A holographic model for soft gluon interactions

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Progress on gravitational physics: 45 years of Belgian-Chilean collaboration.

Purpose of this talk

Construct an effective model for the soft gluon exchanges using BF theories.

2D theory defined on future null infnity

- Color memory effect.
- Leading soft theorem.

Overview of this talk

- Motivation(s)
- Radiative vacua & Effective BF model.
- Wong’s particle & Well-posed action principle.
- Quantum PCM.
- Final Remarks.

Motivation

Enhancement of the symmetries at the boundary of the spacetime.

$(r \rightarrow \infty)$

$\Updownarrow$

Vacuum degeneracy in Gauge theories.
(Energy ~ 0)

Motivation

Enhancement of the symmetries at the boundary of the spacetime.

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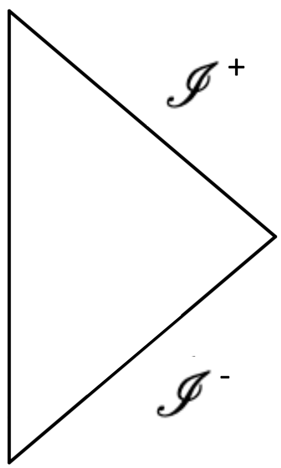

Gravity: BMS supertranslations

Gauge theory: SU(N) angle dependent transformations

Motivation

Vacuum degeneracy in Gauge theories.
(Energy $\sim$ 0)

They do not leave the vacuum invariant

$$\mathcal{L}_\zeta \eta \neq 0 \qquad D_\epsilon A \neq 0$$

BMS Soft SU(N)

Symmetry breaking

$$G_\infty \longrightarrow G_0$$

Motivation

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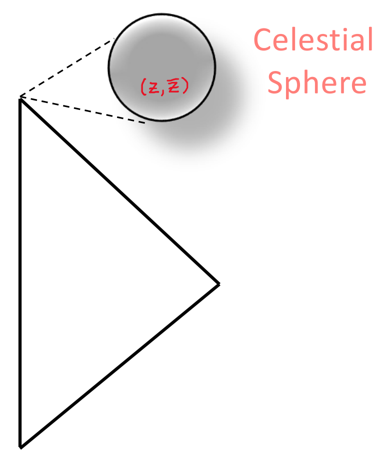
BMS Soft SU(N)

``Pseudo-Goldstone bosons"

$$U(z,\bar{z}) \in \frac{G_\infty}{G_0}$$



Motivation

Non-abelian gauge theories:

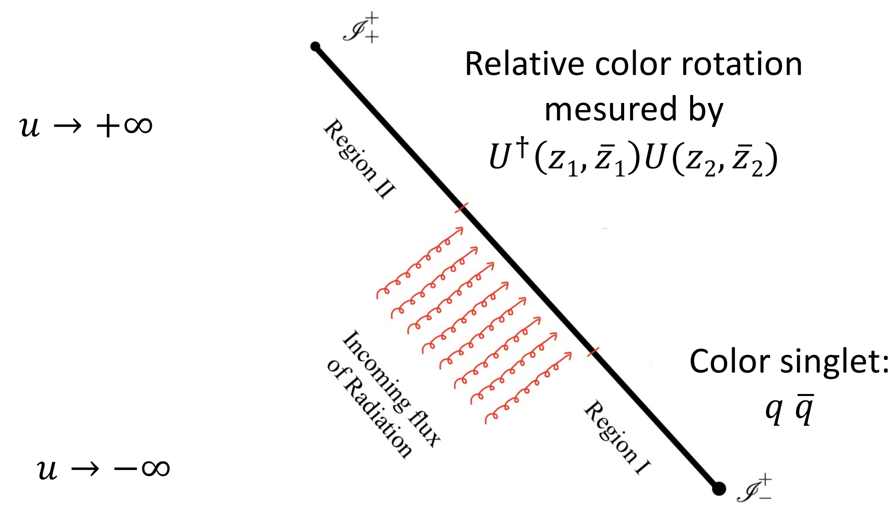
More intricated long wave behavior and richer perturbative structures.

