Beyond General Relativity: causality, cosmology and astrophysics

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PROGRESS ON GRAVITATIONAL PHYSICS
45 years of Belgian-Chilean collaboration
Brussels, April 12, 2023

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w/G. Arciniega, P. Bueno, P. A. Cano, R. A. Hennigar, L. G. Jaime, R. B. Mann, D. Vázquez Rodríguez, A. Vilar López

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General Relativity as a low energy theory

It is generally understood that GR is a low energy effective theory of gravity.

Low energy means low space-time curvature (also large distance): there must be a scale $\ell_\star \sim \Lambda_\star^{-1}$ at which higher-derivative terms arise.

A single combination at every order leads to second order Euler-Lagrange equations [Lovelock, 1971]

$$\mathcal{I} = \int d^D x \sqrt{-\mathrm{g}} \left[R - 2\Lambda + \frac{\lambda}{\Lambda_\star^2} \left(R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) + \cdots \right] .$$

However, the quadratic term is non-trivial if and only if $D \geq 5$ and, in general, the k-th order term is physical for $D \geq 2k + 1$.

I learned (and worked abundantly) on Lovelock theory and its connections to AdS/CFT, cosmology, gravitational phase transitions, Chern-Simons gravity, BPS Chern-Simons p-branes . . . , from my Chilean years.

Outlook

In this talk I will explore three aspects of higher curvature gravities:

- CAUSALITY
  I will suggest that causal higher curvature gravities are stringy.
- COSMOLOGY
  Higher curvature terms lead to geometric inflation.
- ASTROPHYSICS
  Higher curvature terms magnify primordial black holes accretion rate.

Beyond General Relativity: causality issues

Not all local Lorentz invariant Lagrangians are consistent.

For a $U(1)$ gauge field, *e.g.*, [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006]

$$\mathcal{I} = \int d^4x \sqrt{-\mathrm{G}} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{c_1}{\Lambda_\star^4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{c_2}{\Lambda_\star^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \cdots \right] ,$$

the coefficients c_1 and c_2, a priori unconstrained, give the leading S-matrix amplitudes at energies beneath $\Lambda_\star$.

To avoid analiticity and unitarity problems of the S-matrix, and superluminal modes, they must be possitive.

They are indeed positive if obtained by integrating out electrons in QED.

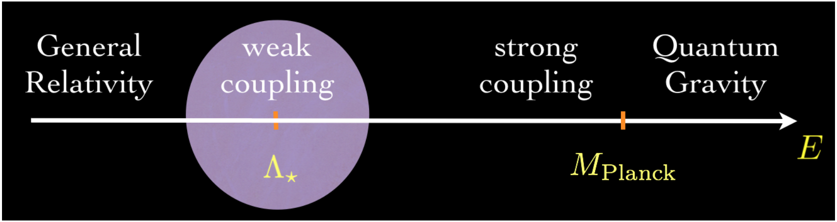
Beyond General Relativity: causality issues

In the case of gravity, the situation is tighter. If we have

$$\mathcal{I} = \int d^D x \sqrt{-\mathrm{G}} \left[R + \frac{c_1}{\Lambda_\star^2} R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda} + \frac{c_2}{\Lambda_\star^4} R_{\mu\nu\sigma\lambda} R^{\sigma\lambda\alpha\beta} R_{\alpha\beta}{}^{\mu\nu} + \cdots \right] ,$$

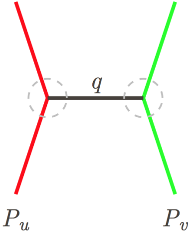
then c_1, $c_2 \simeq 0$ or, else, an infinite tower of massive higher-spin particles must be introduced. [Camanho, Edelstein, Maldacena, Zhiboedov, 2014]

The setup we are dealing with is represented as follows:



Beyond General Relativity: causality issues

The idea is to compute the tree-level 4-point amplitude in the Eikonal limit: $s \simeq P_u P_v$ very large, $t \simeq -\vec{q}^2 \ll s$, and fixed impact parameter $\vec{b}$.



