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Unconventional Unification

Progress on gravitational physics:
45 years of Belgian-Chilean collaboration
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Plan of the talk:

1. Comments on the Standard Model

2. Combining representations
    a. Gravity: SO(d) → SO(d+1)
    b. Dynamical frustration: SO(d+1) → SO(d)
    c. How to accommodate spin-1/2?

3. A simple idea

4. Actions
    a. 3d
    b. 4d

5. SUSY as a contingent symmetry

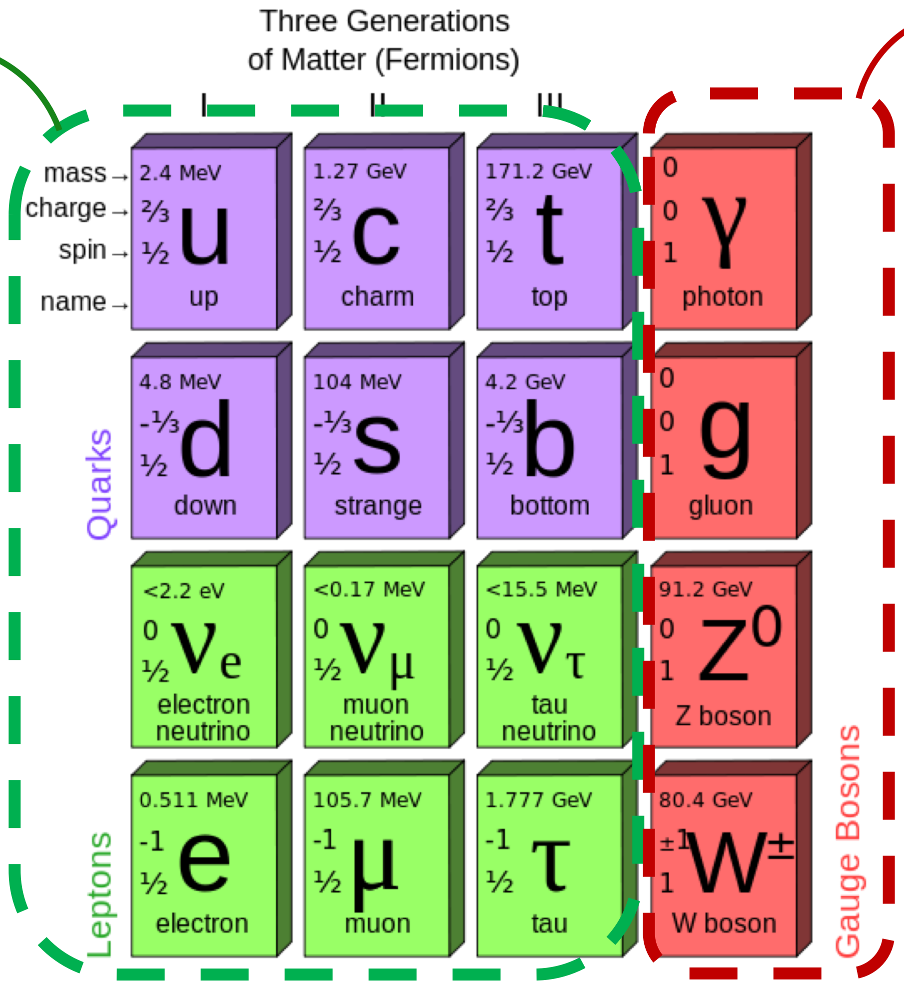
6. Overview

I. Standard Model

Standard Model

Matter (ψ)

- Fermions, s=1/2
- **Gauge vectors (fundamental rep.)**
- Lorentz spinors
- Spacetime scalars (zero forms)
- 1st order field eqs.



Interaction Carriers (A)

- Bosons s=1
- **Gauge connections (adjoint rep.)**
- Lorentz scalars
- Spacetime vectors (one-forms)
- 2nd order field eqs.

(**BEGH boson ?**)

Conventional Grand Unified models tried to replace the SM symmetry group by a (semi)simple Lie group that contains it:

$$\mathbf{G}_{SM} = \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) \subseteq \mathbf{G}_{GUT}$$

- Popular choices over the past 50 years included: $\mathrm{SU}(5)$, $\mathrm{SO}(8)$, $\mathrm{SO}(10)$, supersymmetry (gravity excluded)

- More sophisticated approaches included gravity: supergravity, string theory.