The dynamics of the holographic Goldstone modes controls the IR sector of gauge theories

Motivation

The dynamics of the holographic Goldstone modes controls the IR sector of gauge theories

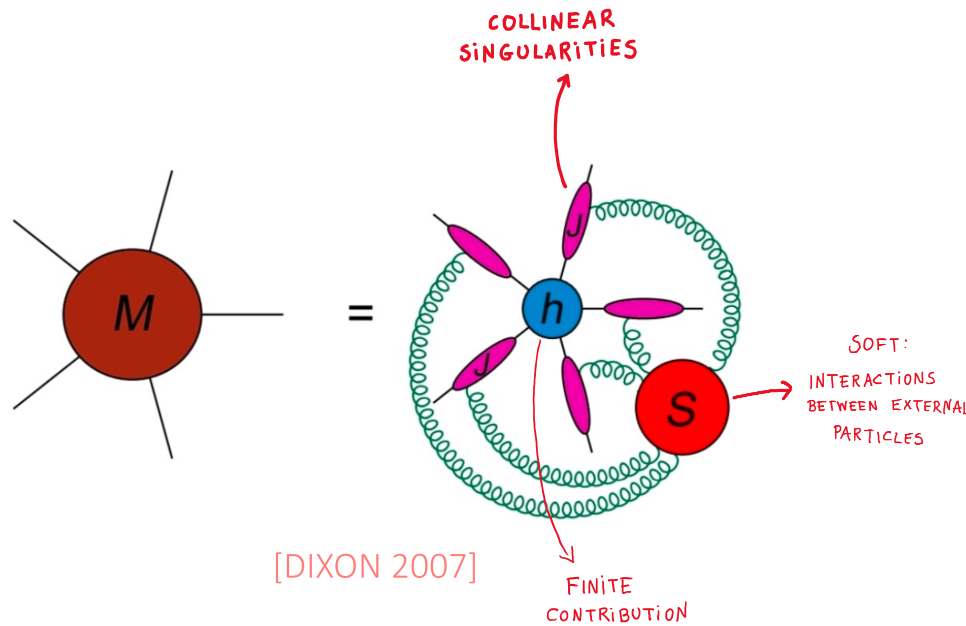
Color memory effect is the transition between different vacua



Motivation

The dynamics of the holographic Goldstone modes controls the IR sector of gauge theories

Exponentiation of the Soft factor in IR regulated gluon amplitudes

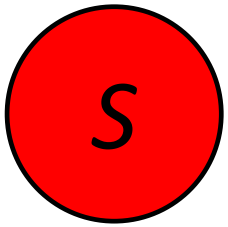


[DIXON 2007]

Motivation

The dynamics of the holographic Goldstone modes controls the IR sector of gauge theories

Exponentiation of the Soft factor in IR regulated gluon amplitudes

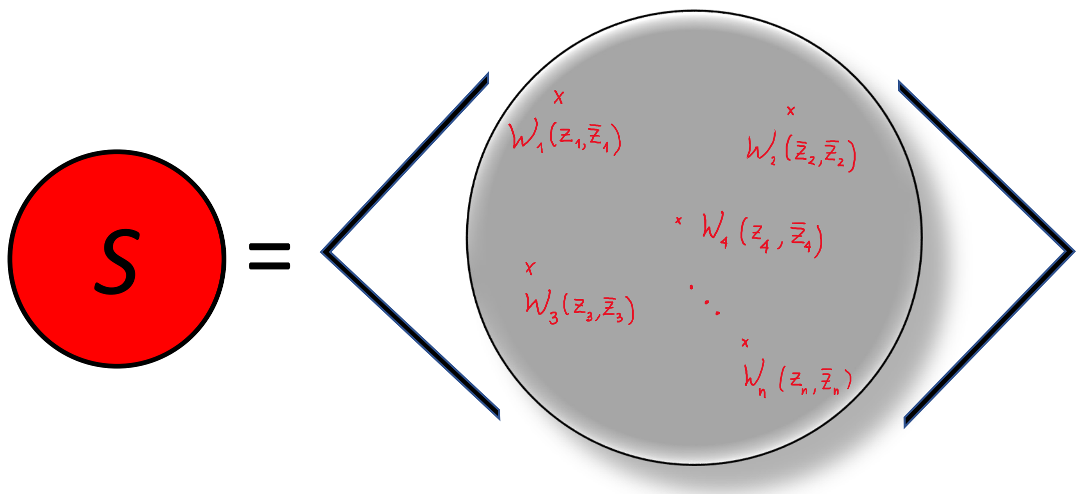


Motivation

The dynamics of the holographic Goldstone modes controls the IR sector of gauge theories

Soft factor is a correlator of “vertex operators”

$$\mathcal{W}_{\mathrm{i}}(z_i, \bar{z}_i) \sim e^{Q_i\, \Phi(z_i, \bar{z}_i)}$$



[NANDE,PATE, STROMINGER 2017], [HIMWICH, NARAYAN, PATE, PAUL, STROMINGER 2020], [MAGNEA 2021], [HG, ROJAS 2021]

