rightarrow 0^+)^{-1} = G_{01} |M^{0\nu\beta\beta}|^2 m_{\beta\beta}^2$

sensitive to absolute neutrino masses,  $m_{\beta\beta} = |\sum U_{ek}^2 m_k|$ , and hierarchy



Matrix elements needed to make sure KamLAND-Zen, PRL117 082503(2016)  
 next generation ton-scale experiments fully explore "inverted hierarchy"

# Outline

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Present status of  $0\nu\beta\beta$  decay nuclear matrix elements

Future prospects for  $0\nu\beta\beta$  nuclear matrix element calculations

Can nuclear structure experiments help with  $0\nu\beta\beta$  decay?

# Calculating nuclear matrix elements

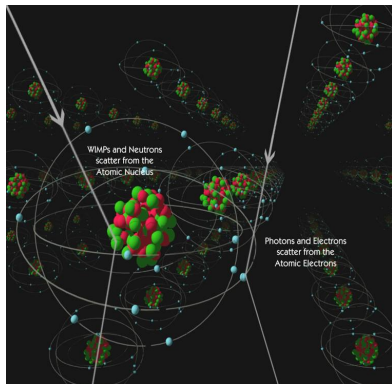
Nuclear matrix elements needed to study fundamental symmetries

$$\langle \text{Final} | \mathcal{L}_{\text{leptons-nucleons}} | \text{Initial} \rangle = \langle \text{Final} | \int dx j^\mu(x) J_\mu(x) | \text{Initial} \rangle$$

- Nuclear structure calculation of the initial and final states:

Shell model Retamosa, Poves, JM, Horoi...  
Energy-density functional Rodríguez, Yao...  
QRPA Vogel, Faessler, Šimkovic, Suhonen...  
Interacting boson model Iachello, Barea...  
Ab initio many-body methods  
Green's Function MC, Coupled-cluster, IM-SRG...

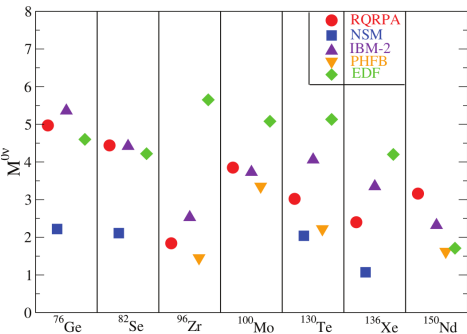
- Lepton-nucleus interaction:  
Study hadronic current in nucleus:  
phenomenological approaches,  
effective theory of QCD



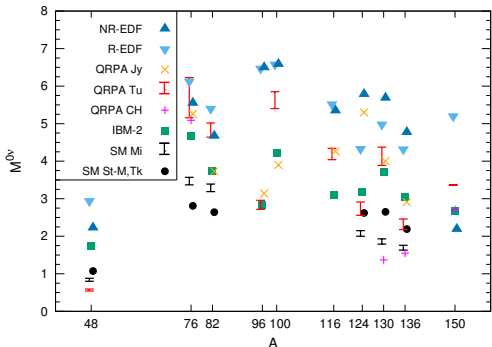
CDMS Collaboration

# $0\nu\beta\beta$ nuclear matrix elements: last 5 years

## Comparison of nuclear matrix element calculations: 2012 vs 2017



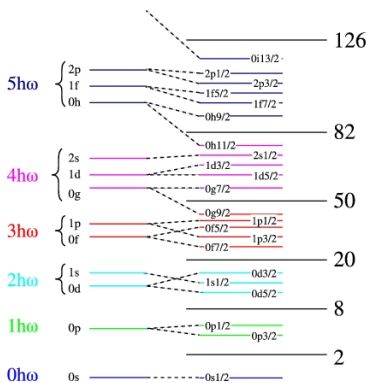
Vogel, J. Phys. G 39 124002 (2012)



Engel, JM, Rep.Prog.Phys. 80 046301(2017)

What have we learned in the last 5 years?

