Infinite-dimensional symmetries of the gauge theories in the light front

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[arXiv: 2304.03211]

HOLOGRAPHYCL

Progress on gravitational physics
45 years of Belgian-Chilean collaboration

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Subject of the talk

We analyse symmetries

in electromagnetism

and Yang-Mills theory

using Hamiltonian formalism

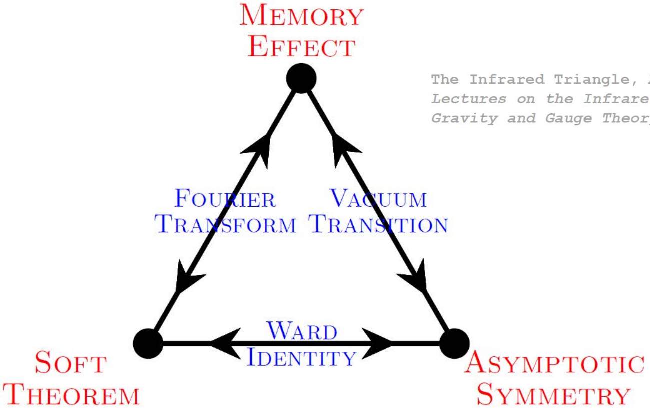
in the null foliation

Outline

1. Infrared structure of gauge theories
2. Hamiltonian analysis of electromagnetism in the null foliation
3. New symmetry generator; Beyond $\mathrm{U}(1)$
4. Extension to Yang-Mills theory
5. Discussion

Infrared structure of gauge theories

IR region of theories with massless particles in asymptotically flat spaces



The Infrared Triangle, A. Strominger, *Lectures on the Infrared Structure of Gravity and Gauge Theory* arXiv:1703.05448

Infrared structure of gauge theories

Motivation

- **Hamiltonian treatment of asymptotic symmetries**

  [Bondi, van der Burg, Metzner 1962; Sachs 1962]

- **BMS symmetry –** infinite-dimensional asymptotic symmetry at the null boundary of 4D asymptotically flat spacetimes

- **Celestial holography**

- Duality between massless 4D asymptotically flat spacetimes and 2D CFT on the celestial sphere

- **Asymptotic symmetries in electromagnetism and Yang-Mills theory**

- 2D realization of soft symmetries in electromagnetism

  [He, Mitra, Porfyriadis, Strominger 2014;
  Nande, Pate, Strominger 2018]

- Extension to Yang-Mills theory

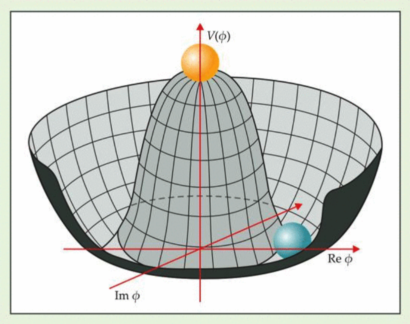
  [Strominger 2014; He, Mitra, Strominger, 2016]

Infrared structure of gauge theories

Vacuum degeneracy in gauge theories $(\omega \to 0)$

$\Leftrightarrow$ Enhancement of symmetries at the boundary of flat spacetime $(r \to \infty)$

- **Goldstone modes**, dominant low-energy excitations
- **Vacuum state** $\quad e^{S[\eta]}\,|A\rangle = |A+\eta\rangle \quad \Leftrightarrow \quad \delta\,|A\rangle = \eta$



Spontaneous symmetry breaking, J. Lykken, M. Spiropulu, *The future of the Higgs boson*, Physics Today 66, 12, 28 (2013)

$\Rightarrow$ **Interest in boundary dynamics of massless particles**

Infrared structure of gauge theories

What to expect at the null infinity?