The eikonal phase shift, in the impact parameter representation, is given by

$$\begin{aligned} \delta(s,\vec{b}) &= \frac{1}{2s}\int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} e^{i\vec{b}\cdot\vec{q}} \mathcal{A}^{[4]}_{\text{tree}}(s,t) \\ &= \frac{1}{2s}\mathcal{A}^{[3]}_{13I}(-i\partial_{\vec{b}})\mathcal{A}^{[3]}_{I24}(-i\partial_{\vec{b}}) \int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} \frac{e^{i\vec{b}\cdot\vec{q}}}{\vec{q}^2} \ . \end{aligned}$$

Beyond General Relativity: causality issues

The vertices $\mathcal{A}^{[3]}_{13I} := \mathcal{A}^{[3]}_{13I}(-i\partial_{\vec{b}})$ and $\mathcal{A}^{[3]}_{I24} := \mathcal{A}^{[3]}_{I24}(-i\partial_{\vec{b}})$ read:

$$\mathcal{A}^{[3]}_{13I} = 2P_u^2 \left[(\vec{e}_1 \cdot \vec{e}_3)^2 + \frac{c_1}{\Lambda_\star^2}(\vec{e}_1 \cdot \vec{e}_3)(\vec{e}_1 \cdot \partial_{\vec{b}})(\vec{e}_3 \cdot \partial_{\vec{b}}) + \frac{c_2}{\Lambda_\star^4}(\vec{e}_1 \cdot \partial_{\vec{b}})^2(\vec{e}_3 \cdot \partial_{\vec{b}})^2 \right]$$

$$\mathcal{A}^{[3]}_{I24} = 2P_v^2 \left[(\vec{e}_2 \cdot \vec{e}_4)^2 + \frac{c_1}{\Lambda_\star^2}(\vec{e}_2 \cdot \vec{e}_4)(\vec{e}_2 \cdot \partial_{\vec{b}})(\vec{e}_4 \cdot \partial_{\vec{b}}) + \frac{c_2}{\Lambda_\star^4}(\vec{e}_2 \cdot \partial_{\vec{b}})^2(\vec{e}_4 \cdot \partial_{\vec{b}})^2 \right]$$

Then, the eikonal phase shift scales linearly, $\delta(s, \vec{b}) \sim s$.

The S-matrix reads

$$S(s, \vec{b}) \simeq e^{i\delta(s,\vec{b})} ,$$

and it has to be analytic and bounded,

$$|S(s, \vec{b})| \leq 1 , \qquad \text{for} \quad \mathbb{Im}\, s \geq 0 ;$$

a sign flip in $\delta(s, \vec{b})$ is not physically admissible.

Beyond General Relativity: causality issues

It is impossible to fulfill this condition for all polarizations of the intervening on-shell gravitons. [Camanho, Edelstein, Maldacena, Zhiboedov, 2014]

In fact, $\delta(s, \vec{b})$ is proportional to the Shapiro delay, which relates this problem to causality violation.

We need to invoke new degrees of freedom at $E \sim \Lambda_\star$

- spin J particles contribute as $\delta(s, \vec{b}) \sim s^{J-1}$.

Massive $J = 2$ would do the job but . . .

- they do not conserve angular momentum!

We need an infinite tower of higher-spin particles with a delicate fine-tuning in their couplings, as in string theory! [D'Appolonio, Di Vecchia, Russo, Veneziano, 2015]

Beyond General Relativity: causality issues

What happens in 3D? Spacetime is locally flat outside the locus of a source; there should be no Shapiro delay.

Topologically Massive Gravity [Deser, Jackiw, Templeton, 1982] instead

$$\mathcal{I}_{\rm TMG} = \int d^3x \left[\sigma\sqrt{-g}R + \frac{1}{2\mu}\epsilon^{\mu\nu\rho}\Gamma^{\sigma}_{\mu\alpha}\left(\partial_\nu\Gamma^{\alpha}_{\rho\sigma} + \frac{2}{3}\Gamma^{\alpha}_{\nu\lambda}\Gamma^{\lambda}_{\rho\sigma}\right) + \frac{1}{2}\sqrt{-g}\,\partial_\mu\phi\partial^\mu\phi\right]$$

has a single massive spin-2 graviton mode with $m_g = \mu > 0$, whose kinetic term is well defined for $\sigma = -1$.