# Configuration space



Nuclear shell model configuration space only keep essential degrees of freedom

- High-energy orbits: always empty
- Configuration space: where many-body problem is solved
- Inert core: always filled

$$H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{eff}|\Psi\rangle_{eff} = E|\Psi\rangle_{eff}$$

$$|\Psi\rangle_{eff} = \sum_{\alpha} c_{\alpha} |\phi_{\alpha}\rangle, \quad |\phi_{\alpha}\rangle = a_{i_1}^{+} a_{i_2}^{+} \dots a_{i_A}^{+} |0\rangle$$

Shell model codes (1 major oscillator shell)  
 $\sim 10^{10}$  Slater dets. [Caurier et al. RMP77 \(2005\)](#)

QRPA calculations suggest  
 larger spaces ( $\gtrsim 2$  major shells) needed

Dimension  $\sim$

$$\binom{(p+1)(p+2)}{N} \binom{(p+1)(p+2)}{Z}$$

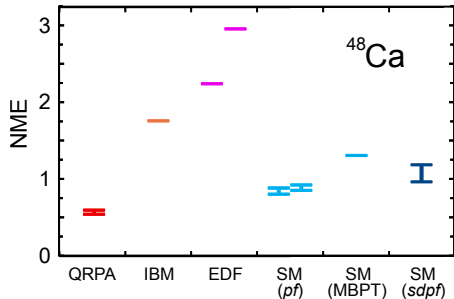
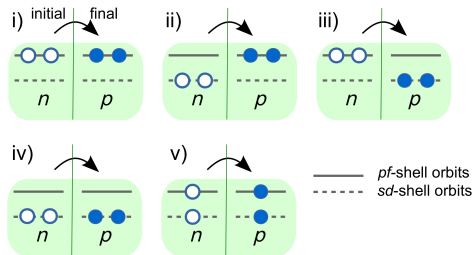


# Shell model configuration space: two shells

For  $^{48}\text{Ca}$  enlarge configuration space from  $pf$  to  $sdpf$

4 to 7 orbitals, dimension  $10^5$  to  $10^9$   
increases matrix elements  
but only moderately 30%

Iwata et al. PRL116 112502 (2016)

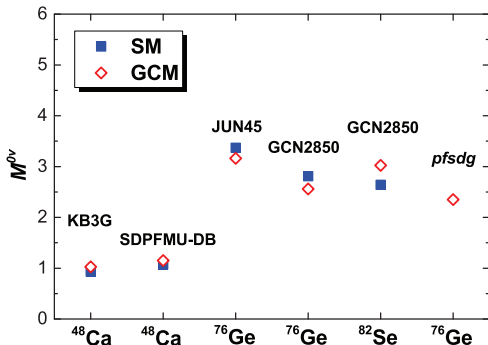


Contributions dominated by pairing 2 particle – 2 hole excitations enhance the  $\beta\beta$  matrix element,

Contributions dominated by 1 particle – 1 hole excitations suppress the  $\beta\beta$  matrix element

## $^{76}\text{Ge}$ matrix element in two shells

Large configuration space calculations in 2 major oscillator shells  
Include all relevant correlations: isovector/isoscalar pairing, deformation  
Many-body approach: generating coordinate method (GCM)



GCM approximates shell model calculation

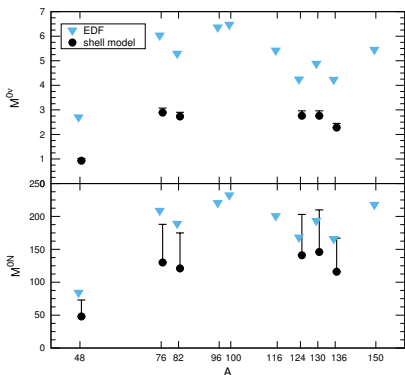
Degrees of freedom, or generating coordinates, validated against exact shell model in small configuration space

Jiao et al. PRC96 054310 (2017)

$^{76}\text{Ge}$  nuclear matrix element in 2 major shells  
very similar to shell model nuclear matrix element in 1 major shell

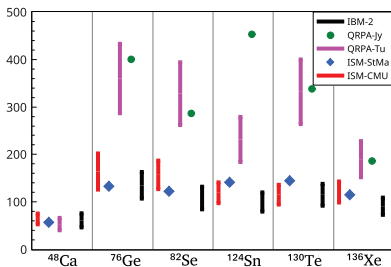
# Heavy-neutrino exchange nuclear matrix elements

Contrary to light-neutrino-exchange, for heavy-neutrino-exchange decay shell model, IBM, and EDF matrix elements agree reasonably!