Standard Model

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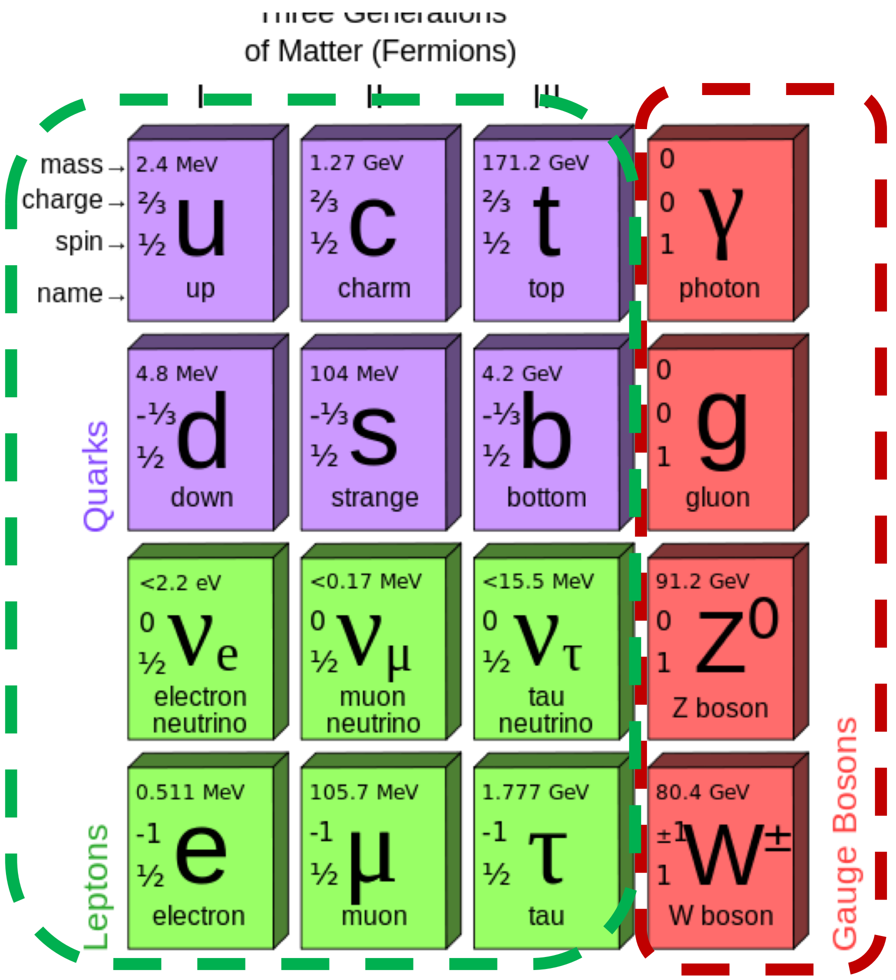


Interaction Carriers (A)

- Bosons S=1
- **Gauge connections (adjoint rep.)**
- Lorentz scalars
- Spacetime vectors (one-forms)
- 2nd order field eqs.

Is it possible to combine these two families as parts of a single object?

2. Combining representations

An example from gravity

In gravity, the spin connection $\omega^{ab} = \omega^{ab}_{\ \ \mu} dx^{\mu}$ (adjoint) and the vielbein $e^{a} = e^{a}_{\ \mu} dx^{\mu}$ (fundamental) can be combined as a connection of a larger group:

$$\left\{ \omega^{ab} \qquad e^{a} \right\} \longrightarrow W^{AB} = \begin{bmatrix} \omega^{ab} & e^{a} \\ -e^{b} & 0 \end{bmatrix}$$

Adjoint SO(3,1)	Fundamental SO(3,1)	Adjoint G_4

$$G_4 = \begin{cases} SO(3,2), & AdS_4 \\ SO(4,1), & dS_4 \end{cases}$$

Equivalently,

$$\boldsymbol{W} = W^{AB} \boldsymbol{J}_{AB} = \tfrac{1}{2}\omega^{ab}\boldsymbol{J}_{ab} + e^{a}\boldsymbol{J}_{a}$$

Perhaps gauge connections ($\boldsymbol{A}$) and matter fields ($\boldsymbol{\psi}$) could also be combined into a single connection of dS_4 or AdS_4 .

However, there are a couple of hurdles...

Hurdle # 1 (The enlarged symmetry might be absent)

(A)dS curvature: $F = dW + W \wedge W = F^{AB} \boldsymbol{J}_{AB}$

where

$$F^{AB} = \begin{bmatrix} R^{ab} \pm e^a e^b & T^a \\ -T^b & 0 \end{bmatrix} \qquad \begin{matrix} + \rightarrow \mathrm{AdS}_4 \\ - \rightarrow \mathrm{dS}_4 \end{matrix}$$

(A)dS-invariant Lagrangian?

$$L = F^{AB} \wedge Q_{ABCD} F^{CD}$$

Invariant tensor
[for SO(3,2) or SO(4,1)]

$$L = F^{AB} \wedge Q_{ABCD} F^{CD}$$

- Q_{ABCD} (A)dS-invariant $\Longrightarrow \int L =$ characteristic class (Chern-Weil theorem)

  $\Longrightarrow$ No dynamics!