Both a scalar particle and the massive graviton crossing a shock wave would experience a Shapiro delay [Edelstein, Giribet, Gómez, Kilicarslan, Leoni, Tekin, 2017]

$$\Delta v = -\frac{2\sigma|p|}{\mu}e^{-\mu b} > 0 \qquad \text{and} \qquad \Delta v = -\frac{3\sigma|p|}{m_g}e^{-m_g b} > 0\ .$$

Unitarity and causality demand negativity of the Newton constant.

Beyond General Relativity: causality issues

Higher curvature gravities have more solutions than perturbative deformations of GR. For instance, [Edelstein, Ghosh, Laddha, Sarkar, 2021]

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \alpha\, R^2 + \beta R_{\mu\nu} R^{\mu\nu}\right) ,$$

with $\beta \leq 0$ and $3\alpha + \beta \geq 0$. For instance

$$ds^2 = -du\, dv + h_0(u, x_i)\, du^2 + \sum_i^{D-2} (dx_i)^2 ,$$

with $h_0(u, x_i) = f(r)\, \delta(u)$, and [Campanelli, Lousto, 1996]

$$f(r) = -\frac{8\pi G\, |P_u|\, \Gamma\left(\frac{D}{2}-1\right)}{\pi^{\frac{D}{2}-1}} \left[\frac{(-2\beta)^{2-\frac{D}{2}}}{\Gamma\left(\frac{D}{2}-1\right)}\left(\frac{r}{\sqrt{-\beta}}\right)^{2-\frac{D}{2}} K_{2-\frac{D}{2}}\left(\frac{r}{\sqrt{-\beta}}\right) - \frac{1}{D-4}\left(\frac{1}{r}\right)^{D-4}\right] ,$$

$K_n(x)$ is the modified Bessel function of the second kind. The Shapiro delay:

$$\Delta v = (\Delta v)_{\rm GR} \times \left(1 - \frac{1}{2^{n-1}\Gamma(n)} x^n K_{-n}(x)\right) > 0 .$$

Beyond General Relativity: geometric inflation

Let us proceed as follows: we would like to write down an action of the form

$$\mathcal{I} = \int d^4x \sqrt{-g} \left(R - 2\Lambda + \sum_{n=2}^{\infty} \lambda_n \, \ell_\star^{2n-2} \, \mathcal{R}_{(n)} \right) ,$$

where $\mathcal{R}_{(n)}$ are polynomials in the Riemann curvature of order n and $L_\star \sim \Lambda_\star^{-1}$ sets the scale at which they become relevant.

The theory has a number of maximally symmetric vacua, depending upon the numerical coefficients λ_n. One of them is connected to the GR vacuum.

If we expand around a vacuum, we find three degrees of freedom: a massless graviton, a ghosty massive graviton and a scalar field.

We can kill the unwanted modes by switching their mass to infinity. This gives algebraic constraints on the $\mathcal{R}_{(n)}$ densities.

Beyond General Relativity: geometric inflation

We will impose the following constraints on the densities $\mathcal{R}_{(n)}$:

(i) right spectrum of massless gravitons in vacuum,

(ii) well-behaved black hole solutions, and

(iii) well-posed initial value problem for cosmology.

For instance, at the quadratic level, there is a unique solution,

$$\mathcal{R}_{(2)} = R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \, .$$

At the cubic level, there is a unique combination $\mathcal{R}_{(3)} := \mathcal{P} - 8\,\mathcal{C}$ [Arciniega, Edelstein, Jaime, 2018], where $\mathcal{P}$ [Bueno, Cano, 2016] and $\mathcal{C}$ [Hennigar, Kubizňák, Mann, 2017] read:

$$\mathcal{P} = 12 R_{\mu\ \nu}^{\ \sigma\ \lambda} R_{\sigma\ \lambda}^{\ \alpha\ \beta} R_{\alpha\ \beta}^{\ \mu\ \nu} + R_{\mu\nu}^{\ \ \ \sigma\lambda} R_{\sigma\lambda}^{\ \ \ \alpha\beta} R_{\alpha\beta}^{\ \ \ \mu\nu} - 12 R_{\mu\nu\sigma\lambda} R^{\mu\sigma} R^{\nu\lambda} + 8 R_{\mu}^{\ \nu} R_{\nu}^{\ \sigma} R_{\sigma}^{\ \mu} \, ,$$

$$\mathcal{C} = R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma}{}_{\alpha} R^{\lambda\alpha} - \frac{1}{4} R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda} R - 2 R_{\mu\nu\sigma\lambda} R^{\mu\sigma} R^{\nu\lambda} + \frac{1}{2} R_{\mu\nu} R^{\mu\nu} R \, .$$