BF models and radiative vacua

Vacuum configurations satisfy (asymptotically)

$$\mathcal{F}_{ij} = \partial_i \mathcal{A}_j - \partial_j \mathcal{A}_i + [\mathcal{A}_i, \mathcal{A}_j] = 0 \qquad x^i = (u, z, \bar{z})$$

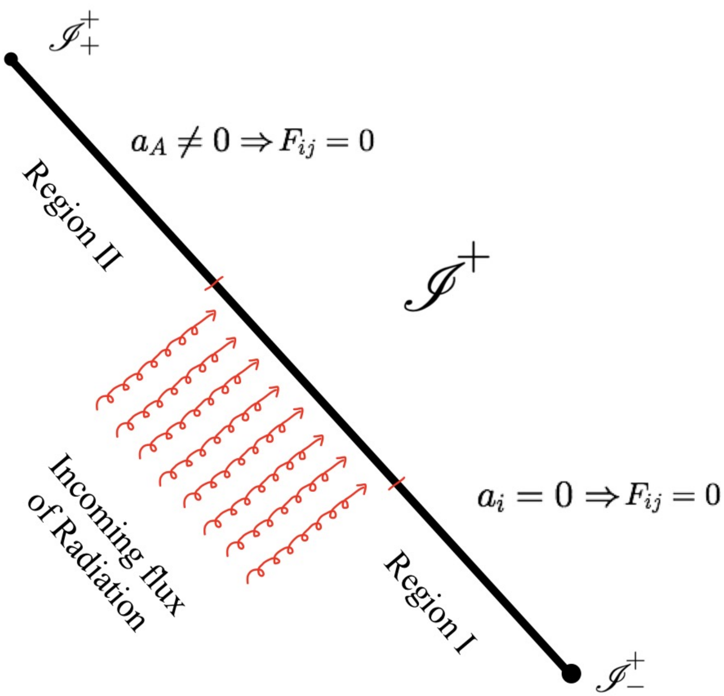
BF models naturally produce these equations: [KAPEC, MITRA 2021] [NGUYEN, SALZER 2020]

$$I_{\rm BF}[\mathcal{B}, \mathcal{A}] = \frac{1}{g^2} \int \mathrm{tr}\,[\mathcal{B} \wedge \mathcal{F}]$$

Dynamics of $\mathcal{B}$ field?

Trivial on-shell dynamics?

BF models and Color memory



Region I & II: well-described by a SU(N) BF model

Incoming radiation: Particle's action carrying color d.o.f.

⇩

Wong's action

BF models and Color memory

Incoming radiation: Particle's action carrying color d.o.f.

⇩

Wong's action

$$I_{\mathrm{p}}[\mathcal{A}, w; \{\lambda_i\}] = \sum_{i=1}^{N} \int d^2z\, du\, \mathrm{tr}\left[\lambda_i w^{-1}(\partial_u + \mathcal{A}_u) w\right] \delta^{(2)}(z - z_i).$$

$$u^i(\tau) = (\tau - \tau_i, z_i, \bar{z}_i)$$

BF models and Color memory

Incoming radiation: Particle's action carrying color d.o.f.

⇩

Wong's action

LIE VALUED CONSTANT

$$I_{\rm p}[\mathcal{A}, w; \{\lambda_i\}] = \sum_{i=1}^{N} \int d^2z\, du\, \mathrm{tr}\left[\lambda_i w^{-1}(\partial_u + \mathcal{A}_u) w\right] \delta^{(2)}(z - z_i).$$

MINIMAL COUPLING TO THE GAUGE FIELD

COLOR DEGREE OF FREEDOM : $w = w(\tau_i)$

PARTICLE'S WORLD-LINE (FIXED)

BF models and Color memory

Equation for $w = w(\tau)$

$$\partial_u \Lambda + [\mathcal{A}_u, \Lambda] = 0 \quad \Lambda = \sum_{k=1}^{N} w \lambda_k w^{-1}$$

Total color charge is parallel-transported along the trajectory of the particle.

$tr[\Lambda^2]$ is constant along u-direction.

