Neutrinoless $\beta\beta$ decay matrix elements: present and future

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Nuclear physics and neutrinoless $\beta\beta$ decay

Neutrinos, dark matter studied in experiments using nuclei

Nuclear matrix elements depend on nuclear structure crucial to anticipate reach and fully exploit experiments

$0\nu\beta\beta$ decay: \[ \left( T_{1/2}^{0\nu\beta\beta} \right)^{-1} \propto \left| M^{0\nu\beta\beta} \right|^2 m_{\beta\beta}^2 \]

Dark matter: \[ \frac{d\sigma_{\chi N}}{dq^2} \propto \left| \sum_i c_i \zeta_i \mathcal{F}_i \right|^2 \]

$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$: Nuclear structure factor
Neutrinoless $\beta\beta$ decay

Lepton-number violation, Majorana nature of neutrinos
Second order process only observable in rare cases with $\beta$-decay energetically forbidden or hindered by $\Delta J$
Next generation experiments: inverted hierarchy

The decay lifetime is

$$T_{1/2}^{0\nu\beta\beta} \left(0^+ \rightarrow 0^+\right)^{-1} = G_{01} \left| M_{0\nu\beta\beta} \right|^2 m_{\beta\beta}^2$$

sensitive to absolute neutrino masses, $m_{\beta\beta} = |\sum U_{\text{ek}}^2 m_k|$, and hierarchy

Matrix elements needed to make sure next generation ton-scale experiments fully explore "inverted hierarchy"

KamLAND-Zen, PRL117 082503(2016)
Outline

Present status of $0\nu\beta\beta$ decay nuclear matrix elements

Future prospects for $0\nu\beta\beta$ nuclear matrix element calculations

Can nuclear structure experiments help with $0\nu\beta\beta$ decay?
Calculating nuclear matrix elements

Nuclear matrix elements needed to study fundamental symmetries

\[
\langle \text{Final} | \mathcal{L}_{\text{leptons-nucleons}} | \text{Initial} \rangle = \langle \text{Final} | \int dx \, j^\mu(x) J_\mu(x) | \text{Initial} \rangle
\]

- **Nuclear structure calculation** of the initial and final states:
  - Shell model Retamosa, Poves, JM, Horoi...
  - Energy-density functional Rodríguez, Yao...
  - QRPA Vogel, Faessler, Šimkovic, Suhonen...
  - Interacting boson model Iachello, Barea...
  - Ab initio many-body methods
    - Green’s Function MC, Coupled-cluster, IM-SRG...

- **Lepton-nucleus interaction:**
  - Study hadronic current in nucleus:
    - phenomenological approaches, effective theory of QCD
$0\nu\beta\beta$ nuclear matrix elements: last 5 years

Comparison of nuclear matrix element calculations: 2012 vs 2017

What have we learned in the last 5 years?
Configuration space

Nuclear shell model configuration space only keep essential degrees of freedom

- High-energy orbits: always empty
- Configuration space: where many-body problem is solved
- Inert core: always filled

\[ H |\psi\rangle = E |\psi\rangle \rightarrow H_{\text{eff}} |\psi\rangle_{\text{eff}} = E |\psi\rangle_{\text{eff}} \]

\[ |\psi\rangle_{\text{eff}} = \sum_{\alpha} c_{\alpha} |\phi_{\alpha}\rangle, \quad |\phi_{\alpha}\rangle = a_{i_1}^+ a_{i_2}^+ \ldots a_{i_A}^+ |0\rangle \]

Shell model codes (1 major oscillator shell)

\( \sim 10^{10} \) Slater dets. Caurier et al. RMP77 (2005)

QRPA calculations suggest larger spaces (\( \gtrsim 2 \) major shells) needed

Dimension \( \sim \)

\[ \left( \frac{(p+1)(p+2)}{N} \right)^{\nu} \left( \frac{(p+1)(p+2)}{Z} \right)^{\pi} \]
Shell model configuration space: two shells

For $^{48}$Ca enlarge configuration space from $pf$ to $sdpf$
4 to 7 orbitals, dimension $10^5$ to $10^9$
increases matrix elements but only moderately 30%

Iwata et al. PRL116 112502 (2016)