- Dynamics requires (A)dS symmetry to be broken
- Cheapest option: (A)dS$_4$ → SO(3,1) ; $Q_{ABCD} \rightarrow \epsilon_{abcd}$

$$\boxed{L = \epsilon_{abcd} F^{ab} \wedge F^{cd} = L_{EH} + cc + E_4}$$

The (A)dS invariance might be a feature of some solutions, but not a gauge symmetry of the theory (action). Contingent symmetry

We will find other examples of contingent symmetries in due course.

Hurdle # 2 (How to include s=1/2 fermions?)

In the previous example both ω^{ab} and e^a are 1-forms.
Fermions in the SM, however, are 0-forms

Observation: s =1/2 fields require a metric

$$\not{\partial} = \Gamma^\mu \partial_\mu , \qquad \{\Gamma^\mu , \Gamma^\nu\} = 2g^{\mu\nu}$$

with $\Gamma^\mu = E^\mu_a \Gamma^a$, where $\{\Gamma^a , \Gamma^b\} = 2\eta^{ab}$, and E^μ_a is the inverse vielbein,

$$E^\mu_a E^\nu_b g_{\mu\nu} = \eta_{ab}.$$

Using the vielbein, one can turn the spin-1/2 field into a spinor 1-form,

$$\chi = e^a \Gamma_a \psi = \Gamma_a \psi \, e^a_\mu dx^\mu .$$

This will be useful in the construction...

3. Unconventional idea

Combine a gauge connection $A^r{}_\mu dx^\mu$, the Lorentz connection $\omega^{ab}{}_\mu dx^\mu$ and a spinor 1-form $\chi^\alpha = \Gamma_a\, \psi\, e^a_\mu dx^\mu$ into a single <u>connection</u> field:

$$\mathcal{A} \sim A^r \mathrm{K}_r + \overline{Q}_\alpha \chi^\alpha + \overline{\chi}_\alpha Q^\alpha + \tfrac{1}{2}\omega^{ab} \mathrm{J}_{ab} + \boxed{\;\dots\;}$$

Internal gauge generators

Complex SUSY generators

Lorentz generators

Needed to close the algebra

This is still looks conventional

- Townsend, MacDowell & Mansouri SUGRA in 4D (1976)
- Achúcarro & Townsend Chern-Simons SUGRA in 3D (1986)

The only technical difference with those SUGRA models is the way the fermion enters.

We take the spinor χ as a composite, $\chi^{\alpha}{}_{\mu} \equiv \left(\Gamma_\mu\right)^{\alpha}{}_{\beta}\psi^{\beta}$ (*Matter Ansatz*)

Standard s $= {}^{1}/_{2}$ *spinor*

where $\Gamma_\mu = \boxed{\Gamma_a}\boxed{e^a_\mu}$

Dirac matrices (tangent space)

$$\{\Gamma_a, \Gamma_b\} = 2\eta_{ab} I$$

Vielbein (soldering form) projects from tangent space onto the spacetime manifold.

The spinor 1-form χ is completely determined by the spin-1/2 field

$$\chi_\mu = \Gamma_\mu \psi \implies \psi = \frac{1}{D}\Gamma^\mu \chi_\mu$$

A generic spinor 1-form belongs to a reducible representation

$$\chi_\mu^\alpha \in 1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{1}{2}$$

In standard Supergravity: $\Gamma^\mu \chi_\mu = 0$ $\rightarrow$ $\chi^\alpha{}_\mu \in 1 \otimes \frac{1}{2} = \frac{3}{2} \oplus \cancel{\frac{1}{2}}$

Here $\chi_\mu = \Gamma_\mu \psi$ and therefore

$$\left(\delta_\nu^\mu - \frac{1}{D}\Gamma_\nu \Gamma^\mu\right)\chi_\mu = 0 \quad \rightarrow \quad \chi^\alpha{}_\mu \in 1 \otimes \frac{1}{2} = \cancel{\frac{3}{2}} \oplus \frac{1}{2}$$

Our approach is the opposite of standard SUGRA. We use the discarded spin-1/2 sector of Supergravity

4. Actions

The action is the integral of a gauge-invariant *D*-form.

$$I = \int L(\mathcal{A}) = \int L(A, \psi, \ldots)$$

There are two standard options:

- Chern-Simons ($D = 2n+1$ only)

$$L_{2n+1} = \left\langle \mathcal{A}(d\mathcal{A})^n + c_1 \mathcal{A}^3 (d\mathcal{A})^{n-1} + \cdots + c_n \mathcal{A}^{2n+1} \right\rangle$$

and

- Yang-Mills (any D)

$$L_{\mathrm{YM}} = \langle \mathcal{F} \wedge \circledast \mathcal{F} \rangle$$

where $\mathcal{F} = d\mathcal{A} + \mathcal{A}\,\mathcal{A}$

$\langle\ \rangle$ = invariant trace, and $\circledast$ ~ Hodge dual

a. Example in 3 dimensions

Consider a connection for an algebra that include

internal U(1) x SU(2) spacetime and fermionic generators

$$\mathcal{A} = iA + A^A \mathrm{T}_A + \tfrac{1}{2}\omega^{ab}\mathrm{J}_{ab} + \bar{\psi}^r_\alpha (\Gamma)^\alpha_\beta Q^\beta_r + \bar{Q}^r_\alpha (\Gamma)^\alpha_\beta \psi^\beta_r$$

$$\left.\begin{array}{l} \boxed{i \to U(1)} \\ \boxed{\mathrm{T}_A \to SU(2)} \\ \boxed{\mathrm{J}_{ab} \to SO(1,2)} \\ \boxed{Q^\beta_r,\ \bar{Q}^r_\alpha \to SUSY} \end{array}\right\}$$ Superalgebra $su(1,2|2)$

The superalgebra contains $SO(1,2)$, $U(1)$, $SU(2)$ and SUSY,

$SO(1,2)$: $$\left[\mathrm{J}^{ab}, \mathrm{J}^{cd}\right] = \eta^{bc}\mathrm{J}^{ad} - \eta^{ac}\mathrm{J}^{bd} + \eta^{ad}\mathrm{J}^{bc} - \eta^{bd}\mathrm{J}^{ac},$$

$SU(2)$: $$\left[\mathrm{T}_i, \mathrm{T}_j\right] = \mathrm{i}\varepsilon_{ij}{}^{k}\mathrm{T}_k, \qquad \left[\mathrm{T}_k, \mathrm{J}^{ab}\right] = 0$$

$U(1)$

SUSY: $$\left\{Q_r^\alpha, \overline{Q}_\beta^s\right\} = i\delta_\beta^\alpha \mathrm{T}_i(\sigma^i)_r^s + \tfrac{1}{2}\delta_r^s \mathrm{J}^{ab}(\Gamma_{ab})^\alpha{}_\beta + \delta_\beta^\alpha\delta_r^s Z$$

$$\left[\mathrm{J}^{ab}, Q_r^\alpha\right] = \tfrac{1}{2}(\Gamma^{ab})^\alpha{}_\beta Q_r^\beta; \quad \left[\mathrm{J}^{ab}, \overline{Q}_\alpha^r\right] = -\tfrac{1}{2}(\Gamma^{ab})^\beta{}_\alpha \overline{Q}_\beta^r,$$

$$\left[\mathrm{T}_i, Q_r^\alpha\right] = i(\sigma_i)_r^s Q_s^\alpha; \quad \left[\mathrm{T}_i, \overline{Q}_\alpha^r\right] = -i(\sigma_i)_s^r \overline{Q}_\alpha^s,$$

The spacetime dimension severely restricts the possible superalgebras

[For real/Majorana spinors, D=3 $\Longrightarrow$ $osp(2|\mathrm{N})$]

D=3

The Chern-Simons form defines a quasi-invariant gauge action for $\mathcal{A}$

$$L = \tfrac{1}{2}\left\langle \mathcal{A} d\mathcal{A} + \tfrac{2}{3}\mathcal{A}\mathcal{A}\mathcal{A} \right\rangle$$

where the bracket is the invariant trace in $su(1,2|2) \oplus u(1)$.

$$L = \overbrace{\tfrac{1}{2} A dA}^{U(1)\text{-}CS} + \overbrace{\mathrm{Tr}[\tfrac{1}{2}\mathbf{A} d\mathbf{A} + \tfrac{1}{3}\mathbf{A}^3]}^{SU(2)\text{-}CS} + \overbrace{\tfrac{1}{2}[\omega^a{}_b d\omega^b{}_a + \tfrac{2}{3}\omega^a{}_b \omega^b{}_c \omega^c{}_a]}^{SO(1,2)\text{-}CS\ (\text{gravity})}$$

$$-\bar{\psi}(\not{\partial} + i\not{A} + \not{\mathbf{A}} - \tfrac{1}{4}\Gamma_a \not{\omega}^{ab}\Gamma_b)\psi |e| d^3x \qquad \text{Dirac}$$

$$-\bar{\psi}\psi\, e^a T_a \qquad \textit{Torsional coupling}\ (\text{mass})$$

This ordinary-looking Lagrangian describes the long wavelength limit of graphene.
The only propagating degree of freedom is the spin-1/2 Dirac fermion.