Song et al. PRC95 024305 (2017)

JM, JPG in print



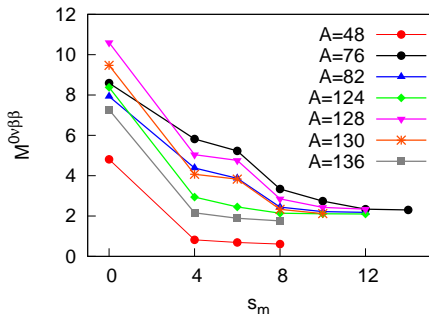
Neacsu et al. PRC100 052503 (2015)

Suggests differences in treating longer-range nuclear correlations dominant in light-neutrino exchange

# Pairing correlations and $0\nu\beta\beta$ decay

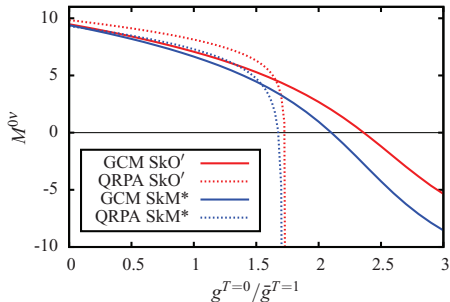
$0\nu\beta\beta$  decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



Caurier et al. PRL100 052503 (2008)

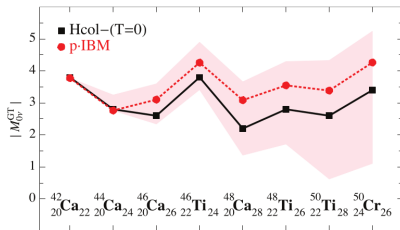
Addition of isoscalar pairing reduces matrix element value



Hinohara, Engel PRC90 031301 (2014)

Related to approximate  $SU(4)$  symmetry of the  $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$  operator

# IBM matrix elements with proton-neutron pairing

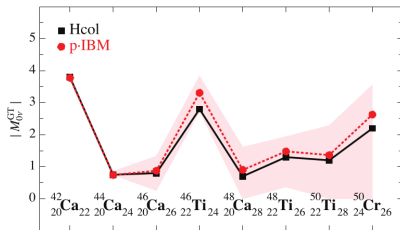


Energy-density functional (EDF) theory and interacting boson model (IBM) calculated nuclear matrix elements do not include explicitly proton-neutron pairing correlations

This effect (partially) accounted for by other degrees of freedom present in these approaches

Include  $p$ -boson ( $L = 1$ ) to IBM in addition to  $s$  and  $d$  bosons ( $L = 0, 2$ )

First IBM results in calcium region suggest nuclear matrix elements could be somewhat reduced



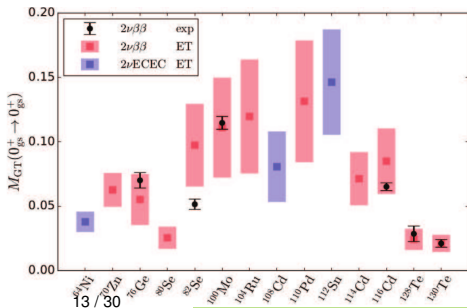
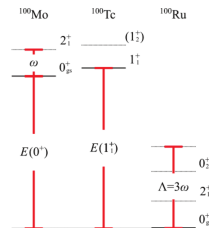
van Isacker et al. arXiv:1708.05925

# Matrix elements: theoretical uncertainty

Systematic uncertainty hard to estimate for phenomenological matrix elements

Effective theory for  $\beta\beta$  decay: spherical core coupled to one nucleon

Couplings adjusted to experimental data, uncertainty given by effective theory (breakdown scale, systematic expansion)



Take  $\beta$  decay data to predict  $2\nu\beta\beta$

$$M^{2\nu\beta\beta} =$$

$$\sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

Good agreement with large errors (leading-order calculations)

Coello-Pérez, JM, Schwenk, arXiv:1708.06140

# Outline

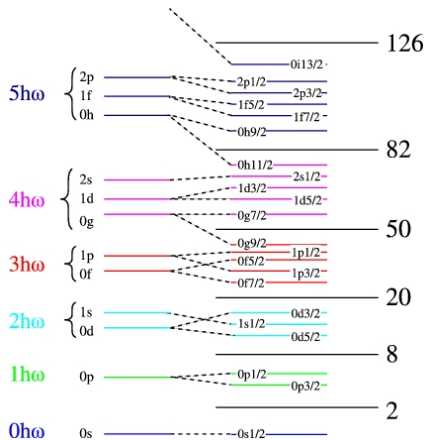
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Present status of  $0\nu\beta\beta$  decay nuclear matrix elements

Future prospects for  $0\nu\beta\beta$  nuclear matrix element calculations

Can nuclear structure experiments help with  $0\nu\beta\beta$  decay?

# Ab initio methods: No core shell model



No core shell model

Many-body wave function  
linear combination of  
Slater Determinants  
from single particle states in the basis  
(3D harmonic oscillator)

$$|i\rangle = |n_i l_i j_i m_j m_t\rangle$$

$$|\phi_\alpha\rangle = a_{i1}^+ a_{j2}^+ \dots a_{kA}^+ |0\rangle$$

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\phi_{\alpha}\rangle$$

$$H|\Psi\rangle = E|\Psi\rangle$$

$$\text{Dim} \sim \binom{(\rho+1)(\rho+2)_{\nu}}{N} \binom{(\rho+1)(\rho+2)_{\pi}}{Z}$$

Dimensions increase  
combinatorially...