- Realisation of a canonical analysis in a null foliation [Dirac 1949]
- The induced metric on a null hypersurface is degenerate
- Double-null foliation $= 2 + 2$ formalism in GR, complicated symplectic structure, difficult to quantize [d'Inverno, Smallwood 1980]
- Ashtekar variables in GR, simpler symplectic structure, but still difficult to quantize [Ashtekar 1986, 1987]

Peculiarities of the light-front dynamics in the Minkowski space

- **Light-cone coordinates** $x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}}$; Time coordinate $u = x^-$
- **Increased number of isometries** of the surface $u = \mathrm{const.}$ compared to $t = x^0 = \mathrm{const.}$ (one more because of degenerated direction)
- **Dispersion equation** for a massive scalar

  $p^2 = m^2 \quad \Rightarrow$ Energy $E = p^- = \frac{(p^{\perp})^2 + m^2}{2p^+}$

  $\Rightarrow$ Consequences: $p^+ > 0$ and trivial physical vacuum, $p^{\mu}_{\mathrm{vac}} = 0$

Infrared structure of gauge theories

- **Nontrivial effects on the light front** are contained in the zero modes [Yamawaki 1998]
- **Boundary conditions** in the light front formalism
- **Light-cone actions** are first order in velocities [Steinhardt 1980]

Kinetic term $T = -\frac{1}{2}\,(\partial\phi)^2 = \dot{\phi}\,\partial_+\phi - (\nabla_\perp\phi)^2$

$\Rightarrow$ The canonical momentum $\pi = \partial_+\phi$ is not invertible

★ New constraint $\chi \equiv \pi - \partial_+\phi \approx 0$

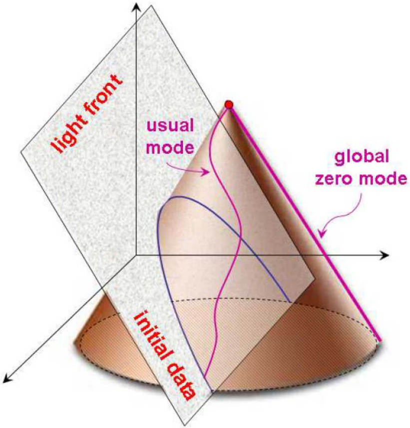
★ It does not commute with itself, $\{\chi(x), \chi(x')\}_{u=u'} = -2\partial_+\delta(x - x')$

$\Rightarrow$ Reduction of the phase space: elimination $\chi = 0$

- **Global zero mode** in massless theories [Alexandrov, Speziale 2015]

- A massless particle worldline is parallel to the light front hypersurface, not determined by the initial data
- It has vanishing energy, $E = p^- \to 0$, $p^\perp = 0$ (soft particles)

Global zero mode



Global zero mode, *First order gravity on the light front*, S. Alexandrov, S. Speziale, Phys.Rev.D 91 (2015) 6, 064043

Hamiltonian analysis of electromagnetism in the null foliation

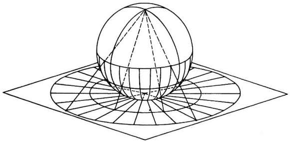
Null foliated reference frame

- **Minkowski metric in $D = 4$ in the spherical coordinates** (t, r, y^A)

$$M_4 : \quad \mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2$$

$$\mathbb{S}^2 : \quad \mathrm{d}\Omega^2 = \gamma_{AB}(y)\, \mathrm{d}y^A \mathrm{d}y^B$$

Stereographic projection, T. Apostol, *Mathematical Analysis* (1973)



- **Time coordinate** $\quad u = t - \epsilon r\,, \quad -1 \leq \epsilon \leq 1$

$\epsilon = 1$	retarded time
$\epsilon = 0$	proper time of a massive particle
$\epsilon = -1$	advanced time

- **Coordinates on** $\mathbb{S}^2$: **stereographic projection** $(\theta, \varphi) \to y^A = (z, \bar{z})$

$$z = \mathrm{e}^{\mathrm{i}\varphi} \cot \frac{\theta}{2}\,, \quad \bar{z} = \mathrm{e}^{-\mathrm{i}\varphi} \cot \frac{\theta}{2}$$