Beyond General Relativity: geometric inflation

Let us then consider the action

$$\mathcal{I} = \int d^4x \sqrt{-g} \left(R - 2\Lambda + \sum_{n=3}^{\infty} \lambda_n \, L_\star^{2n-2} \, \mathcal{R}_{(n)} \right) ,$$

and a FLRW spacetime ansatz

$$ds^2 = -dt^2 + a(t)^2 \left(dr^2 + r^2 d\Omega^2 \right) ;$$

the equations of motion for $a(t)$ are second order. It is suitable to trade t by N,

$$\frac{\mathrm{d}}{\mathrm{d}t} = H \frac{\mathrm{d}}{\mathrm{d}N} , \qquad \text{where} \quad H := \frac{\dot{a}}{a} ,$$

since $a = a_{\rm init}\, e^N$. The set of Friedmann equations read:

$$3F(H) = \frac{1}{M_{\rm Pl}^2} \rho , \qquad -H' \, F'(H) = \frac{1}{M_{\rm Pl}^2} (\rho + P) ,$$

and $\rho' + 3(\rho + P) = 0$, where

$$F(H) := H^2 + L_\star^{-2} \sum_{n=3}^{\infty} (-1)^n \lambda_n \, (L_\star H)^{2n} .$$

Beyond General Relativity: geometric inflation

It was recently shown that the cosmology of a T-dual invariant theory is driven by such function! [Hohm, Zwiebach, 2019]

Albeit in the string frame the coefficients $\{\lambda_n\}$ must be computed, in the Einstein frame $F(H)$ satisfies a second order non-linear differential equation [Krishnan, 2019].

Cosmology is entirely dictated by $F(H)$ —as in Lovelock's black holes [Camanho, Edelstein, 2011] and cosmology [Camanho, 2015]—; that is,

$$L_\star^{-1} \qquad \& \qquad \{\lambda_n\}$$

Ambiguities in the definition of $\mathcal{R}_{(n)}$ densities: $\mathcal{R}^A_{(n)}$ and $\mathcal{R}^B_{(n)}$ differing by $\mathcal{T}^{AB}_{(n)}$ make no contribution to the field equations for the classes of metrics considered here!

This is tantamount to an ambiguity in the $\{\lambda_n\}$.

Reasonable constraints on $F(H)$ include positive-mass black holes [Bueno, Cano, 2017], and absence of local extrema (which would lead to singularities).

Beyond General Relativity: geometric inflation

If we truncate at cubic order, the action

$$\mathcal{I} = \int d^4x \sqrt{-g} \left(R - 2\Lambda + \lambda_3 \, L_\star^4 \, \mathcal{R}_{(3)} \right) + \mathcal{I}_{\text{radiation}} \,,$$

is unique, and leads to accelerated expansion [Arciniega, Edelstein, Jaime, 2018].

By the way, this theory also has interesting holographic applications such as in the computation of 3d CFTs on squashed spheres [Bueno, Cano, Hennigar, Mann, 2018]:

$$\mathcal{F}_{S^3_\epsilon} = \mathcal{F}_{S^3_0} - \frac{\pi^4 C_T}{6} \epsilon^2 \left[1 - \frac{t_4}{630} \epsilon + \mathcal{O}(\epsilon^2) \right] \,,$$

where C_T and t_4 are given by two- and three-point functions of $T_{\mu\nu}$.

If we truncate at $n_{\max}$, the limit $a \to 0$ implies acceleration at early times,

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = 1 - \frac{\ddot{a}}{aH^2} \sim \frac{2}{n_{\max}} \,.$$

the expansion being polynomial rather than exponential.

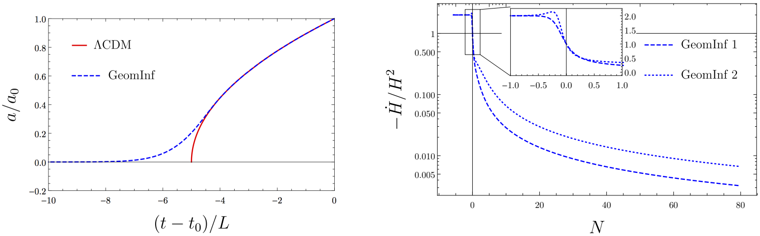
Beyond General Relativity: geometric inflation

We shall not truncate the series. The full tower of higher-curvature terms becomes relevant before reaching the singularity.