Field equations:

$$\delta A: \qquad F^A = \tfrac{i}{2}\varepsilon_{abc}\overline{\psi}\mathrm{T}^A\Gamma^a\psi e^b e^c \qquad (1)$$

$$(F^A_{\mu\nu} = \varepsilon_{\mu\nu\lambda} j^{\lambda A})$$

$$\delta\omega^{ab}: \qquad R^{ab} = -2\overline{\psi}\psi e^a e^b \Longrightarrow DT^a = 0 \qquad (2)$$

$$\delta\overline{\psi}: \qquad [\not{\partial} + i\not{A} - \tfrac{1}{4}\Gamma^a\omega_{ab}\Gamma^b + \mu]\psi = 0 \qquad (3)$$

Fermion mass

$$\delta e^a: \qquad \overline{\psi}\varepsilon_{abc}\Gamma^c[\overleftarrow{d}e^b - e^b\overrightarrow{d} + 2iAe^b]\psi = 2\overline{\psi}\psi T_a \qquad (4)$$

- Standard equations for CS $SU(2)$, gravity and spin ½ in 2+1 dimensions.
- ψ acquires "mass" from torsion: $\mu=\eta_{ab}e^a_\mu T^b_{\nu\lambda}\varepsilon^{\mu\nu\lambda}$

$$DT^a = 0 \Rightarrow T_a = \tfrac{1}{6}\mu\varepsilon_{abc}e^b e^c, \quad \mu = \text{const}$$

The fermion mass is an *effect of the background*, *not a parameter in the action.*

b. Example in 4 dimensions

Required by the algebra

- Minimal susy extension of *su*(2), *so*(3,1) leads to *osp*(4|2):

$$\mathcal{A} = \underbrace{A_r \mathbf{K}^r + A\mathbf{Z}}_{\boxed{\begin{array}{c} SU(2)\mathrm{x}U(1) \\ \text{Internal symmetry} \end{array}}} + \bar{Q}_\mathrm{i}\Gamma\psi^\mathrm{i} + \overline{\psi}_\mathrm{i}\Gamma Q^\mathrm{i} + \underbrace{f^a \mathbf{J}_a + \tfrac{1}{2}\,\omega^{ab}\mathbf{J}_{ab}}_{\boxed{\begin{array}{c} SO(3,2) \\ \text{anti - de Sitter} \end{array}}}$$

Complex Dirac spinor

$$\left\{\bar{Q}^\mathrm{i}_\alpha, Q^\beta_\mathrm{j}\right\} = i\delta^\beta_\alpha\left[(\sigma^r)^\mathrm{i}{}_\mathrm{j}\,\mathbf{K}_r + \delta^i_j\mathbf{Z}\right] + \left(\tfrac{1}{2}(\Gamma^a)^\beta{}_\alpha\,\mathbf{J}_a - \tfrac{1}{2}(\Sigma^{ab})^\beta{}_\alpha\mathbf{J}_{ab}\right)\delta^\mathrm{i}_\mathrm{j}$$

- Curvature:

$$\begin{aligned} \mathcal{F} &= d\mathcal{A} + \mathcal{A}\,\mathcal{A} \\ &= F_r\ \mathbf{K}^r + F\mathbf{Z} + \bar{Q}_\mathrm{i}\,\mathcal{F}^\mathrm{i} + \overline{\mathcal{F}}_\mathrm{i}\,Q^\mathrm{i} + F^a\mathbf{J}_a + \tfrac{1}{2}F^{ab}\mathbf{J}_{ab} \end{aligned}$$

P.D.Alvarez, P.Pais, JZ, Phys.Lett. **B735** (2014) 314

The superalgebra $su(2,2|2)$ contains $SO(1,2)$, $U(1)$, $SU(2)$ and SUSY:

$so(3,2)$ $$\begin{cases} [\mathbf{J}^{ab}, \mathbf{J}^{cd}] = \eta^{ad}\mathbf{J}^{bc} - \eta^{ac}\mathbf{J}^{bd} + \eta^{bc}\mathbf{J}^{ad} - \eta^{bd}\mathbf{J}^{ac} \\ [\mathbf{J}^{ab}, \mathbf{J}^{c}] = \eta^{ac}\mathbf{J}^{b} - \eta^{bc}\mathbf{J}^{d}, \qquad [\mathbf{J}^{a}, \mathbf{J}^{b}] = \mathbf{J}^{ab} \end{cases} \qquad \begin{bmatrix} \mathbf{J}^{ab} & \mathbf{J}^{a} \\ -\mathbf{J}^{b} & 0 \end{bmatrix} = \mathbf{J}^{AB}$$