Hamiltonian analysis of electromagnetism in the null foliation

- **Minkowski metric $\mathfrak{g}_{\mu\nu}$ in the coordinates $x^\mu = (u, r, y^A)$:**

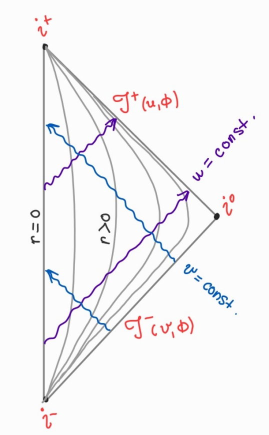
$$\boxed{\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\epsilon\,\mathrm{d}u\mathrm{d}r + \left(1-\epsilon^2\right)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2}$$

Jacobian $\quad \sqrt{\mathfrak{g}} = r^2\sqrt{\gamma}$

- **$\mathbb{S}^2$ metric in the complex coordinates**

$$\gamma_{AB} = \begin{pmatrix} 0 & \gamma_{z\bar{z}} \\ \gamma_{z\bar{z}} & 0 \end{pmatrix}$$

$$\sqrt{\gamma} = \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$



Hamiltonian analysis of electromagnetism in the null foliation

- **Electromagnetic action in the background $\mathfrak{g}_{\mu\nu}$**

$$I[A] = -\frac{1}{4e^2}\int \mathrm{d}^4x\,\sqrt{\mathfrak{g}}\,F^{\mu\nu}F_{\mu\nu} \qquad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

- **Canonical momenta** $\pi^\mu = -\frac{1}{e^2}\sqrt{\mathfrak{g}}\,F^{u\mu}$

In components:

$$\begin{aligned} \pi^u &= 0 && \dot{A}_u \quad \times \\ \pi^r &= \frac{r^2}{e^2}\sqrt{\gamma}\,F_{ur} && \dot{A}_r \quad \checkmark \\ \pi^A &= -\frac{1}{e^2}\sqrt{\gamma}\gamma^{AB}\left[(\epsilon^2-1)F_{uB} - \epsilon F_{rB}\right] && \dot{A}_B \quad ? \end{aligned}$$

- The limit $\epsilon^2 \to 1$ is discontinuous
- The action in the light-cone ($\epsilon^2 = 1$) has an additional constraint

Hamiltonian analysis of electromagnetism in the null foliation

In the Bondi reference frame ($\epsilon^2 = 1$)

- **Primary constraints** $\pi^u \approx 0\,, \quad \chi^A \equiv \epsilon\pi^A - \frac{1}{e^2}\sqrt{\gamma}\gamma^{AB}F_{rB} \approx 0$

- **Total Hamiltonian** [Dirac 1964]

$$\mathcal{H}_T = \frac{e^2(\pi^r)^2}{2r^2\sqrt{\gamma}} + \frac{e^2\tilde{\pi}_A\pi^A}{2\sqrt{\gamma}} + \frac{\sqrt{\gamma}}{4e^2r^2}\tilde{F}^{AB}F_{AB} - A_u\partial_i\pi^i + \lambda_u\pi^u + \lambda_A\chi^A$$

- Hamiltonian multipliers A_u, λ_u, λ_A incorporate constraints

- **Matching conditions**

- Two Hamiltonians on the future ($\epsilon = +1$) and past ($\epsilon = +1$) light cones satisfy the antipodal matching conditions near the boundary i^0:

$$\mathcal{H}_T|_{\mathcal{J}^-_+} = \mathcal{H}_T|_{\mathcal{J}^+_-}$$

Hamiltonian analysis of electromagnetism in the null foliation

Auxiliary symplectic matrix

$$\left\{\chi^A(x), \chi^B(x')\right\} = \Omega^{AB}(x,x') \equiv -\frac{2\epsilon}{e^2}\sqrt{\gamma}\gamma^{AB}\partial_r\delta^{(3)}$$