Well-posed variational principle

Effective variational principle

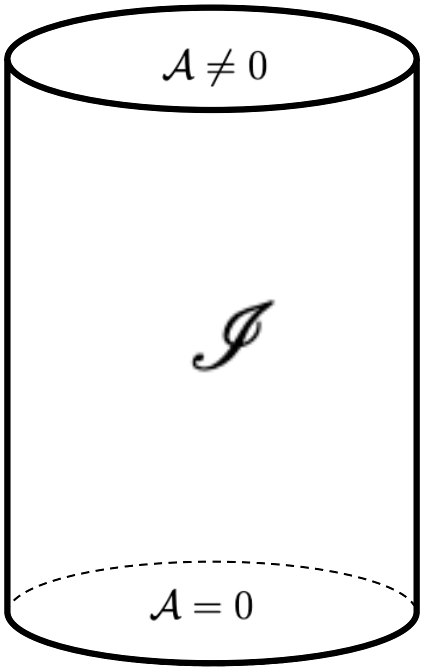
$$I_{\text{eff}}[\mathcal{B},\mathcal{A},w] = I_{\text{BF}}[\mathcal{B},\mathcal{A}] + I_{\text{p}}[\mathcal{A},w;\{\lambda_i\}]\,.$$

First variation produces:

$$\delta(I_{\text{eff}}[\mathcal{B},\mathcal{A}]) = (\text{field equations}) - \frac{1}{g^2}\int_{\partial\mathscr{I}} d^2z\,\epsilon^{AB}\text{tr}[\mathcal{B}_A\,\delta\mathcal{A}_B]$$

We introduce (the simplest yet non-trivial) boundary condition

$$\boxed{\mathcal{B}_A = \sqrt{\gamma}\epsilon_A{}^B\mathcal{A}_B}$$



Symmetry breaking

Gauge symmetries are broken by the boundary condition

$$\mathcal{A} \to h^{-1}\mathcal{A}h + h^{-1}dh \quad \mathcal{B} \to h^{-1}\mathcal{B}h$$

From local transformation $h(x)$ $\in SU(N)$ down to constant group element h.

Mimicking "spontaneous" symmetry breaking

$$G_\infty \longrightarrow G_0$$

Reduced dynamics

Incorporating boundary condition $$I[\mathcal{B},\mathcal{A}] = I_{\text{eff}}[\mathcal{B},\mathcal{A}] + \frac{1}{2g^2}\int_{\partial\mathscr{I}} d^2z\,\sqrt{\gamma}\gamma^{AB}\text{tr}[\mathcal{A}_A\,\mathcal{A}_B]\,.$$

We are interested in the sector $\mathcal{F}_{ij} = 0$

$$I_{\text{red}}[U;\{\lambda_i\}] = \int_{\partial\mathscr{I}} d^2z\,\left(\frac{1}{g^2}\,\text{tr}[-\partial_{\bar{z}}U^{-1}\partial_z U] + \sum_{i=1}^{N}\delta^{(2)}(z-z_i)\int du\;\text{tr}\left[\lambda_i\,\mathcal{U}^{-1}\partial_u\mathcal{U}\right]\right)$$

SU(N) PRINCIPAL CHIRAL MODEL

(HARD) PARTICLE CONTRIBUTION

Reduced dynamics

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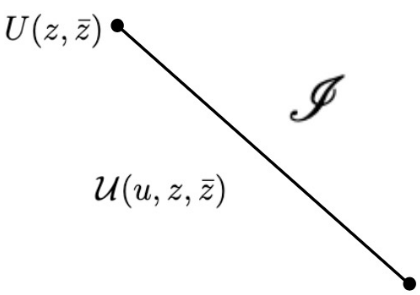
BOUNDARY FIELD

EXTENSION ON $\mathscr{I}$ ($\sim$ WZ TERM)



Classical field equations

$$\partial_z(U^{-1}\partial_{\bar{z}}U) + \partial_{\bar{z}}(U^{-1}\partial_z U) = -g^2 \sum_{i=1}^{N} \lambda_i\, \delta^{(2)}(z - z_i)\,.$$

It approximates the the u-component of Yang-Mills equations:

$$-\partial_u \left(F^{(2)}_{ru} + D^A\, A^{(0)}_A \right) = g^2_{\mathrm{YM}}\, j^{(2)}_u + \gamma^{AB}[A^{(0)}_A, \partial_u A^{(0)}_B]\,,$$

FLAT CONNECTION

QUADRATIC CONTRIBUTION

Classical field equations

$$\partial_z(U^{-1}\partial_{\bar{z}}U) + \partial_{\bar{z}}(U^{-1}\partial_z U) = -g^2 \sum_{i=1}^{N} \lambda_i \, \delta^{(2)}(z - z_i) \, .$$

It approximates the the u-component of Yang-Mills equations:

$$D^A \left(U^{-1}\partial_A U\right) = -g_{\mathrm{YM}}^2 \int_{u_i}^{u_f} du j_u^{(2)} \qquad j_u^{(2)} = \gamma^{-1/2} \sum_i \lambda_i \, \delta(u - u_i)\delta^{(2)}(z - z_k)$$