# Coupled Cluster, In-Medium SRG

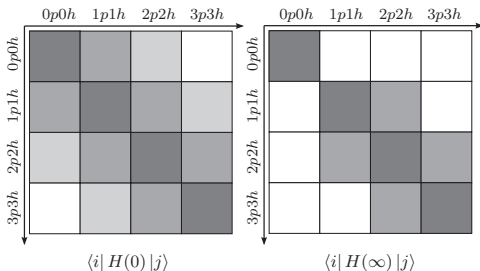
Coupled Cluster method: operators (correlations) acting on reference impose no particle-hole excitations present in the reference state

$$|\Psi\rangle = e^{-(T_1+T_2+T_3\cdots)} |\Phi\rangle$$

$$\text{with } T_1 = \sum_{\alpha, \bar{\alpha}} t_{\alpha}^{\bar{\alpha}} \{a_{\bar{\alpha}}^{\dagger}, a_{\alpha}\}, T_2 = \sum_{\alpha\beta, \bar{\alpha}\bar{\beta}} t_{\alpha\beta}^{\bar{\alpha}\bar{\beta}} \{a_{\bar{\alpha}}^{\dagger} a_{\bar{\beta}}^{\dagger}, a_{\alpha} a_{\beta}\}, \dots$$

$$\text{solve } \langle \Phi_{\alpha}^{\bar{\alpha}} | e^{\sum T_i} H e^{-\sum T_i} |\Phi\rangle = 0, \langle \Phi_{\alpha\beta}^{\bar{\alpha}\bar{\beta}} | e^{\sum T_i} H e^{-\sum T_i} |\Phi\rangle = 0$$

In-medium similarity  
renormalization group method:  
apply a similarity (unitary)  
transformation  
to decouple reference state  
from particle-hole excitations



# Ab initio many-body methods

Oxygen dripline using chiral NN+3N forces correctly reproduced  
ab-initio calculations treating explicitly all nucleons  
excellent agreement between different approaches

No-core shell model  
(Importance-truncated)

In-medium SRG

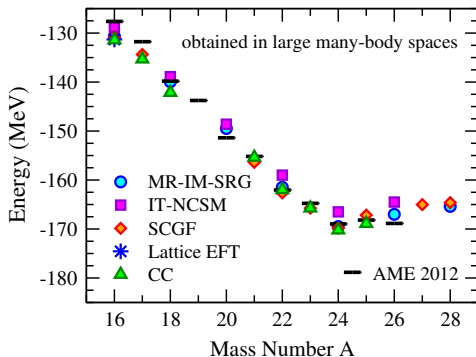
Hergert et al. PRL110 242501(2013)

Self-consistent Green's  
function

Cipollone et al. PRL111 062501(2013)

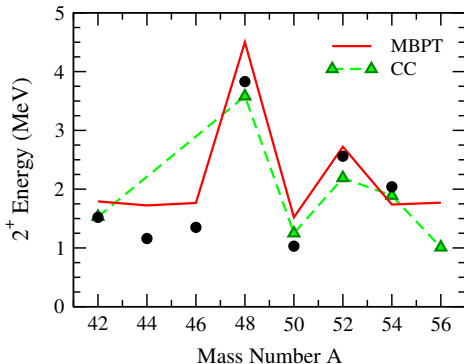
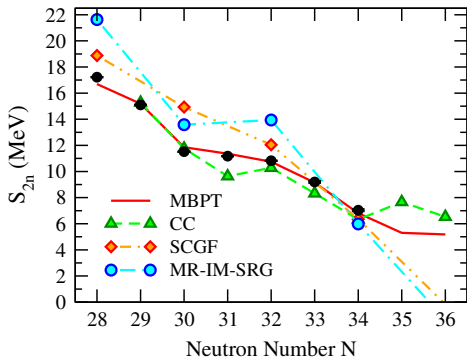
Coupled-clusters

Jansen et al. PRL113 142502(2014)

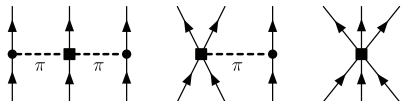


# Calcium isotopes with NN+3N forces

Calculations with NN+3N forces predict shell closures at  $^{52}\text{Ca}$ ,  $^{54}\text{Ca}$



$^{51-54}\text{Ca}$  masses [TRIUMF/ISOLDE]  
 $^{54}\text{Ca}$   $2_1^+$  excitation energy [RIBF,RIKEN]