- If Ω^{AB} is not invertible: χ^A are **first class** (generate symmetries)
- If Ω^{AB} is invertible: χ^A are **second class** (eliminate redundant fields)

One possibility

- Ω^{AB} is invertible because γ^{AB} is invertible $\Rightarrow \chi^A$ are second class
- Reduced phase space $\chi^A = 0$ [Goldberg 1991, Majumdar 2022]

Second possibility

- Ω^{AB} is invertible, but its inverse is not unique
  - Ω^{AB} is infinite-dimensional matrix and it has zero modes

$$\int d^3x'\,\Omega^{AB}V'_B = -\frac{2\epsilon}{e^2}\sqrt{\gamma}\gamma^{AB}\partial_r V_B = 0 \quad \Rightarrow \quad V_B = V_B(y)$$

Hamiltonian analysis of electromagnetism in the null foliation

Symplectic matrix

$$\left\{\chi^A(x), \chi^B(x')\right\} = \Omega^{AB}(x,x') \equiv -\frac{2\epsilon}{e^2}\sqrt{\gamma}\gamma^{AB}\partial_r\delta^{(3)}$$

- If Ω^{AB} - invertible: χ^A are **first class** (generate symmetries)
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One possibility

- Ω^{AB} is invertible because γ^{AB} is invertible $\Rightarrow \chi^A$ are second class
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Other possibility

- Ω^{AB} is invertible, but its inverse is not unique
- Ω^{AB} is infinite-dimensional matrix and it has zero modes

$\Rightarrow \chi^A_{(0)}(y)$ **is first class constraint** (r-independent part of the constraint)

Hamiltonian analysis of electromagnetism in the null foliation

Consistency conditions

- **Conservation of constraints during their evolution**

$$\dot{\pi}^u = 0 \quad \Rightarrow \quad \chi = \partial_i \pi^i \approx 0 \quad \text{(differential Gauss law)}$$

$$\dot{\chi}^A = 0 \quad \Rightarrow \quad \text{differential equation in the multiplier}$$

- **The multiplier λ_A is partially determined**

$$\partial_r \lambda_A = -\frac{\epsilon e^2}{2\sqrt{\gamma}}\,\partial_r \tilde{\pi}_A - \frac{1}{2r^2}\,\nabla^B F_{AB} + \frac{\epsilon e^2}{2r^2}\,\partial_B\left(\frac{\pi^r}{\sqrt{\gamma}}\right)$$

$$\lambda_A = \bar{\lambda}_A + \Lambda_A(y)$$

- $\bar{\lambda}_A$ – determined part of λ_A
- A free function $\Lambda_A(y)$ is due to the zero modes of Ω^{AB}

Hamiltonian analysis of electromagnetism in the null foliation

Summary of the constraints

Primary constraints:	$\pi^u\,, \quad \chi^A = \epsilon\pi^A - \frac{1}{e^2}\sqrt{\gamma}\gamma^{AB}F_{rB}$
Secondary constraint:	$\chi = \partial_i \pi^i\,.$

Nature of the constraints

- π^u – first class, A_u is a multiplier in the Hamiltonian
- χ – first class, differential Gauss law, $\pi^i = \sqrt{\gamma}\,E^i$
- $\chi^A_{(0)}$ – first class, r-independent part of the constraint
- $\chi^A_{(n)}$ $(n \geq 1)$ – second class, coefficients of the Taylor expansion in $1/r$
- We have to expand all the fields asymptotically in the vicinity of the boundary $r = const \rightarrow \infty$.