Contributions dominated by pairing 2 particle – 2 hole excitations enhance the $\beta\beta$ matrix element,
Contributions dominated by 1 particle – 1 hole excitations suppress the $\beta\beta$ matrix element
76 Ge matrix element in two shells

Large configuration space calculations in 2 major oscillator shells
Include all relevant correlations: isovector/isoscalar pairing, deformation
Many-body approach: generating coordinate method (GCM)

GCM approximates shell model calculation

Degrees of freedom, or generating coordinates, validated against exact shell model in small configuration space

Jiao et al. PRC96 054310 (2017)

76 Ge nuclear matrix element in 2 major shells
very similar to shell model nuclear matrix element in 1 major shell
Heavy-neutrino exchange nuclear matrix elements

Contrary to light-neutrino-exchange, for heavy-neutrino-exchange decay shell model, IBM, and EDF matrix elements agree reasonably!

Song et al. PRC95 024305 (2017)
JM, JPG in print

Neacsu et al. PRC100 052503 (2015)

Suggests differences in treating longer-range nuclear correlations dominant in light-neutrino exchange
Pairing correlations and $0\nu\beta\beta$ decay

$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities

Addition of isoscalar pairing reduces matrix element value

Caurier et al. PRL100 052503 (2008)

Hinohara, Engel PRC90 031301 (2014)

Related to approximate $SU(4)$ symmetry of the $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$ operator
Energy-density functional (EDF) theory and interacting boson model (IBM) calculated nuclear matrix elements do not include explicitly proton-neutron pairing correlations. This effect (partially) accounted for by other degrees of freedom present in these approaches. Include \( p \)-boson \((L = 1)\) to IBM in addition to \( s \) and \( d \) bosons \((L = 0, 2)\). First IBM results in calcium region suggest nuclear matrix elements could be somewhat reduced.
Matrix elements: theoretical uncertainty

Systematic uncertainty hard to estimate for phenomenological matrix elements

Effective theory for $\beta\beta$ decay: spherical core coupled to one nucleon

Couplings adjusted to experimental data, uncertainty given by effective theory (breakdown scale, systematic expansion)

Take $\beta$ decay data to predict $2\nu\beta\beta$

$$M^{2\nu\beta\beta} = \sum_k \langle 0^+_f | \sum_n \sigma_n \tau_n^- | 1^+_k \rangle \langle 1^+_k | \sum_m \sigma_m \tau_m^- | 0^+_i \rangle \frac{E_k - (M_i + M_f)/2}{E_k}$$

Good agreement with large errors (leading-order calculations)

Coello-Pérez, JM, Schwenk, arXiv:1708.06140
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Present status of $0\nu\beta\beta$ decay nuclear matrix elements

Future prospects for $0\nu\beta\beta$ nuclear matrix element calculations

Can nuclear structure experiments help with $0\nu\beta\beta$ decay?
Ab initio methods: No core shell model

No core shell model

Many-body wave function
linear combination of
Slater Determinants
from single particle states in the basis
(3D harmonic oscillator)

$$|i\rangle = |n_i l_j m_j m_{ti}\rangle$$

$$|\phi_\alpha\rangle = a_{i_1}^+ a_{j_2}^+ \cdots a_{k_\alpha}^+ |0\rangle$$

$$|\Psi\rangle = \sum_\alpha c_\alpha |\phi_\alpha\rangle$$

$$H |\Psi\rangle = E |\Psi\rangle$$

Dimensions increase combinatorially...
Coupled Cluster, In-Medium SRG

Coupled Cluster method: operators (correlations) acting on reference impose no particle-hole excitations present in the reference state

\[ |\psi\rangle = e^{-(T_1 + T_2 + T_3 \cdots)} |\Phi\rangle \]

with \( T_1 = \sum_{\alpha, \bar{\alpha}} t_{\alpha}^{\bar{\alpha}} \{ a^{\dagger}_{\alpha}, a_{\alpha} \} \), \( T_2 = \sum_{\alpha \beta, \bar{\alpha} \bar{\beta}} t_{\alpha \beta}^{\bar{\alpha} \bar{\beta}} \{ a^{\dagger}_{\alpha} a^{\dagger}_{\beta}, a_{\alpha} a_{\beta} \} \), \( \cdots \)

solve \( \langle \Phi_{\alpha} | e^{\sum T_i H} e^{\sum - T_i} | \Phi \rangle = 0 \), \( \langle \Phi_{\alpha \beta} | e^{\sum T_i H} e^{\sum - T_i} | \Phi \rangle = 0 \)