Hebeler et al. ARNPS 65 457 (2015)

# Ab initio $0\nu\beta\beta$ decay matrix elements?

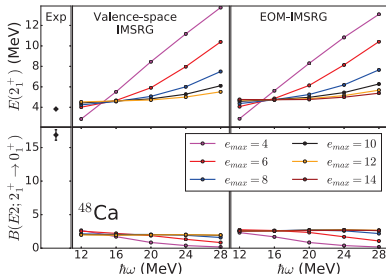
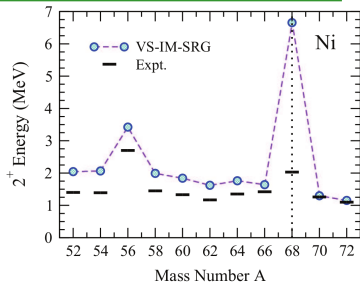
Nuclei up to  $A \sim 70$   
 explored with ab initio approaches  
 Limited by good chiral nuclear force

Challenge for ab initio  $0\nu\beta\beta$  decay:  
 (unitary) transformation:  $H' = U^\dagger H U$   
 applied to operators:  $O' = U^\dagger O U$   
 First electromagnetic transition results  
 $\Rightarrow 0\nu\beta\beta$  decay matrix elements next

$^{48}\text{Ca}$  ab initio  $0\nu\beta\beta$  decay  
 nuclear matrix element  
 ready very soon: stay tuned!

Simonis et al. PRC96 014303 (2017)

Parzuchowski et al. PRC96 034324 (2017)

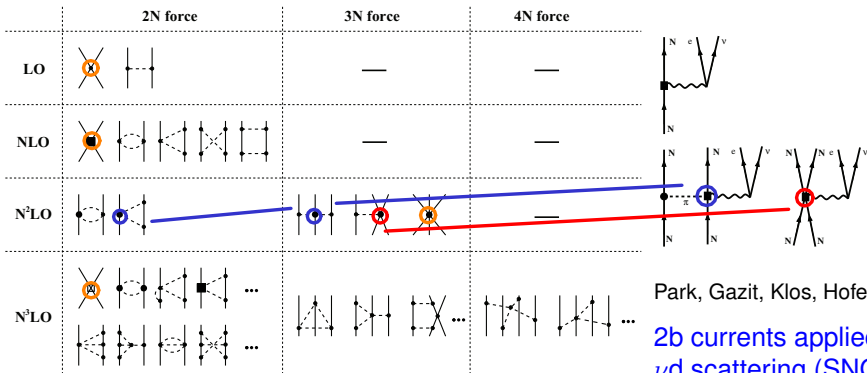


# Chiral effective field theory

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents



Park, Gazit, Klos, Hoferichter...

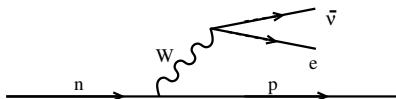
2b currents applied to  $\nu d$  scattering (SNO),

$^3\text{H}$   $\beta$ -decay,  $\mu$  moment...

Weinberg, van Kolck, Kaplan, Savage, Epelbaum, Kaiser, Meißner...

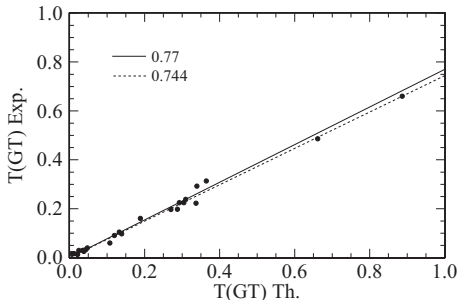
# Gamow-Teller transitions: quenching

Single  $\beta$  decays well described by nuclear structure (shell model)



$$\langle F | \sum_i g_A^{\text{eff}} \sigma_i \tau_i^- | I \rangle$$

$$g_A^{\text{eff}} = qg_A, \quad q \sim 0.7 - 0.8.$$



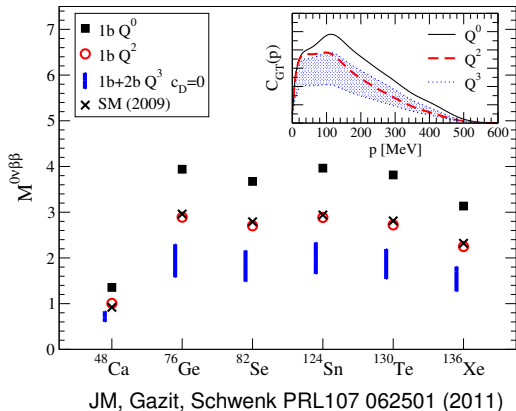
Martínez-Pinedo et al. PRC53 2602 (1996)

Theory needs to “quench” Gamow-Teller operator to reproduce Gamow-Teller lifetimes: problem in nuclear many-body wf or operator?