$su(2)$: $$[\,\mathbf{T}_i\,,\,\mathbf{T}_j\,] = i\,\epsilon_{ijk}\,\mathbf{T}_k\,, \qquad [\,\mathbf{T}_i\,,\mathbf{J}^{ab}] = 0 = [\,\mathbf{T}_i\,,\mathbf{J}^{a}]$$

susy: $$\{\mathbf{Q}^{\alpha}_{r}\,,\ \bar{\mathbf{Q}}^{s}_{\beta}\} = i\,\delta^{\alpha}_{\beta}(\sigma^{i})^{s}{}_{r}\mathbf{T}_i + \tfrac{1}{2}\delta^{s}_{r}(\Gamma^{AB})^{\alpha}_{\beta}\,\underbrace{\mathbf{J}_{AB}}_{so\,(3,2)} + \delta^{s}_{r}\delta^{\alpha}_{\beta}\underbrace{\mathbf{Z}}_{U(1)}$$

$$[\mathbf{J}^{AB}, \mathbf{Q}^{\alpha}_{r}] = \tfrac{1}{2}(\Gamma^{AB})^{\alpha}_{\beta}\mathbf{Q}^{\beta}_{r}, \qquad [\mathbf{J}^{AB}, \bar{\mathbf{Q}}^{r}_{\alpha}] = -\tfrac{1}{2}(\Gamma^{AB})^{\beta}_{\alpha}\bar{\mathbf{Q}}^{r}_{\beta}$$

$$[\mathbf{T}_i\,, \mathbf{Q}^{\alpha}_{r}] = -i(\sigma_I)^{s}{}_{r}\mathbf{Q}^{\alpha}_{s}\,, \qquad [\mathbf{T}_i\,, \bar{\mathbf{Q}}^{r}_{\alpha}] = -i(\sigma_I)^{r}{}_{s}\bar{\mathbf{Q}}^{s}_{\alpha}$$

$$[\mathbf{Z}\,, \mathbf{Q}^{\alpha}_{r}] = i\mathbf{Q}^{\alpha}_{r}\,, \qquad [\mathbf{Z}\,, \bar{\mathbf{Q}}^{r}_{\alpha}] = -i\bar{\mathbf{Q}}^{r}_{\alpha}$$

[For real/Majorana spinors, D=4 $\Longrightarrow$ $osp(4|\mathrm{N})$]

Curvatures:

$$F = dA - \frac{i}{4}\bar{\psi}_i \not{e}\, \not{e}\, \psi^i \qquad U(1)$$

$$F_r = dA_r + \frac{1}{2}\epsilon_r{}^{st} A_s A_t - \frac{i}{2}\bar{\psi}\not{e}\sigma_r\not{e}\psi \qquad SU(2)$$

$$\mathcal{F}^i = d(\not{e}\psi^i) + iA_r(\sigma^r)^i{}_j\psi^j + \frac{1}{4}\Omega^{AB}\Gamma_{AB}\psi^i \qquad SUSY$$

$$\left.\begin{aligned} F^a &= df^a + \frac{1}{2}\omega^a{}_b f^b + \frac{1}{2}\bar{\psi}_i \not{e}\,\Gamma^a \not{e}\,\psi^i \\ F^{ab} &= R^{ab} + f^a f^b - \bar{\psi}_i \not{e}\,\Gamma^{ab} \not{e}\,\psi^i \end{aligned}\right\} SO(3,2)$$

For $D = 2n$, the only invariant $2n$-forms without involving the Hodge dual are *characteristic classes* (Chern-Weil theorem).

No locally $SO(3,2)$-invariant gravity in $D = 4$ [Townsend/MacDowell-Mansouri frustration]

➔ $SO(3,2)$ is broken down to $SO(3,1)$

➔ Local SUSY must also be broken

➔ The largest surviving local symmetry is $U(1) \times SU(2) \times SO(3,1)$

- $\nexists$ $Osp(4|2)$ or $SO(3,2)$-*invariant traces in* 4*D. The biggest symmetry group with an invariant trace is* $SU(2)\times U(1)\times SO(3,1)$ ➔ *Largest gauge symmetry of the action.*

$U(1)\times SU(2)\times SO(3,1)$ - invariant trace

Lagrangian: $L_4 = \langle \mathcal{F} \wedge^{\circledast} \mathcal{F} \rangle$

- *Hodge dual* $\circledast = \begin{cases} * & \text{Internal} \\ \Gamma_5 & \text{Fermions} \\ \epsilon_{abcd} & \text{Spacetime} \end{cases}$

$$L_4 = -\tfrac{1}{4}F_r \wedge {}^*F^r + F \wedge {}^*F + \mathcal{F}_i\, \Gamma_5\, \overline{\mathcal{F}}^i + \tfrac{1}{2}\epsilon_{abcd}F^{ab}F^{cd}$$

- f^a *is an auxiliary field* ➔ *Townsend's choice:* $f^a_\mu = \sigma\, e^a_\mu$