In-medium similarity renormalization group method: apply a similarity (unitary) transformation to decouple reference state from particle-hole excitations
Ab initio many-body methods

Oxygen dripline using chiral NN+3N forces correctly reproduced ab-initio calculations treating explicitly all nucleons. Excellent agreement between different approaches.

No-core shell model (Importance-truncated)
In-medium SRG
Hergert et al. PRL110 242501(2013)

Self-consistent Green’s function
Cipollone et al. PRL111 062501(2013)

Coupled-clusters
Jansen et al. PRL113 142502(2014)
Calculations with NN+3N forces predict shell closures at $^{52}\text{Ca}$, $^{54}\text{Ca}$

$^{51-54}\text{Ca}$ masses [TRIUMF/ISOLDE]

$^{54}\text{Ca}$ $2^+_1$ excitation energy [RIBF,RIKEN]

Hebeler et al. ARNPS 65 457 (2015)
Ab initio $0\nu\beta\beta$ decay matrix elements?

Nuclei up to $A \sim 70$ explored with ab initio approaches
Limited by good chiral nuclear force

Challenge for ab initio $0\nu\beta\beta$ decay:
(unitary) transformation: $H' = U^\dagger H U$
applied to operators: $O' = U^\dagger \ O \ U$
First electromagnetic transition results
$\Rightarrow 0\nu\beta\beta$ decay matrix elements next

$^{48}\text{Ca}$ ab initio $0\nu\beta\beta$ decay
nuclear matrix element
ready very soon: stay tuned!

Simonis et al. PRC96 014303 (2017)
Parzuchowski et al. PRC96 034324 (2017)
## Chiral effective field theory

**Chiral EFT**: low energy approach to QCD, nuclear structure energies  
Approximate chiral symmetry: pion exchanges, contact interactions  
Systematic expansion: nuclear forces and electroweak currents

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Park, Gazit, Klos, Hoferichter...  
2b currents applied to $\nu d$ scattering (SNO),  
$^3H$ $\beta$-decay, $\mu$ moment...  

Weinberg, van Kolck, Kaplan, Savage, Epelbaum, Kaiser, Meißner...
Gamow-Teller transitions: quenching

Single $\beta$ decays well described by nuclear structure (shell model)

$$\langle F | \sum_i g_A^{\text{eff}} \sigma_i \tau_i^- | I \rangle$$

$$g_A^{\text{eff}} = q g_A, \quad q \sim 0.7 - 0.8.$$

Theory needs to “quench“ Gamow-Teller operator to reproduce Gamow-Teller lifetimes: problem in nuclear many-body wf or operator?

This puzzle has been the target of many theoretical efforts:

Arima, Rho, Towner, Bertsch and Hamamoto, Wildenthal and Brown...
Nuclear matrix elements with 1b+2b currents

2b currents reduce matrix elements \( \sim 20\% - 50\% \)

Momentum transfer \( p \sim m_\pi \), reduces quenching \( \downarrow \)

Smaller quenching \( q = 0.96 \ldots 0.92 \)

Ekström et al. PRL113 262504 (2014)
Improved (ab initio) calculations needed
Green’s function Monte Carlo

NN forces do not reproduce binding energies and spectra: need 3N forces

Good agreement with 3N forces
$0\nu\beta\beta$ decay matrix elements in very light nuclei

Variational Monte Carlo (VMC) $0\nu\beta\beta$ decay matrix elements in $A \leq 12$


Larger/smaller matrix elements given by same/different nuclear isospin

VMC free from "quenching": reproduce $\beta$ decay with $\sigma_T + 2b$ (small)

Anchor for other ab initio calculations that can extend to heavier nuclei
Outline

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Can nuclear structure experiments help with $0\nu\beta\beta$ decay?
Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...