This puzzle has been the target of many theoretical efforts:

Arima, Rho, Towner, Bertsch and Hamamoto, Wildenthal and Brown...

# Nuclear matrix elements with 1b+2b currents

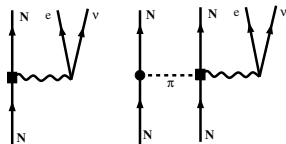


Smaller quenching  $q = 0.96...0.92$

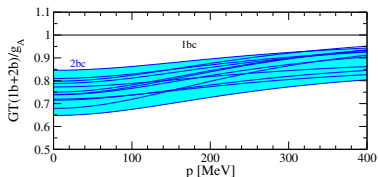
Ekström et al. PRL113 262504 (2014)

Improved (ab initio) calculations needed

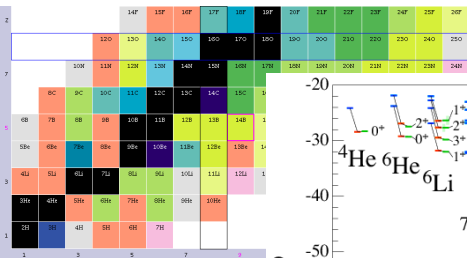
2b currents  
reduce matrix elements  
 $\sim 20\% - 50\%$



Momentum transfer  $p \sim m_\pi$ ,  
reduces quenching  $\downarrow$

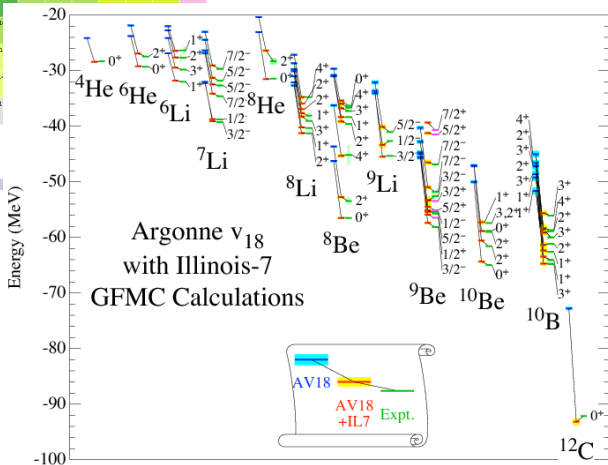


# Green's function Monte Carlo



Green's function Monte Carlo

NN forces  
 do not reproduce  
 binding energies  
 and spectra:  
 need 3N forces  
  
 Good agreement  
 with 3N forces

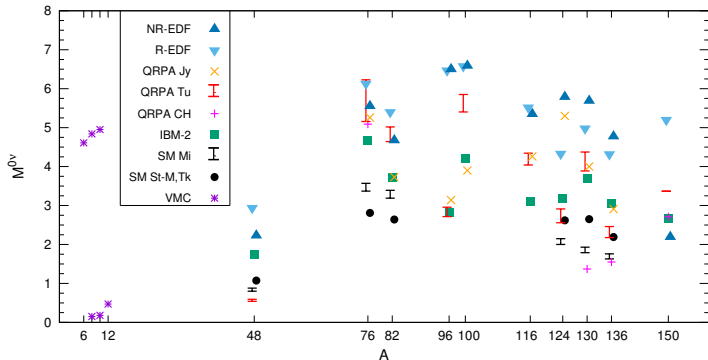




# $0\nu\beta\beta$ decay matrix elements in very light nuclei

Variational Monte Carlo (VMC)  $0\nu\beta\beta$  decay matrix elements in  $A \leq 12$

Pastore et al. arXiv:1710:05026



Larger/smaller matrix elements given by same/different nuclear isospin

VMC free from "quenching": reproduce  $\beta$  decay with  $\sigma\tau + 2b$  (small)