➔ *Conformal (scale) symmetry is broken*

4D Lagrangian

$$L = \Big\{ -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad \text{Maxwell / YM}$$

$$+ \tfrac{i}{2}[\bar{\psi}\overleftarrow{\nabla}\psi - \bar{\psi}\overrightarrow{\nabla}\psi] + \bar{\psi}\Gamma_5\Gamma_a T^a\psi \qquad \text{Dirac}$$

$$-\sigma^{-2}\boxed{[(\bar{\psi}\psi)^2 - (\bar{\psi}\Gamma_5\psi)^2]}\Big\}\sqrt{-g}\,d^4x \qquad \text{Nambu-Jona Lasinio}$$

$$-\tfrac{1}{16}\varepsilon_{abcd}[R^{ab} - \sigma e^a e^b][R^{cd} - \sigma e^c e^d] \qquad \text{Einstein-Hilbert + cc}$$

- Standard couplings: $\nabla_\nu = \partial_\nu - iA_\nu + \tfrac{1}{4}\Gamma_{ab}\omega_\nu^{ab} - \tfrac{i\mu}{2}\Gamma_\nu$
- No $\partial_\mu\partial_\nu\psi$ - terms: fermions behave as standard matter
- Cosmological constant $\Lambda \sim -\sigma^2$
- Newton's constant $G \sim \sigma^{-1}$

✓ *Phenomenology for low energy, 4D theory*

Other 4D U-SUSY models

Dimensional reduction from 5D CS.
Y.Gómez, J.Helayel-Neto, Phys.Lett. **B777** (2018)275

Superconformal:

- $so(3{,}1)\times su(2)\times u(1) \subseteq \mathrm{su}(2{,}2|2)$ JHEP07(2020)205
  [Here chiral symmetry is broken by the AdS vacuum: $\Lambda \neq 0 \;\; \Rightarrow \psi_L \neq \psi_R$]
- $so(3{,}1)\times su(\mathrm{N})\times u(1) \subseteq \mathrm{su}(2{,}2|\mathrm{N})$ JHEP07(2021)176, JHEP02(2022)111

Extended superconformal:

- $so(3{,}1)\times su(5)_5\times su(5)_{10} \subseteq \mathrm{su}(2{,}2|d_\mathrm{n})$ [Georgi–Glashow]
  J.Math.Phys.**63**(2022)042304, JHEP02(2023)050

5. SUSY as a contingent symmetry

Underlying supersymmetry

External (spacetime) symm

$[J, J] \sim J$

$[J, T] = 0$

Internal gauge symm.

$[T, T] \sim T$

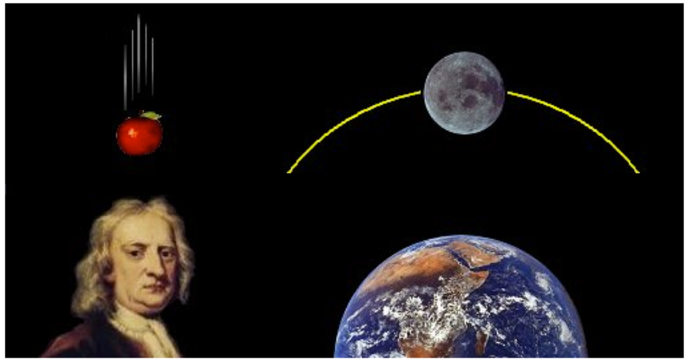
$[J, Q] \sim Q$

$\{Q, Q\} \sim T + J$

$[T, Q] \sim Q$

The fermionic sector belongs to fundamental irreps of the internal and spacetime gauge groups: matter couples to all gauge fields including gravity.

➔ Fermions provide a bridge between internal and spacetime symmetries



Spacetime

Matter (fermions) couples to both

Internal

Does this mean that the action is supersymmetric?

SUSY transformation
$$\begin{cases} \delta A^i_\mu = \frac{i}{2}\left[\bar{\epsilon}\Gamma_\mu\sigma^i\psi + \bar{\psi}\Gamma_\mu\sigma^i\epsilon\right] \\ \delta\omega^{ab}_{\ \ \mu} = \bar{\epsilon}\Gamma_\mu\Gamma^{ab}\sigma^i\psi + \bar{\psi}\Gamma^{ab}\Gamma_\mu\sigma^i\epsilon \\ \delta\psi = \frac{1}{D}\nabla\epsilon\,, \qquad \delta e^a_{\ \mu} = 0 \end{cases}$$

For the algebra to close, the system should also be invariant under the AdS boosts, but they are no longer symmetries:

- The spacetime invariance of the action is $SO(3,1)$, not $SO(3,2)$
- f^a is no longer a gauge field but an auxiliary field
- The action is not supersymmetric

SUSY transformations can leave the vacuum ($\psi = 0$) unchanged, provided

$$\left(\delta_\nu^\mu - \frac{1}{D}\Gamma_\nu \Gamma^\mu\right) \nabla_\mu \epsilon = 0 \,.$$

This generically requires the background to admit Killing spinors [$\nabla\epsilon = 0$].