The scale factor will grow faster than any polynomial near $a = 0$. For instance:

$$F(H) = H^2\left(1 + \lambda_4 (L_\star H)^6\, e^{(L_\star H)^4}\right) \quad \Rightarrow \quad a(t) \sim \left[e^{-(3|t|/L_\star)^{4/3}}\right]^{1/4}$$

when $t \to -\infty$. The growth is quasi-exponential. Features are model-independent [Arciniega, Bueno, Cano, Edelstein, Hennigar, Jaime, 2018].



This exponential growth gracefully connects at late times with Einstein gravity, as long as the new energy scale, $L_\star^{-1}$, is high enough.

Beyond General Relativity: geometric inflation

The details depend on the choice of parameters $\{\lambda_n\}$ and energy scale $L_\star^{-1}$, but the general message is: an inflationary epoch seems unavoidable.

When $\epsilon = 0$, $H = H_0$ is a root of $F(H)$ and the expansion is pure de Sitter. For the cubic theory, this is unstable [Pookkillath, De Felice, Starobinsky, 2020].

Naively it seems that the inflaton field is unnecessary [Arciniega, Bueno, Cano, Edelstein, Hennigar, Jaime, 2019]. Let us revisit this issue. The first Friedmann equation reads

$$3H^2\,(1 + G(H)) = \frac{1}{M_{\rm Pl}^2}\,\rho\,, \qquad \text{so that} \qquad G(H) := \frac{F(H)}{H^2} - 1\,.$$

and accelerated expansion arises when $G(H) \gg 1$. Late time connection to GR needs $G(H \to 0) \to 0$. There are two stages: [Edelstein, Vázquez Rodríguez, Vilar López, 2020]

- $G(H) \gg 1$, a higher-curvature era at which accelerated expansion happens,
- $G(H) \ll 1$, a GR-ish late time universe.

Let us define $H_{\rm end}$ as the value of the Hubble parameter for which $G(H_{\rm end}) = 1$.

Beyond General Relativity: geometric inflation

The energy density at the end of inflation is $\rho_{\rm end} \sim M_{\rm Pl}^2 H_{\rm end}^2$. Assuming $\rho \sim a^{-3(1+w)}$, if we allow for N e-folds,

$$\rho_{\rm init} \sim \rho_{\rm end} e^{3(1+w)N} \sim M_{\rm Pl}^2 H_{\rm end}^2 e^{3(1+w)N} \, .$$

Demanding $\rho_{\rm init} < M_{\rm Pl}^4$,

$$H_{\rm end} < e^{-\frac{3}{2}(1+w)N} M_{\rm Pl} \, .$$

But $H_{\rm end}$ sets the scale of corrections to GR. Astrophysical tests imply [Hennigar, Poshteh, Mann, 2018], $L_\star \lesssim 10^8$m. Thereby $H_{\rm end} \gtrsim 10^{-43} M_{\rm Pl}$, since $|\lambda_3| \sim \mathcal{O}(1)$.

For radiation and $N = 60$, we obtain $H_{\rm end} < 7.7 \times 10^{-53} M_{\rm Pl}$ to avoid super-Planckian energy densities at the beginning of inflation! This is ruled out!

The exponential expansion is so extreme that it needs tiny energy densities at the end of inflation. This in turn means that corrections to GR have to be relevant up until these tiny energy densities, but this conflicts with observational constraints.

An obvious way out: inflation driven by something which does not dilute so fast!