Anchor for other ab initio calculations that can extend to heavier nuclei

# Outline

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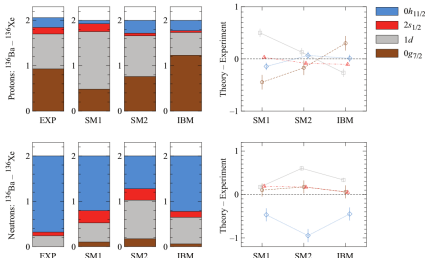
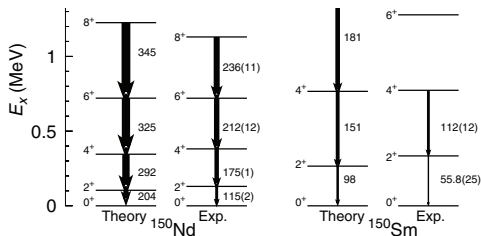
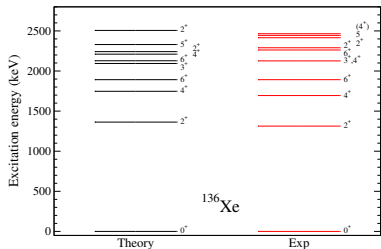
Present status of  $0\nu\beta\beta$  decay nuclear matrix elements

Future prospects for  $0\nu\beta\beta$  nuclear matrix element calculations

Can nuclear structure experiments help with  $0\nu\beta\beta$  decay?

# Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...



Schiffer et al. PRL100 112501(2009)

Kay et al. PRC79 021301(2009)

...

Szwec et al., PRC94 054314 (2016)

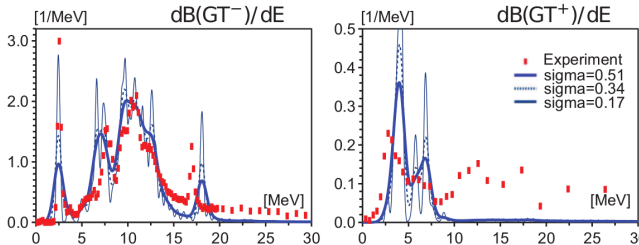
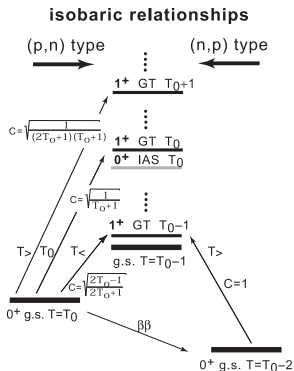
Rodríguez et al. PRL105 252503 (2010)

...

Vietze et al. PRD91 043520 (2015)

# Gamow-Teller strength distributions

Gamow-Teller (GT) distributions well described by theory (quenched)



Iwata et al. JPSCP 6 03057 (2015)

$$\langle 1_f^+ | \sum_i [\sigma_i \tau_i^\pm]^{\text{eff}} | 0_{gs}^+ \rangle, \quad [\sigma_i \tau_i^\pm]^{\text{eff}} \approx 0.7 \sigma_i \tau_i^\pm$$

$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

Freckers et al.

NPA916 219 (2013)

25 / 30

# Double Gamow-Teller strength distribution

Measurement of Double Gamow-Teller (DGT) resonance  
in double charge-exchange reactions  $^{48}\text{Ca}(pp,nn)^{48}\text{Ti}$  proposed in 80's

Auerbach, Muto, Vogel... 1980's, 90's

Recent experimental plans in RCNP, RIKEN ( $^{48}\text{Ca}$ ), INFN Catania

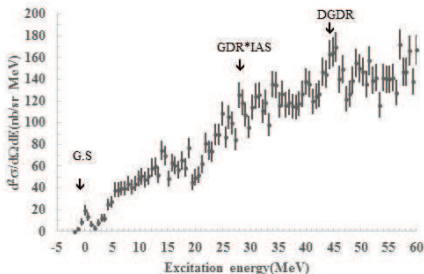
Takaki et al. JPS Conf. Proc. 6 020038 (2015)

Capuzzello et al. EPJA 51 145 (2015), Takahisa, Ejiri et al. arXiv:1703.08264

Promising connection to  $\beta\beta$  decay,  
two-particle-exchange process,  
especially the (tiny) transition  
to ground state of final state

Two-nucleon transfers related to  
 $0\nu\beta\beta$  decay matrix elements

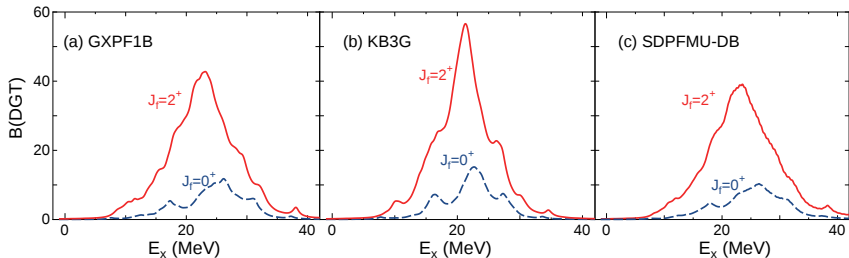
Brown et al. PRL113 262501 (2014)