Hamiltonian analysis of electromagnetism in the null foliation

Standard asymptotic conditions of the fields [Strominger 2014]

$$
\begin{aligned}
A_u &= \mathcal{O}(\tfrac{1}{r})\,, & A_r &= \mathcal{O}(\tfrac{1}{r^2})\,, & A_A &= \mathcal{O}(r^0)\,, \\
\pi^u &= 0\,, & \pi^r &= \mathcal{O}\left(r^0\right)\,, & \pi^A &= \mathcal{O}(\tfrac{1}{r^2})
\end{aligned}
$$

- **Boundary fields**: $A_{(0)A}, \quad \pi^r_{(0)}$

Hamiltonian analysis of electromagnetism in the null foliation

Summary

1^{st} class constraints	Parameters	Generators	Charges
$\pi^u,\ \chi = \partial_i \pi^i$	$\theta_u,\ \theta$	$G[\theta]$	$Q[\theta]$
$\chi^A_{(0)}$	$\eta_A(y)$	$S[\eta]$	$Q_s[\eta]$

- **Smeared generators**

$$G[\theta] = \int \mathrm{d}^3x \, \left(\theta\, \partial_i \pi^i + \theta_u \pi^u\right) \quad \text{standard U(1) symmetry}$$

$$S[\eta] = \int \mathrm{d}^3x \, \eta_A \chi^A \quad \text{asymptotic symmetry}$$

- **Only first class constraints contribute to** $S[\eta]$

$$S[\eta] = \int \mathrm{d}^3x \, \eta_{(0)A} \chi^A_{(0)} + 0$$

A new symmetry generator

- **Transformation law of the fields**

$$\begin{aligned} \delta_\theta A_\mu &= -\partial_\mu \theta\,, & \delta_\eta A_\mu &= \epsilon\, \eta_A\, \delta^A_\mu \\ \delta_\theta \pi^\mu &= 0\,, & \delta_\eta \pi^\mu &= \tfrac{1}{e^2}\, \delta^\mu_r \sqrt{\gamma}\, \nabla_A \eta^A \end{aligned}$$

- $G[\theta]$ generates standard gauge transformations, $\delta A_\mu = -\partial_\mu \theta$ because $\theta_u = -\dot{\theta}$ [Castellani 1974]

★ η_A changes the boundary fields only

$$\delta_\eta A_{(0)A} = \epsilon\, \eta_A\,, \quad \delta_\eta A_{(n)A} = 0\,, \quad n \geq 1 \qquad \text{(similarly for } \pi^r)$$

- **Improper transformations**: $\theta_{(0)}$, η^A [Benguria, Cordero, Teitelboim 1977]

- They act on the boundary fields $A_{(0)A}$ and $\pi^r_{(0)}$

A new symmetry generator

Improved generators and charges

- **Improved generators**

$$G_Q[\theta] = G[\theta] + Q[\theta] \quad (Q[\theta] = \text{surface term})$$
$$S_Q[\eta] = S[\eta] + Q_s[\eta] \quad (Q_s[\theta] = \text{surface term})$$

- **Differentiability**

- Boundary terms are chosen so that $\delta G_Q[\theta]$, $\delta S_Q[\eta]$ are well-defined

- **Charges**

$$Q[\theta] = -\oint d^2y\, \theta\, \pi^r$$
$$Q_s[\eta] = \frac{1}{e^2} \oint d^2y\, \sqrt{\gamma}\, \eta^A A_A$$

- Infinite number of asymptotic global charges (Laurent coefficients).

A new symmetry generator

Charge algebra

- **Reduced phase space**: $G_Q[\theta] = Q[\theta],\ S_Q[\eta] = Q_s[\eta]$
- **Abelian charge algebra**

$$\boxed{\begin{aligned} \{Q[\theta_1], Q[\theta_2]\} &= 0 \\ \{Q_s[\eta_1], Q_s[\eta_2]\} &= 0 \\ \{Q[\theta], Q_s[\eta]\} &= C[\theta,\eta] \end{aligned}}$$

- **Central charge** $C[\theta,\eta] = \frac{1}{e^2}\oint \mathrm{d}^2y\,\sqrt{\gamma}\,\eta^A\partial_A\theta \neq 0$
- Holographic conjugate pairs on $\mathbb{S}^2$ [Donnay,Puhm, Strominger 2019]

$$\{Q[\theta], Q_s[\eta]\} = C[\theta,\eta] \quad \leftrightarrow \quad \{q,p\} = 1$$