Beyond General Relativity: geometric inflation

A scalar field in a slow-roll regime has $w \approx -1$: its energy density does not dilute at all!

$$\mathcal{I}_{\rm scalar} = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} m^2 \phi^2 \right] .$$

This matter sector contributes to the generalized Friedmann equations with

$$\rho = \frac{1}{2} H^2 \phi'^2 + \frac{1}{2} m^2 \phi^2 \, , \qquad P = \frac{1}{2} H^2 \phi'^2 - \frac{1}{2} m^2 \phi^2 \, ,$$

and the energy-momentum conservation. Let us first explore

$$\mathcal{I} = \frac{M_{\rm Pl}^2}{2} \int d^4 x \sqrt{-g} \left(R - \beta \, L_\star^4 \, \mathcal{R}_{(3)} \right) + \mathcal{I}_{\rm scalar} \, ,$$

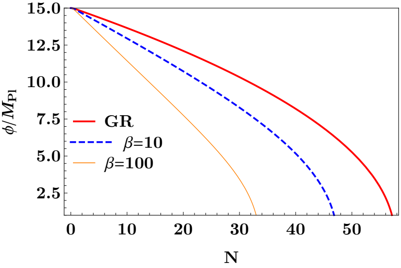
whose Friedmann equation reads:

$$3H^2 \left(1 + \beta H^4 \right) = \frac{1}{2} H^2 \phi'^2 + \frac{1}{2} m^2 \phi^2 \, .$$

H and m are given in units of $L_\star^{-1}$, while ϕ is in Planck units. It is easy to check that the cubic theory is less efficient as an inflationary theory.

Beyond General Relativity: geometric inflation

This is shown by the evolution of the scalar field (we take $\tilde{\phi} = 15M_{\rm Pl}$, $m = 0.1L_\star^{-1}$)



Let us quickly explore two possibilities:

- Including radiation. This will lead to a hybrid scenario where ordinary inflation follows a sort of bandoneon-like geometric inflation.
- Pushing the inflationary regime to higher energies, $\Lambda_{\rm inf} \gg L_\star^{-1}$. This leads us to a remarkable small free field inflation scenario.

Beyond General Relativity: geometric inflation

If we bring in radiation into the picture,

$$3H^2\left(1+\beta H^4\right)=\tilde{\rho}\, e^{-4N}+\frac{1}{2}H^2\phi'^2+\frac{1}{2}m^2\phi^2\ ,$$

where $\tilde{\rho}=\xi^2 M_{\rm Pl}^{-4}\rho_{\rm init}$ and $\xi=L_\star M_{\rm Pl}$.

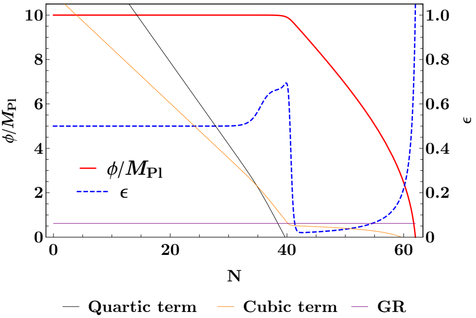
Consider a period of expansion where radiation dominates the right-hand side and the cubic term the left-hand side.

This regime stops either when the GR term equals the cubic one ($N=N_{\rm GR}$ for which $H(N_{\rm GR})=H_{\rm end}$), or when the scalar field energy density becomes dominant ($N=N_{\rm s}$ such that, in a slow-roll regime, $\frac{1}{2}m^2\phi(N_{\rm s})^2=\tilde{\rho}\, e^{-4N_{\rm s}}$), whatever comes first.

The full (mostly analytic) investigation of all the relevant regimes for the cubic and quartic theories was performed in [Edelstein, Vázquez Rodríguez, Vilar López, 2020].

I will present them just by showing you a plot.

Beyond General Relativity: geometric inflation



- The super-Planckian problem of the initial energy density is marginally solved.
- The scalar field stays pretty much constant while Geometric inflation is active! Its value can be reduced, but $\tilde{\phi} \geq \sqrt{2} M_{\rm Pl}$.
- Ordinary inflation is the last stage at the end of the bandoneon-like cascade: it smoothly connects with a reheating era.

Beyond General Relativity: geometric inflation

Let us come back to an $F(H)$ theory coupled to a scalar field. Calling K_ϕ and V_ϕ the kinetic and potential energies, the ϵ-parameter is:

$$\epsilon = \frac{6F(H)}{HF'(H)} \frac{K_\phi}{K_\phi + V_\phi} \, .$$

A fast-growing $F(H)$ will produce accelerated expansion.