Quantum PCM

The soft sector should be described by

$$I[U] = -\frac{1}{g^2}\int_{\partial\mathscr{I}} d^2z\,\mathrm{tr}[\partial_{\bar z}U^{-1}\partial_z U]$$

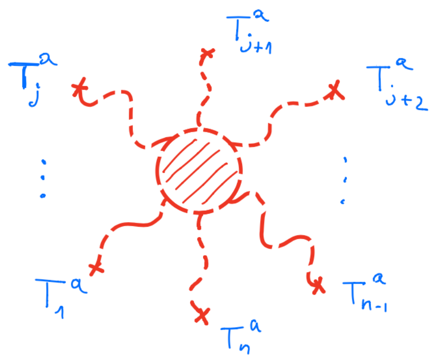
Left and right multiplication symmetries $U(z,\bar z) \to h_L\, U(z,\bar z)\, h_R$

Noether currents: $J_A^{(L)}[\theta_L] = -\frac{1}{g^2}\mathrm{tr}\left[\theta_L U^{-1}\partial_A U\right]\ , \quad J_A^{(R)}[\theta_R] = -\frac{1}{g^2}\mathrm{tr}\left[\theta_R\partial_A U U^{-1}\right]\ .$

Ward identities → Soft theorems

$$\langle(\partial_{\bar{w}} J^a_w + \partial_w J^a_{\bar{w}})\; U(z_1,\bar{z}_1)\cdots U(z_n,\bar{z}_n)\rangle \;\; = \sum_{k=1}^{n} \delta^{(2)}(w - z_k)\mathbf{T}^a_k \langle U(z_1,\bar{z}_1)\cdots U(z_k,\bar{z}_k)\cdots U_n(z_n,\bar{z}_n)\rangle$$

Color operator: $\mathbf{T}^a_k U(z_i,\bar{z}_i) = \delta_{ik} T^a_k U(z_i,\bar{z}_i)$



Global SU(N) invariance implies color conservation

$$\sum_{k=1}^{n} \mathbf{T}^a_k = 0\,.$$

Ward identities → Soft theorems

$$\langle(\partial_{\bar{w}} J_w^a + \partial_w J_{\bar{w}}^a)\, U(z_1,\bar{z}_1)\cdots U(z_n,\bar{z}_n)\rangle = \sum_{k=1}^{n} \delta^{(2)}(w-z_k)\mathbf{T}_k^a \langle U(z_1,\bar{z}_1)\cdots U(z_k,\bar{z}_k)\cdots U_n(z_n,\bar{z}_n)\rangle$$

Shadow transform: $$S_z^a = \int d^2w\, \frac{J_{\bar{w}}^a}{(z-w)^2}\,, \quad S_{\bar{z}}^a = \int d^2w\, \frac{J_w^a}{(\bar{z}-\bar{w})^2}\,.$$

Soft theorems: $$\langle S_z^a\, U(z_1,\bar{z}_1)\cdots U(z_n,\bar{z}_n)\rangle = \sum_{k=1}^{n} \frac{\mathbf{T}_k^a}{z-z_k} \langle U(z_1,\bar{z}_1)\cdots U(z_n,\bar{z}_n)\rangle$$

↳ SIMILAR FOR $S_{\bar{z}}^a$

Final Remarks

Prescription to holographically reproduce color memory and leading soft theorem.

BF model+boundary conditions

Different boundary conditions?

Final Remarks

Prescription to holographically reproduce color memory and leading soft theorem.

BF model+boundary conditions

Closely related model: CS based on $\mathrm{SU}(N) \oplus \mathfrak{su}(N)_{\text{abelian}}$

$$[T_a, T_b] = f_{ab}{}^c T_c\,, \quad [P_a, T_b] = f_{ab}{}^c P_c\,, \quad [P_a, P_b] = 0\,.$$

$$I_{\text{CS}}[\mathbf{A}] = \frac{1}{g^2} \int_{\mathscr{I}} \text{tr}\,[\mathcal{B} \wedge \mathcal{F}] + \frac{k}{g^2} I_{\text{CS}}[\mathcal{A}]$$

Final Remarks

Prescription to holographically reproduce color memory and leading soft theorem.

BF model+boundary conditions

Closely related model: CS based on $\mathrm{SU}(N) \oplus \mathfrak{su}(N)_{\mathrm{abelian}}$

PCM model adquires a beta function at one-loop. It is important to understand the consequences of it on the IR behavior.

Similar findings obtained here apply to gravity [Nguyen, Salzer 2020]

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THANKS!

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