There might exist SUSY-invariant states (e.g., BPS vacua), similar to Poincaré or AdS-invariant ones. But SUSY would not be an invariance of the action.

Local SUSY could be an approximate symmetry for some configurations or in asymptotic regions, like Poincaré or AdS invariance.

➔ SUSY would be a *contingent* symmetry (depends on the vacuum).

6. Overview

Ingredients (Input):

- $\mathrm{Ad}_{\mathrm{G}} + \mathrm{Fund}_{\mathrm{G}} \hookrightarrow \mathrm{Ad}_{\widehat{\mathrm{G}}}\,;\ \mathrm{G} \subseteq \widehat{\mathrm{G}}$

- Superconnection:

$$\mathcal{A} = [\,\tfrac{1}{2}\omega^{ab}\mathbf{J}_{ab} + \cdots\,] + [\bar{\boldsymbol{Q}}^{\alpha}\chi_{\alpha} + \bar{\chi}^{\alpha}\boldsymbol{Q}_{\alpha}] + [\,A^{K}\mathbf{T}_{K}\,]$$

$$\begin{bmatrix}\text{spacetime}\\ \text{symmetry}\end{bmatrix} \qquad \begin{bmatrix}\text{charged}\\ \text{fermion}\end{bmatrix} \qquad \begin{bmatrix}\text{internal}\\ \text{symmetry}\end{bmatrix}$$

- Matter ansatz: $\chi^{\alpha}_{\mu} = (\Gamma_{\mu})^{\alpha}{}_{\beta}\ \psi^{\beta},\ \left(\delta^{\mu}_{\nu} - \frac{1}{D}\Gamma_{\nu}\Gamma^{\mu}\right)\chi_{\mu} \equiv 0$

- Invariant trace for the largest subgroup: $\langle \ldots \rangle$

- Hodge dual $\circledast$ (required for even D)

The role of SUSY: a *guiding principle*

- It connects spacetime and internal groups that can be combined
- Superalgebra fixes gauge couplings
- Brings in gravity
- Supersymmetry algebra is eventually broken down to

  (Internal gauge group) x (Lorentz group)
- Invariance under entire supergroup for some vacua: BPS states

Consequences of the construction:

- All fields are part of the same superconnection $\mathcal{A}$

$$\left.\begin{array}{r} \mathbb{F}\ \text{(matter)} \ \ \text{- sections} \\ \mathbb{B}\ \text{(interactions)} \ \ \text{- connections} \end{array}\right\}$$

$\boxed{\textit{Packaged into a single gauge connection}}$

- Only standard kinetic terms (Yang-Mills, Dirac, Chern-Simons)
- Only $s = ½,1$ fundamental fields (s =0, 3/2, 2 can be composite)
- Not all internal and spacetime symmetries can be combined
- Only standard gauge couplings ($\sim\bar{\psi}\,\not{A}\,\psi$ ✓, $\bar{\psi}\,\not{A}_1\not{A}_2\not{A}_3\psi$ ✗)
- (Bare) coupling constants and masses are fixed
- No SUSY pairs, no matching d.o.f., no hidden sectors
- Only Lorentz & internal symmetries are gauge symmetries (SUSY is contingent)

Nice things about this approach:

- All fields are part of the same connection $\mathcal{A}$
- Given D and the spacetime signature the algebra is very restricted
- Only $s = ½,1$ fundamental fields (s =0, 3/2, 2 can be composite)
- Gravity is unavoidably included
- General covariance automatically built in
- Right kinetic terms, right couplings
- Breaks symmetry by frustration, not spontaneously
- The respected symmetry is always [local Lorentz] x [internal gauge]
- SUSY is a contingent symmetry at most
- Provides a N-JL term in 4D → mass gap, neutrino oscillations
- It is falsifiable (it can be proven wrong)
- It is simple!

Challenges/open questions:

- Observable effects (e.g., in graphene)
- Classical solutions
- Topologically nontrivial vacua
- Renormalizability
- Neutrino masses
- Proton decay
- Anomalies
- Hierarchy
- Matter ansatz in SUGRA:

$$\bar{\chi}_\mu \Gamma^{\mu\nu\rho} \nabla_\nu \chi_\rho = \frac{(D-1)(D-2)}{2} \bar{\psi} \not{\partial} \psi$$

There might be a U-SUSY lurking in every SUGRA theory…

This could be a model for the microscopic world at the current experimentally accessible energies. We do not pretend to have the ultimate description of nature at the most fundamental level.

At that level, differentiability of spacetime and Lorentz invariance may not exist, and new interactions could be relevant. But spacetime is currently well described by a smooth four-dimensional manifold of Lorentzian signature and matter is accurately described by Fermions in irreps of SO(3,1).

Thanks!