But we need to connect with GR. This is at the root of the constraint $\tilde{\phi} \geq \sqrt{2} M_{\rm Pl}$, which is behind the higher-curvature terms upsetting inflation in the absence or radiation!

We can solve both problems by pulling apart $\Lambda_{\rm inf} \gg L_\star^{-1}$ [Edelstein, Mann, Vázquez Rodríguez, Vilar López, 2020].

Again, I will skip the details of all relevant regimes and present them just by showing some plots.

We can identify the relevant features of $F(H)$ to entail a proper inflationary model.

Beyond General Relativity: geometric inflation

Let us summarize the relevant features of $F(H)$:

- For small H, $F(H) \sim H^2 + \mathcal{O}(H^6)$. We cannot have an H^4 term in four dimensions for GQTG gravity [Hennigar, Kubizňák, Mann, 2017].
- We require an almost flat region of $F(H)$, whose only purpose is to separate the inflating part of the function from the GR one.
- Finally, we require a region in which $F(H)$ grows fast, and where inflation will be produced.

We can considered the Gaussian model

$$F(H) = H^2 \left[\frac{1}{2 + (L_\star H)^2} + \frac{1}{4} \left(1 - \frac{1}{\Gamma c_\sigma^2} \right) \frac{(L_\star H)^2}{1 + (L_\star H)^4} \right] \left[1 + e^{\frac{1}{2c_\sigma^2}} \, e^{-\frac{((L_\star H)^2 - \Gamma)^2}{2\Gamma^2 c_\sigma^2}} \right] .$$

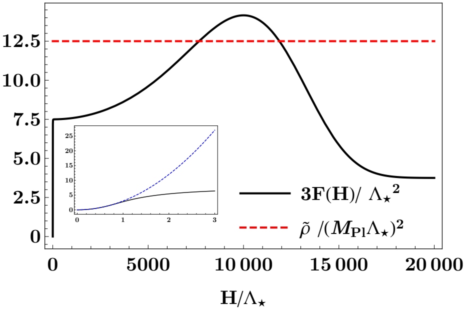
$L_\star$ sets the energy scale at which corrections to GR arise, $\Lambda_\star = L_\star^{-1}$.

Beyond General Relativity: geometric inflation

The center of the Gaussian is located at $\Lambda_{\rm inf} = \sqrt{\Gamma}\Lambda_\star$; $\Gamma \gg 1$ ($\Gamma = 10^8$) guarantees we are separating the inflationary regime from the GR one.

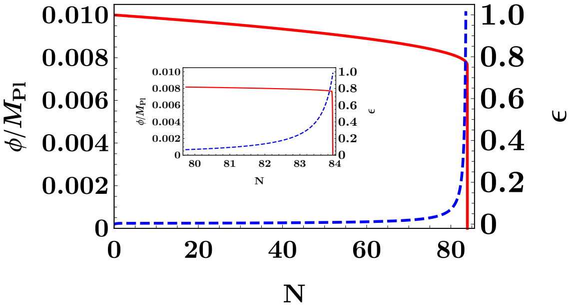
$$F(H) = H^2 \left[\frac{1}{2 + (L_\star H)^2} + \frac{1}{4}\left(1 - \frac{1}{\Gamma c_\sigma^2}\right) \frac{(L_\star H)^2}{1 + (L_\star H)^4} \right] \left[1 + e^{\frac{1}{2c_\sigma^2}} e^{-\frac{((L_\star H)^2 - \Gamma)^2}{2\Gamma^2 c_\sigma^2}} \right] .$$

c_σ determines the relative width of the Gaussian; we set $c_\sigma \lesssim 1$ ($c_\sigma = 0.7$).



We choose $m = 500\, L_\star^{-1}$ and an initial value $\tilde{\phi} = 0.01\, M_{\rm Pl}$. We can see that the large scale separation, $\Lambda_{\rm inf} \gg \Lambda_\star$ does the job!

We obtain $N = 84$ e-folds of inflation before the system enters the flat part of $F(H)$, where we get quick dissipation and eventually connection with the GR regime.



All of the accelerating expansion happens in the fast-growing part of $F(H)$, with an almost de Sitter expansion ($\epsilon \approx 0$).

Beyond General Relativity: black hole accretion

Let us investigate the effects of strong gravity on the surrounding matter. Nowhere else is this interplay more dramatic than in an accretion scenario.