# $^{48}\text{Ca}$ Double Gamow-Teller distribution

Calculate with shell model  $^{48}\text{Ca } 0_{\text{gs}}^+$  Double Gamow-Teller distribution

$$B(\text{DGT}^-; \lambda; i \rightarrow f) = \frac{1}{2J_i + 1} \left| \left\langle {}^{48}\text{Ti} \left\| \left[ \sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^- \right]^{(\lambda)} \right\| \left| {}^{48}\text{Ca}_{\text{gs}} \right\rangle \right|^2$$



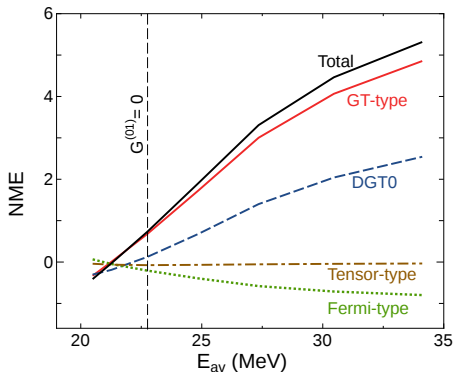
Shell model calculation with Lanczos strength function method

Double GT resonances in one and two shells rather similar result

Shimizu, JM, Yako, arXiv:1709.01088

# $^{48}\text{Ca}$ double GT resonance and $0\nu\beta\beta$ decay

Correlation between Double Gamow-Teller resonance in  $^{48}\text{Ca}$  and  $0\nu\beta\beta$  decay nuclear matrix element



Energy of DGT resonance with accuracy to  $\sim 1\text{MeV}$ , can give insight on the value of  $0\nu\beta\beta$  decay nuclear matrix element

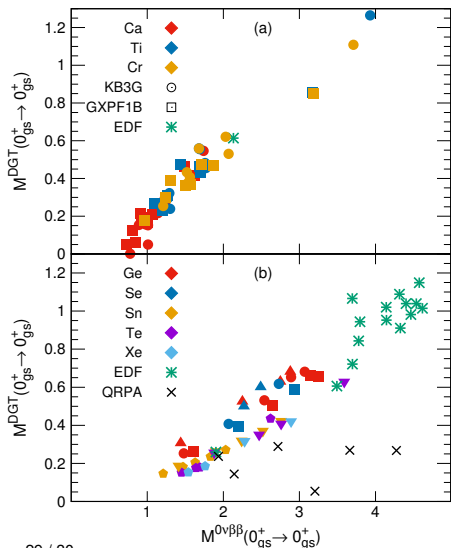
$$E_{av} = \frac{\sum_f E_f B(DGT^-, i \rightarrow f)}{\sum_f B(DGT^-, i \rightarrow f)}$$

Might be feasible in near future

Shimizu, JM, Yako, arXiv:1709.01088

In progress: sensitivity to other nuclear structure correlations

# Double Gamow-Teller and $0\nu\beta\beta$ decay



DGT transition to ground state

$$M^{\text{DGT}} = \langle F_{\text{gs}} || [\sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^-]^0 || I_{\text{gs}} \rangle|^2$$

very good linear correlation  
with  $0\nu\beta\beta$  decay  
nuclear matrix elements

Correlation holds  
across wide range of nuclei,  
from Ca to Ge and Xe

Common to shell model and  
energy-density functional theory  
 $0 \lesssim M^{0\nu\beta\beta} \lesssim 5$   
disagreement to QRPA

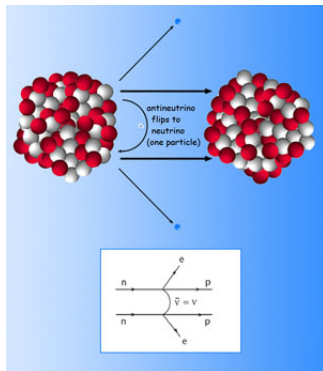
Shimizu, JM, Yako, arXiv:1709.01088



# Summary

Reliable nuclear matrix elements needed to plan and fully exploit impressive experiments looking for neutrinoless double-beta decay

- Matrix element differences between present calculations, factor 2 – 3
- $^{48}\text{Ca}$  and  $^{76}\text{Ge}$  matrix elements in large configuration space increase  $\sim 30\%$ , missing correlations introduced in IBM, EDF
- First ab initio calculations with 2b currents small matrix elements (due to light nuclei?) with no additional "quenched" needed, stay tuned for ab initio  $^{48}\text{Ca}$  matrix elements
- Double Gamow-Teller transitions can give insight on  $0\nu\beta\beta$  matrix elements



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