Schiffer et al. PRL100 112501(2009)
Kay et al. PRC79 021301(2009)
Szwec et al., PRC94 054314 (2016)
Rodríguez et al. PRL105 252503 (2010)
Vietze et al. PRD91 043520 (2015)
Gamow-Teller strength distributions

Gamow-Teller (GT) distributions well described by theory (quenched)

\[
\langle 1^+_f | \sum_i [\sigma_i \tau_i^{\pm}]^{\text{eff}} | 0^+_{\text{gs}} \rangle, \quad [\sigma_i \tau_i^{\pm}]^{\text{eff}} \approx 0.7 \sigma_i \tau_i^{\pm}
\]

\[
M^{2\nu\beta\beta} = \sum_k \frac{\langle 0^+_f | \sum_n \sigma_n \tau_n^- | 1^+_k \rangle \langle 1^+_k | \sum_m \sigma_m \tau_m^- | 0^+_i \rangle}{E_k - (M_i + M_f)/2}
\]

- Freckers et al.
  - NPA916 219 (2013)

- Iwata et al. JPSCP 6 03057 (2015)
Double Gamow-Teller strength distribution

Measurement of Double Gamow-Teller (DGT) resonance in double charge-exchange reactions $^{48}\text{Ca}(pp,nn)^{48}\text{Ti}$ proposed in 80’s
Auerbach, Muto, Vogel... 1980’s, 90’s

Recent experimental plans in RCNP, RIKEN ($^{48}\text{Ca}$), INFN Catania

Promising connection to $\beta\beta$ decay, two-particle-exchange process, especially the (tiny) transition to ground state of final state

Two-nucleon transfers related to $0\nu\beta\beta$ decay matrix elements
Brown et al. PRL113 262501 (2014)
**48Ca Double Gamow-Teller distribution**

Calculate with shell model $^{48}$Ca $0^+_{gs}$ Double Gamow-Teller distribution

$$B(DGT^-; \lambda; i \rightarrow f) = \frac{1}{2J_i + 1} \left| \left\langle 48Ti \left| \sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^- \right| 48^{\text{Ca}}_{gs} \right\rangle \right|^2$$

Shell model calculation with Lanczos strength function method

Double GT resonances in one and two shells rather similar result

Shimizu, JM, Yako, arXiv:1709.01088
\( {\text{\(^{48}\)Ca double GT resonance and } 0\nu\beta\beta \text{ decay}} \)

Correlation between Double Gamow-Teller resonance in \(^{48}\text{Ca}\) and \(0\nu\beta\beta\) decay nuclear matrix element

Energy of DGT resonance with accuracy to \(\sim 1\text{MeV}\), can give insight on the value of \(0\nu\beta\beta\) decay nuclear matrix element

\[ E_{av} = \frac{\sum_i E_i B(DGT^-, i \rightarrow f)}{\sum_i B(DGT^-, i \rightarrow f)} \]

Might be feasible in near future

Shimizu, JM, Yako, arXiv:1709.01088

In progress: sensitivity to other nuclear structure correlations
Double Gamow-Teller and $0^\nu\beta\beta$ decay

DGT transition to ground state

$$M^{\text{DGT}} = \langle F_{gs} \mid [\sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^- ]^0 \mid I_{gs} \rangle^2$$

very good linear correlation with $0^\nu\beta\beta$ decay nuclear matrix elements

Correlation holds across wide range of nuclei, from Ca to Ge and Xe

Common to shell model and energy-density functional theory

$0 \lesssim M^{0^\nu\beta\beta} \lesssim 5$

disagreement to QRPA

Shimizu, JM, Yako, arXiv:1709.01088
Summary

Reliable nuclear matrix elements needed to plan and fully exploit impressive experiments looking for neutrinoless double-beta decay

- Matrix element differences between present calculations, factor 2 – 3
- $^{48}$Ca and $^{76}$Ge matrix elements in large configuration space increase $\sim 30\%$, missing correlations introduced in IBM, EDF
- First ab initio calculations with 2b currents small matrix elements (due to light nuclei?) with no additional "quenching" needed, stay tuned for ab initio $^{48}$Ca matrix elements
- Double Gamow-Teller transitions can give insight on $0\nu\beta\beta$ matrix elements
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