In wind accretion, a massive gravitational object accretes as it moves through a gas cloud at supersonic speeds [Hoyle, Lyttleton, 1939] [Bondi, Hoyle, 1944].

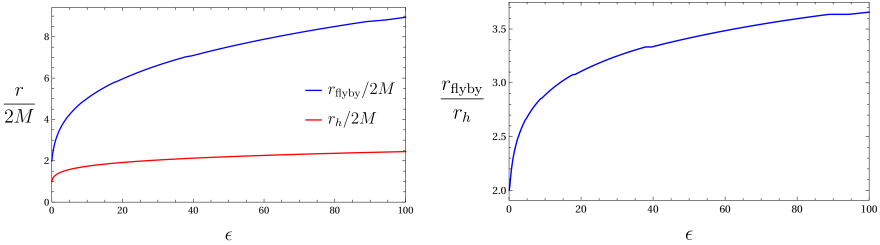
When the relative motion between the gas cloud and the accretor can be neglected, we talk about spherical accretion [Bondi, 1952].

We want to explore the higher curvature corrections to these in the context of

- Primordial black hole accretion (and abundance).
- Supernovae triggered by PBHs.

Let us compute the fly-by radius for a cubic black hole of mass M, [Edelstein, Rivadulla Sánchez, Rodríguez Moris, Tejeda, *to appear*]

$$\mathcal{I} = \int d^4x \sqrt{-g} \left(R + \lambda_3 \, L_\star^4 \, \mathcal{R}_{(3)} \right) .$$



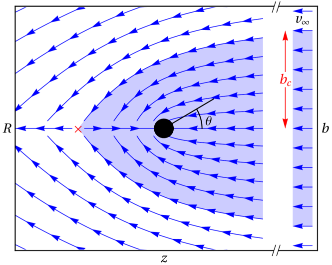
We use

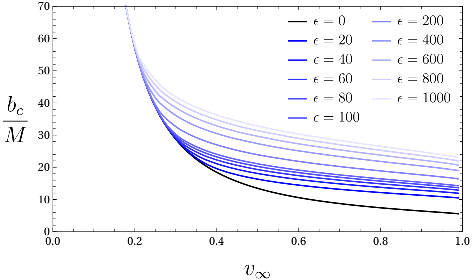
$$\epsilon = \lambda_3 \, \frac{L_\star^4}{M^4} .$$

Let us now turn to the ballistic accretion: relativistic wind by a static black hole.

Beyond General Relativity: black hole accretion

Infinitely far away the wind has a constant density ρ_∞ and velocity v_∞.



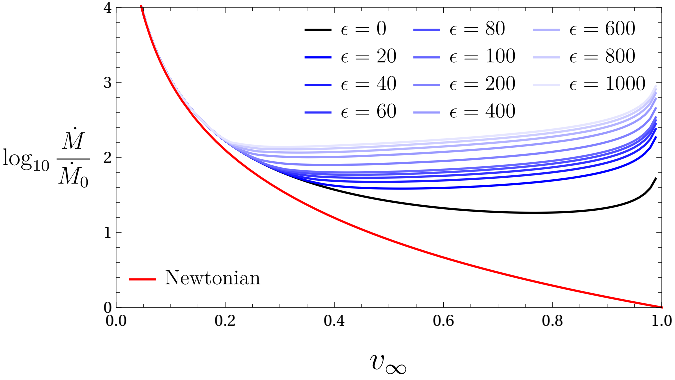


The material accreted is therefore inside the cylinder with radius b_c. We can integrate its flow to obtain the accretion rate,

$$\dot{M}_{\rm HL} = \pi b_c^2 \, \rho_\infty v_\infty \gamma_\infty \ .$$

Let us show the accretion rate normalized by $\dot{M}_0 = 4\pi M^2 \rho_\infty$ for different values of ϵ.

Beyond General Relativity: black hole accretion



More important for relatively large values of v_∞, particularly for small ϵ.

Primordial black holes traversing a carbon-oxygen white dwarf can trigger a detonation which produces normal thermonuclear supernovae (SNe Ia) [Steigerwald, Tejeda, 2021].

Beyond General Relativity: black hole accretion

The cubic terms tend to increase the contrast in density and decrease the aperture angle of the shock cone.

TO BE CONTINUED. . .