$Q[\theta]$ – conformally soft photon mode

$Q_s[\eta]$ – Goldstone current

A new symmetry generator

Mode expansion of the charge algebra

- **Laurent series**

$$\psi(z,\bar{z}) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \frac{\psi_{nm}}{z^{n+h}\bar{z}^{m+\bar{h}}}$$

- The powers $(h,\bar{h})$ are related to the spin of the tensor ψ

- Scalars $\pi^r : (0,0)$

- Vectors $A_z : (1,0)$, $A_{\bar{z}} : (0,1)$

Generators	*Parameters*
$G_{nm} = 4\pi^2\, \pi_{1-n,1-m}$	θ_{nm}
$S_{nm} = -\frac{4\pi^2}{e^2}\, A_{-n,-m}$	$\bar{\eta}_{nm}$
$\bar{S}_{nm} = -\frac{4\pi^2}{e^2}\, \bar{A}_{-n,-m}$	η_{nm}

A new symmetry generator

- **Algebra** (non vanishing brackets only)

$$\begin{aligned} \{G_{nm}, S_{kl}\} &= \kappa n\, \delta_{n+k,0}\delta_{m+l,0} \\ \{G_{nm}, \bar{S}_{kl}\} &= \kappa m\, \delta_{n+k,0}\delta_{m+l,0} \end{aligned}$$

- **Level of the algebra:** $\kappa = \frac{4\pi^2}{e^2}$
- **Change of the basis**: $(G_{nm}, S_{nm}, \bar{S}_{nm}) \to (R_{nm}, J_{nm}, \bar{J}_{nm})$
- **Generalization of the Kac-Moody algebra**

$$\begin{aligned} \{J_{nm}, J_{kl}\} &= \kappa\,(n-m)\; \delta_{n+k,0}\delta_{m+l,0} \\ \{\bar{J}_{nm}, \bar{J}_{kl}\} &= -\kappa\,(n-m)\; \delta_{n+k,0}\delta_{m+l,0} \\ \{R_{nm}, J_{kl}\} &= \kappa n\, \delta_{n+k,0}\delta_{m+l,0} \\ \{R_{nm}, \bar{J}_{kl}\} &= \kappa\, m\, \delta_{n+k,0}\delta_{m+l,0} \\ \{R_{nm}, R_{kl}\} &= \kappa\,(n+m)\, \delta_{n+k,0}\delta_{m+l,0} \end{aligned}$$

A new symmetry generator

Abelian Kac-Moody subalgebras

- **We obtain six Abelian KM algebras** $\{j_n, j_m\} = \kappa n \, \delta_{n+m,0}$

Currents j_n	*Levels*
J_{n0}, J_{0n}	$\kappa, -\kappa$
$\bar{J}_{n0}, \bar{J}_{0n}$	$-\kappa, \kappa$
R_{n0}, R_{0n}	κ, κ

- **Non vanishing mixed brackets**: $\{R_{n0}, J_{m0}\}, \{R_{0n}, \bar{J}_{0m}\} \neq 0$
- Each KM algebra is naturally generated by a current that is a holomorphic or anti-holomorphic function.
- $\{J_{00}, \bar{J}_{00}, R_{00}\}$ span the global Abelian algebra $\mathrm{U}(1)^2$

A new symmetry generator

Beyond $\mathrm{U}(1)$ – conformal symmetry

- Conformal plane – a realization of conformal symmetry described by Virasoro algebra
- Virasoro algebra – obtained from KM algebra using the Sugawara construction [Sugawara 1967]
- **Four classical Virasoro generators** $L_n = \frac{1}{2\kappa} \sum_k j_k j_{n-k}$
- **Four classical Virasoro algebras** $\{L_n, L_m\} = (n-m)\, L_{n+m}$
- We can have all six Virasoro generators/subalgebras, but the full algebra becomes nonlinear.
- Quantization will introduce central extensions.

A new symmetry generator

Comment

- We can also construct more inequivalent Virasoro algebras
- *Example:* Inspired by the Poincaré charge $Q[i_\xi A_{(0)}] \sim \oint \mathrm{d}^2 y\, \pi^r_{(0)} A_{(0)A} \xi^A$, where ξ are the Killing vectors of the background Minkowski metric $\mathfrak{g}_{\mu\nu}$
- Virasoro generators $(K_n, \bar{K}_n)$:

$$K_n = \frac{1}{\kappa} \sum_k G_{k0} S_{n-k,0} = \frac{1}{2\kappa} \sum_k (J_{k0} + \bar{J}_{k0})\,(R_{n-k,0} - J_{n-k,0})$$

$$\{K_n, K_m\} = (n-m)\, K_{n+m}\,, \qquad \{\bar{K}_n, \bar{K}_m\} = (n-m)\, \bar{K}_{n+m}$$

- The generators $(K_n, \bar{K}_n)$ are independent from $(L_n, \bar{L}_n, \mathcal{L}_n, \bar{\mathcal{L}}_n)$
- *There is a richer symmetry structure than a usual CFT*

Extension to Yang-Mills theory

Yang-Mills theory

$$I[A] = -\frac{1}{4g^2} \int \mathrm{d}^4 x \sqrt{\mathfrak{g}}\, F_a^{\mu\nu} F_{\mu\nu}^a$$

- **Constraints** $(\pi_a^u, \chi_a, \chi_a^A)$ non-Abelian generalization
- **Constraint algebra**

$$\begin{aligned}
\{\chi_a, \chi_b'\} &= f_{ab}{}^{c}\, \chi_c\, \delta^{(3)} \\
\left\{\chi_a, \chi_b'^{A}\right\} &= f_{ab}{}^{c}\, \chi_c^A\, \delta^{(3)} \\
\left\{\chi_a^A, \chi_b'^{B}\right\} &= \Omega_{ab}^{AB}(x, x')
\end{aligned}$$

- **Non-Abelian symplectic matrix**

$$\Omega_{ab}^{AB}(x, x') = -\frac{2\epsilon}{g^2} \sqrt{\gamma}\gamma^{AB} \left(g_{ab}\, \partial_r + f_{abc}\, A_r^c\right) \delta^{(3)}$$

Extension to Yang-Mills theory

- **Zero mode**

- Solution of $\partial_r V_A = -[A_r, V_A]$, with the bdry. condition $V_A|_{r\to\infty} = V_{(0)A}(y)$

  $\Rightarrow V_A(x) = U V_{(0)A}(y) U^{-1}, \qquad U = \mathrm{e}^{\int_r^\infty \mathrm{d}r\, A_r}$

- **Charges**

  $Q[\theta] = -\oint \mathrm{d}^2 y\, \theta^a \pi_a^r\,, \quad Q_s[\eta] = \frac{1}{g^2} \oint \mathrm{d}^2 y\, \sqrt{\gamma}\, \eta_A^a A_a^A$

- **Symmetry transformations**

  $\delta_{\theta,\eta} A_u^a = \theta_u^a, \qquad \delta_{\theta,\eta} A_r^a = -D_r \theta^a, \qquad \delta_{\theta,\eta} A_A^a = -D_A \theta^a + \epsilon\, \eta_A^a$

- **Non-Abelian charge algebra**

$\{Q[\theta_1], Q[\theta_2]\}$	$= Q[[\theta_1, \theta_2]]$	$\longrightarrow$ Q is non-Abelian
$\{Q[\theta], Q_s[\eta]\}$	$= Q_s[[\theta, \eta]] + \frac{1}{g^2} \oint \mathrm{d}^2 y\, \sqrt{\gamma}\, \eta_a^A \partial_A \theta^a$	
$\{Q_s[\eta_1], Q_s[\eta_2]\}$	$= 0$	$\longrightarrow$ Q_s is Abelian

Extension to Yang-Mills theory

- **Mode algebra**

$$\begin{aligned}
\left\{G^a_{nm}, G^b_{kl}\right\} &= f^{ab}_{\ \ c}\, G^c_{n+k,m+l} \\
\left\{G^a_{nm}, S^b_{kl}\right\} &= f^{ab}_{\ \ c}\, S^c_{n+k,m+l} + \kappa n g^{ab} \delta_{n+k,0}\delta_{m+l,0} \\
\left\{G^a_{nm}, \bar{S}^b_{kl}\right\} &= f^{ab}_{\ \ c}\, \bar{S}^c_{n+k,m+l} + \kappa m g^{ab} \delta_{n+k,0}\delta_{m+l,0}
\end{aligned}$$

- **Level** $\kappa = \frac{4\pi^2}{g^2}$
- One can apply the Sugawara method again: $\{K_n, K_m\} = (n-m)K_{n+m}$
- Symmetries at the asymptotic null boundary, described by KM algebras and Virasoro algebras, are general features of 4D gauge theories
- *Nonlinear algebra of Virasoro generators generically appears*

Discussion

Degrees of freedom count

- Dirac formula $\quad \mathrm{d.o.f.} = N - N_{1^{st}\mathrm{class}} - \frac{1}{2} N_{2^{nd}\mathrm{class}}$

- Electromagnetism

A_μ	$N = 4$
$\pi^u, \; \chi = \partial_i \pi^i$	$N_{1^{st}\mathrm{class}} = 2$
χ_A	$N_{2^{nd}\mathrm{class}} = 2$

- $\mathrm{d.o.f.} = 4 - 2 - \frac{1}{2}2 = 1$ **WRONG!!** $\mathrm{d.o.f.} = 2$

- The Dirac formula is applicable only when the multipliers are either arbitrary (1^{st} class constraints) or fully determined (2^{nd} class constraints).

  [Henneaux, Teitelboim 1992]

- It fails when the multipliers satisfy a differential equation.

- In the null foliation: $\partial_r \lambda^A = f^A \quad \Rightarrow \quad \lambda^A = \Lambda^A(y) + \bar{\lambda}^A$

Discussion

Asymptotic conditions

- Invariance of boundary conditions under Poincaré transformatons is not straighforward
- Hamiltonian treatment at spatial infinity needs additional **parity conditions** to ensure invariance under boosts

- Electromagnetism [Henneaux, Troessaert 2018]
- Yang-Mills [Tanzi, Giulini 2020]

- Null-slices foliated standard b.c. are invariant under Poincaré group

- Electromagnetism [Bunster, Gomberoff, Pérez 2018]
- Yang-Mills [He, Mitra 2009]

Discussion

Poincaré transfromations

- We found several Kac-Moody algebras, but not all of them are related to the global Poincaré symmetry in 4D spacetime.
- Canonical generator of 4D Poincaré transformations

$$P[\xi] = \int \mathrm{d}^3x \, T^u{}_\mu \xi^\mu$$

- Differentiability of this generator is ensured by adding the boundary term $Q[\theta] + Q_s[\eta]$ with the parameters $\theta = i_\xi A_{(0)}$ and $\eta_A = \xi^u \Lambda_A$.

Symplectic structure

- Symplectic form $\omega = \int \mathrm{d}^3x \, \delta A_\mu \wedge \delta\pi^\mu$ is invariant under gauge and Poincaré transformations, for instance $i_{X_\theta}\omega = -\delta G_Q[\theta]$ and $i_{X_\eta}\omega = -\delta S_Q[\eta]$

$$X_\theta = \int \mathrm{d}^3x \left(\theta^u \frac{\delta}{\delta A_u} - \partial_i\theta \, \frac{\delta}{\delta A_i} \right), \quad X_\eta = \int \mathrm{d}^3x \left(\frac{\sqrt{\gamma}}{e^2} \nabla_A \eta^A \frac{\delta}{\delta \pi^r} + \epsilon \eta_A \frac{\delta}{\delta A_A} \right)$$

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